Eurasian Economic Review (2022) 12:315–332
https://doi.org/10.1007/s40822-022-00211-x

ORIGINAL PAPER

Volatility dependent smooth transitions and abrupt switches: why they are needed for better forecasting the FX rates

Arif Orçun Söylemez[1]

Received: 8 December 2020 / Revised: 20 April 2021 / Accepted: 10 February 2022 /
Published online: 29 March 2022

Abstract
Exchange rate prediction is a problematic area. There still does not exist a structural exchange rate model that can consistently predict future exchange rates better than a driftless random walk model. This paper draws attention to two important points regarding this problem. First of all, many structural exchange rate models inherently depend on the uncovered interest parity (UIP) rule. However, empirical evidence almost universally rejects UIP. Therefore, this paper firstly questions whether it would be possible to improve the predictions of UIP by converting it into a nonlinear form since the workhorse UIP specification has traditionally been linear. Secondly, this paper also discusses that a nonlinear transformation is indeed a necessity given that exchange rates typically follow meandering time paths. Inspired by a Bank for International Settlements (BIS) report, UIP model is estimated by volatility-dependent Threshold autoregression (TAR) and Smooth transition regression (STR) specifications using a dataset on two popular carry trade currencies against the US dollar and the Japanese yen. Estimations clearly favor TAR and STR over a linear specification. Although random walk model remains as the champion, results are still indicative of the usefulness of a volatility-dependent regime switching framework for improving the prediction performances of various other structural models that are dependent on UIP.

Keywords Threshold regressions · Regime switching · Exchange rate prediction · Random walk model

JEL Classification F31 · G15 · G12

✉ Arif Orçun Söylemez
orcun.soylemez@marmara.edu.tr

[1] Marmara University, Economics Faculty, Istanbul, Turkey

1 Introduction

One of the basic premises that the entire behavioral finance literature depends on is the mood switches of the investors (López-Cabarcos et al., 2020; Lux, 1995). According to the results of many empirical and experimental studies, investors can go from a state of complete euphoria to an opposite state of despair conditional on various factors (Hu & McInish, 2013; Livanas, 2011).

The following excerpt in that sense is highly intriguing for it clearly provides us with an example of how these kinds of swings in the mental states of investors can affect the foreign exchange market. The excerpt is an illuminating text also for it hints that risk can be an important factor provoking the shifts in the investors' sentiment.

> Foreign exchange markets experienced a substantial increase in volatility in August 2007... Prior to August, historically low volatility and large interest rate differentials had underpinned cross-border capital flows that put downward pressure on funding currencies, ..., and supported high-yielding currencies. (As a result of the heightened volatility) there was a substantial reassessment... as the... problems in financial markets became more apparent. In this environment, (other) factors... which have an important bearing on the future path of monetary policy, became more of a focal point for market sentiment than the prevailing level of interest rates.

Bank for International Settlements (BIS) (2008), *78th Annual Report, July 2008*

Foreign exchange determination has been a long-standing problem of the international finance literature since Meese and Rogoff (1983) have shown in a seminal paper that a driftless random walk model is the champion of the out-of-sample foreign exchange rate predictions. To be more articulate, determining the future exchange rates had become an important research topic in the late 1970s and early 1980s when the foreign exchange rates were let to float freely in the international markets following the collapse of the Bretton System in 1971. Meese and Rogoff (1983) have taken six of the most popular models of exchange rate determination at the time—three structural and three purely time series models—and tested their out-of-sample predictive powers. However, so ingeniously, they have not only compared those models against each other but also against a random walk model. The result has been a shocking finding, which remains to be valid still today after almost four decades and a myriad of papers written in an attempt to overturn it: the simple driftless random walk model has been the best predictor of the future exchange rates with respect to both mean squared prediction error (MSPE) and mean absolute error (MAE) criteria. However, most of the models that have been tried so far had linear specifications, including the structural models in the Meese and Rogoff (1983). But the lines in the above excerpt are suggestive of a nonlinear regime-switching process. This point will be the basis of this paper anyway: can we improve our existing linear foreign exchange models by converting them to nonlinear as regime-switching models? And what kind of regime switches should we consider, i.e. abrupt or slow? This paper is an attempt to answer these questions but if we return to the cornerstone finding of Meese and Rogoff (1983), it was a shocking finding because a driftless random walk model cannot be used as a prediction model due to a technical reason, i.e. random walk processes are the discrete time counterparts of the continuous Brownian motion processes and it is a well-known fact about the Brownian motion processes that they cannot be predicted. Even in the case that one insists on using the driftless random walk process as a prediction model, there is basically no useful information that she could extract from this simple model above that of what she would have extracted if she had thrown dices. We can easily see the reason why if we recall the specification of the driftless random walk model as follows.

$$s_{t+1} = s_t + \varepsilon_t \tag{1}$$

In Eq. 1, s_{t+1} and s_t are the spot exchange rates at time t and $t + 1$, respectively. 'ε' is a white-noise error term distributed around zero with time-invariant variance σ. It should be now clear why we cannot use this model as a prediction model. Assume that we want to predict the spot exchange rate at time $t + 1$. Since, we know s_t at time t, $E(s_t) = s_t$. Plus, we know ε is distributed around zero symmetrically which means that we are not making systematic estimation errors. That is why $E(\varepsilon_t)_t = 0$. In return, our best prediction for s_{t+1} would be equal to s_t. In plain English, the best prediction we can make using the random walk model is equal to the spot exchange rate today. Plus, the very same model also tells us that there is fifty percent chance that the spot exchange rate tomorrow will be higher than the spot exchange rate today and there is fifty percent chance that it will be lower than the spot exchange rate today since ε is symmetrically distributed around zero. In sum, the random walk model carries no more information that can be used for predictive purposes than a pair of dice. It is, therefore, really miserable that the random walk model has repeatedly and almost universally beaten all the other models, both the structural or purely time series, since 1983 and remained as the best prediction model of exchange rates. Therefore, this paper is interested in whether we can improve our existing structural prediction models.

The rest of the paper is organized as follows. Some of the leading structural foreign exchange models are reviewed in detail in Sect. 2. In Sect. 3, two different regime-switching models are introduced, namely the threshold autoregressive model and smooth transition model. Dataset and estimation outputs are presented in Sect. 4. Section 5 concludes.

2 Foreign exchange rate determination models

A review of some of the leading structural models that have been empirically tried so far in the existing literature over different cross-sectional and time series datasets with a hope of surpassing the performance of random walk model would be a good departure point given the interests of this paper. For this purpose, the three structural models that have been included in the paper of Meese and Rogoff (1983) will be revisited in this section. The structural models they employed were the flexible-price

(Frenkel–Bilson) and sticky-price (Dornbusch–Frankel) monetary models, and a sticky-price model augmented with the current account balances (Hooper–Morton).

The most general specification, covering all these three structural models, was as follows.

$$s = a_0 + a_1(m - m^*) + a_2(y - y^*) + a_3(i_s - i_s^*) + a_4(\pi^e - \pi^{e*}) + a_5\overline{TB} + a_6\overline{TB}^* + u \tag{2}$$

where s stands for the log value of the Dollar price of the foreign currency. They used data for Dollar/Pound, Dollar/German Mark, Dollar/Yen, and trade-weighted Dollar exchange rates in their paper by the way. As for the set of explanatory variables, $m - m^*$ stands for the log differences in the supplied amount of money in the US and the foreign country. The star signs on top of some variables indicates that these are the foreign country observations for that variable. The second explanatory variable, $y - y^*$, stands for the log value differences in the GDP growth rates of the US and the foreign country. The third ($i_s - i_s^*$) and the fourth ($\pi^e - \pi^{e*}$) variables, respectively, indicate the differences in the short-run interest rates and the expected inflation rates in the US and the foreign country. Please note that these are not log-transformed as the previous variables. Finally, $\overline{TB}$ indicates the cumulated trade balance of the US, while $\overline{TB}^*$ is its counterpart for the foreign country. 'u' is the error term.

Equation 2 is a catch-all specification for all the three structural models, namely the Frenkel–Bilson, Dornbusch–Frankel, and Hooper–Morton models. Equation 2 becomes the Frenkel–Bilson model if a_4, a_5, and a_6 are set equal to zero. When only a_5 and a_6 are set equal to zero, Eq. 2 boils down to the Dornbusch–Frankel model. Without any restrictions imposed on the coefficients, Eq. 2 is named the Hooper–Morton model.

The underlying model that is necessary for validating all these three models above is the uncovered interest parity model, which relates the forward premium in exchange rates to the interest rate differences in the countries and is as stated below.

$$i - i^* = s_{t+1} - s_t \tag{3}$$

In this specification, $i - i^*$ is the interest rate difference between the home and foreign country while s_{t+1} and s_t stand for the log values of the spot exchange rates at time $t + 1$ and at present, respectively, where the exchange rates are expressed as the home currency worth of a unit of foreign currency. Equation 3 is also known as the Fama (1984) equation. The idea leading to the derivation of Eq. 3 rests on the assumption that no long-lasting arbitrage opportunity could persist in financial markets. Although no-long-lasting-arbitrage seems like a highly reasonable assumption at first glance, Eq. 3 hardly makes sense in practice. To see the reason of that, assume that the home country interest rate has increased but not the foreign country interest rate. As a result, the right hand side (RHS) of Eq. 3. increases as well. For the sake of equality, left hand side (LHS) should have increased proportionately. On the LHS, 's_t' is the spot exchange rate. Therefore, it is given today and its value cannot change. That is why the equality can be maintained only if s_{t+1} increases. Since s_{t+1} is the log-transformed value of the future price of the foreign currency, a higher

s_{t+1} means appreciation of the foreign currency. That means, whenever home country yields increase, foreign currency should appreciate. However, this is not realistic. In most cases, investors, at least in their euphoric states of mood, would chase the yield. That is why, we cannot make generalizable claims that hold at all times in reality (Kilian & Taylor, 2003). Investors, in their euphoric states, would pour money into the home country financial markets once the yields become higher there and hence cause the appreciation of the home currency. This opposite reality holds at least when the volatility levels in the financial markets tend to be low according to the BIS excerpt at the beginning of this article. Tests of the uncovered interest parity rule with real life data almost universally report empirical failure of the rule anyway (Burnside, 2019; Chinn & Meredith, 2005). Because the uncovered interest parity fails in reality, which means that the rule of no-financial arbitrage does not suffice to establish a stable prediction model for the exchange rates, there exist need for developing alternative models. One such alternative, known as the purchasing parity rule, based this time on the assumption of no-long-lasting-commodity-arbitrage rather than no-financial-arbitrage as in the case of uncovered interest parity, is developed by the help of the following Fisher equation.

$$i = r + \Delta p^{e}_{t+1} \tag{4}$$

where i is the nominal interest rate in the home country as before, r is the real interest rate, and Δp^{e}_{t+1} is the expected inflation. Writing the same for the foreign country as $i^{*} = r^{*} + \Delta p^{e*}_{t+1}$ and then reducing from the home country equation, we end up with the following notation.

$$i - i^{*} = (r - r^{*}) + \left(\Delta p^{e}_{t+1} - \Delta p^{e*}_{t+1}\right) \tag{5}$$

In Eq. 5, $(r - r^{*})$, i.e. the real interest rate difference between countries, could be set equal to zero on the average for countries like the US and the UK sharing comparable sovereign risk ratings. Using the uncovered interest parity rule, we can also replace $i - i^{*}$ with $s_{t+1} - s_{t}$. Then, we end up with the following equation, which is known as the purchasing power parity (PPP) rule.

$$s_{t+1} - s_{t} = \left(\Delta p^{e}_{t+1} - \Delta p^{e*}_{t+1}\right) \tag{6}$$

The trick of replacing $i - i^{*}$ with $s_{t+1} - s_{t}$ is what turns the difference of country-wise Fisher equations into an exchange rate prediction model. However, this trick is dependent on the uncovered interest parity. That is why, once we do this trick, we transfer all the empirical problems of the uncovered interest parity rule to this new rule. Therefore, it is hardly surprising that the PPP rule holds weakly and only in the medium to long-run but not in the short-run at all according to a myriad of empirical literature (Taylor & Taylor, 2004). The weakness of the PPP model has implications for the strength of the Dornbusch–Frankel and Hooper–Morton models since PPP model is nested within these broader models (see the fourth term in Eq. 2).

Given the poor empirical results for the uncovered interest parity and purchasing power parity rules, there surely exists need for developing alternative structural models for predicting foreign exchange rates. Based on the idea that the relative amount of money supplied in two countries should affect their currencies' exchange rate, a monetary model of exchange rates can be developed. Let M/P be the money supply and $L^d = i^{-\lambda}Y^{\beta}$ be the money demand in the home country. Money market equilibrium, then, dictates $M/P = i^{-\lambda}Y^{\beta}$. In this notation, -$\lambda$ is a negative number since people would demand less cash if the yields rise and β is a positive number since people would demand more cash if their incomes rise. In order to keep the following notations simple, and without loss of generality, let us assume that λ and β are the same across the countries. Then, we can write the following for the foreign country: $M^*/P^* = i^{*-\lambda}Y^{*\beta}$. After log-linearizing these two equilibria, and taking λ s as numeraire, one ends up with the following equations.

$$m - p = -i + \beta y \tag{7}$$

$$m^* - p^* = -i^* + \beta y^* \tag{8}$$

Subtracting Eq. 8 from Eq. 7 results in the following notation.

$$i - i^* = \beta(y - y^*) - (m - m^*) + (p - p^*) \tag{9}$$

In order to convert Eq. 9 to a foreign exchange prediction model, one could replace $i - i^*$ with $s_{t+1} - s_t$ again just like we did in converting the Fisher equations into purchasing power parity model. The following resulting model is then known as the monetary model of exchange rates.

$$\Delta s_{t+1} = \beta(y - y^*) - (m - m^*) + (p - p^*) \tag{10}$$

Of course, once again the trick of replacing $i - i^*$ with $s_{t+1} - s_t$ transfers all the empirical problems of the uncovered interest parity rule to this new model. Once we add trade balance data into the monetary model, we obtain a modified form of the Hooper–Morton model. Remember that the Hooper–Morton model was the most comprehensive model in Meese and Rogoff's (1983) paper nesting the other two structural models. In brief, uncovered interest parity rule stems as the centrally important model of exchange rate prediction for all these models since all of them depend on its validity. That said, Saadon and Sussman (2018) argue that the UIP equation could be recovered from the PPP, which is opposite of our discussion above. Furthermore, they claim that the UIP could better hold if we have used forward-looking inflation-indexed assets' interest rates in estimating the UIP. However, as we have showed above, in fact it is the UIP serving as the backbone of the PPP rather than the other way round. That is why, this article primarily focuses on the question of whether we could improve the uncovered interest parity rule.

3 TAR vs. STR models

Exchange rates typically follow meandering time paths. That is indeed such a widely observed feature of the exchange rates that one should almost always attempt to model exchange rates as nonlinear stationary rather than linear stationary processes (Ahmad & Glosser, 2011). As meandering series, high volatility in the short-run and stability in the long-run is a staple of exchange rates. That characteristic is indeed highly suggestive for a regime-switching pattern (Johansson, 2001). Take for example that investors tend to assume so much risk in a state of the world, pushing prices to arbitrarily high levels which has no fundamental ground. Then another state of the world kicks in and investors become highly precautious. They start to pay attention to fundamentals and adjust their portfolios to reduce their risk exposure. In that second state of the world, exchange rates might recede to a shadow price (to a hypothetical price, which is supported by economic fundamentals). In such a scenario, it would be possible to record high volatility in the short-run and stability in the long-run just as has been recorded in real life for many exchange rate series. The BIS excerpt in the introduction section also revealed a regime switching pattern for exchange rates in that same spirit. That is why, using the description in the BIS excerpt, we will create volatility-dependent regime switching specifications for exchange rates in this paper using two popular regime-switching models: TAR and STR models.

TAR models are powerful tools capturing the nonlinear stationarity of the exchange rates. TAR models assume abrupt switches between the regimes but in certain cases it may be more appropriate to assume smooth transitions as well. Another kind of regime switching models, known as STR models, allow smooth transitions.

A typical logistic STR (LSTR) model for a dependent variable of y_t is defined as follows,

$$y_t = x_t^{'}\beta_0 + x_t^{'}\beta_1 G_{1t} + \varepsilon_t \tag{11}$$

where x_t is the vector of explanatory variables, β's are the parameter vectors and ε_t is a white-noise error. G_{1t} is the transition function and it is defined as follows.

$$G_{1t} = G(r_t, \gamma, c) = (1 + exp\{-\gamma(r_t - c)\})^{-1} \tag{12}$$

In this notation above, $\gamma > 0$. As γ approaches infinity, the logistic transition function G_{1t} approaches the indicator function and the logistic smooth transition regression (LSTR) model turns into a switching regression model. In the univariate case with an autoregressive term, it becomes a threshold autoregressive regression (TAR) model. As aforesaid, threshold models assume abrupt switches between the regimes. Smooth transition models, on the other hand, allow for relatively slower transitions from one regime to the other. As one can tell easily, TAR model is in fact nothing but a special case of STR. That is why an STR model would be able to capture a TAR-like process with an extremely steep G_{1t} if the true nature of regime switches is abrupt. However, a TAR model would not be able to behave like an STR if the regime switches are slow. In short, STR, as the general case, can mimic TAR dynamics, while TAR, as the restricted case, cannot mimic the dynamics of a smooth transition process. It might therefore look as if using an STR model would be a better choice. However, this is not a sure thing due to potential overfitting problems.

4 Dataset and estimation results

These two models are estimated in this paper using monthly data from the two most important carry trade currencies, i.e. Australian dollar and New Zealand dollar against the vehicle currency of the world, i.e. the US dollar and one of the most significant funding currency, i.e. the Japanese yen. As the interest rates, monthly observations of the 3-month interbank rates are retrieved from the Fred database of St. Louis FED. Interest rate differentials and the exchange rates are presented in the Fig. 1. Observations that include the US currency and the US interest rates extend from January 1990 to August 2020 while the observations that include the Japanese currency and the Japanese interest rates extend from December 2003 to August 2020. Chicago Board Options Exchange's (CBOE) Volatility Index (VIX) is used as the regime switching parameter. VIX is a real-time market index representing the market's expectation of 30-day implied volatility on the S&P 500 index options. It provides a measure of market risk and indicates the investors' sentiments. VIX is also known as the "fear gauge" or "fear index" and investors, research analysts, and portfolio managers look to VIX values as a way to measure market risk, fear, and stress before they take investment decisions.

The exact specifications of the models that are estimated with these data are as follows.

STR model:

$$s_{t+1} - s_t = \alpha_1 + \beta_1(i - i^*)_t + [(1 + exp\{-\gamma(VIX_t - c)\})^{-1}(\alpha_2 + \beta_s(i - i^*)_t)]$$

TAR model:

$$s_{t+1} - s_t = \begin{cases} \alpha_1 + \beta_1(i - i^*)_t + \varepsilon_{t+1} if r_t > c \\ \alpha_2 + \beta_s(i - i^*)_t + v_{t+1} if r_t \leq c \end{cases}$$

where r and c are the switching variable and its threshold value, respectively. In this paper, as aforesaid, VIX is going to be tried as the switching variable since regime switches are assumed to be volatility dependent in line with the claims stated in the excerpt from the BIS report at the beginning due to the worldwide popularity of VIX as a generally accepted global volatility measure with an established ability to trigger sentiment shifts in the markets. The threshold value of VIX is chosen in an iterative manner. The VIX observations, having been sorted out first, are trimmed by 20% from the top and the bottom. The remaining observations are then tried one by one as the potential c value. In each try, the estimations have produced a regime-1 sum of squared residuals (SSR) and a regime-2 SSR value. The sum of these SSRs is recorded as the total SSR for each VIX value. The VIX value that has been able to produce the minimum total SSR is selected as the c value.

Fama equation model:

$$s_{t+1} - s_t = (i - i^*)_t$$

The estimation results from all these three models for the four currency pairs, i.e., AUD/USD, NZD/USD, AUD/JPY and NZD/JPY, are presented in Tables 1, 2, and 3 at the end. The estimated outputs are based on the first 300 observations for the currency

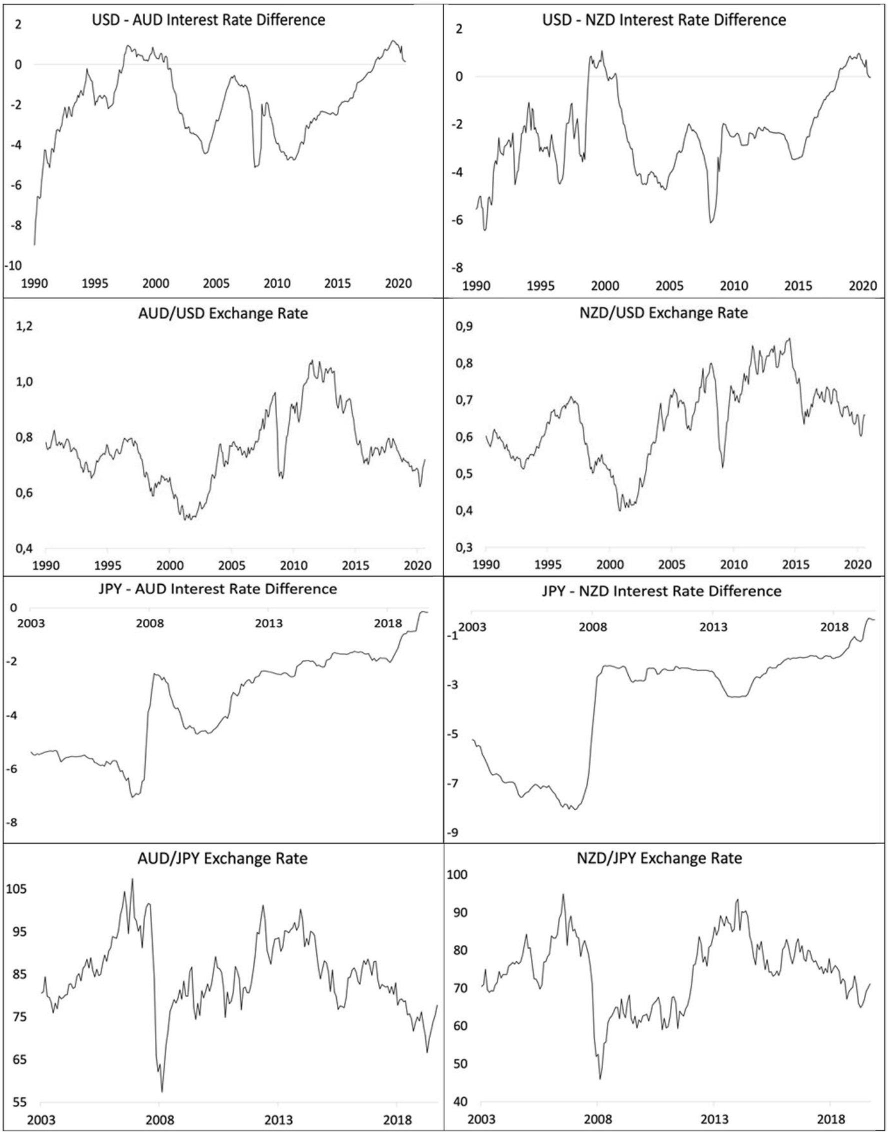

Fig. 1 Interest rate differences and exchange rates. Source: FRED Database of St. Louis FED

pairs including the USD, leaving the remaining 68 observations out-of-sample. That means, all the observations from January 1990 to December 2014 were used to train the models and the remaining observations from January 2015 to August 2020 were spared for out-of-sample testing. For the currency pairs involving the Japanese yen, all the observations from December 2003 to June 2017 were used to train and the remaining observations from July 2018 to August 2020 were used for testing. The datasets were split into training and testing subsets according to Guyon's (1997) methodology after allowing for a significance level of 0.01 with a Chernoff bound constant of 2.5.

The TAR, STR, and linear model estimation results for the exchange rates are presented in Tables 1, 2, and 3. When one looks at the adjusted R-squared values, the superiority of the TAR model in the two cases including the New Zealand dollar (i.e. NZD/USD and NZD/JPY) is seen while STR happens to be the superior model in the remaining two cases including the Australian dollar (i.e. AUD/USD and AUD/JPY). The noteworthy difference between these two currencies is that the STR models that are estimated for NZD/USD and NZD/JPY produced higher gamma estimates with lower threshold values for VIX around 20. The higher gamma estimates signal swift regime switches. That is the superiority of TAR over STR for NZD/USD and NZD/JPY makes sense. As for the AIC and SBC model selection criterion, STR seems to be offering better specification for NZD/USD only and TAR is better in the remaining three cases. In the AUD/USD case, even the linear model is able to surpass the STR model with respect to the AIC and SBC criterion. However, the out-of-sample prediction powers of the models are what really matters. Root mean squared errors (RMSE), mean absolute errors (MAE), and mean absolute percentage errors (MAPE) are estimated to compare the out-of-sample predictive powers of these three models and the random walk. Random walk model serves as the benchmark model of prediction. The RMSE, MAE and MAPE results are presented in Table 4 at the end. Estimated numbers in Table 4 serve as a clear proof of the superiority of the random walk model as the best predictor of the exchange rates studied in this paper. The second-best model after random walk is TAR in three out of the four cases according to MAE and MAPE. STR surpasses TAR and becomes the obvious second-best model only in the case of NZD/USD. According to the RMSE score, TAR and STR is almost the same for AUD/USD. TAR is the clear winner over STR for AUD/JPY and NZD/JPY. Linear model performs so poorly in all cases.

Finally, as for whether the UIP is holding or not, we need to understand that a significant negative coefficient indicates support for UIP given the definition of our exchange rates. A significant but positive relationship indicates a stable relationship between the interest rate differences and exchange rates although this stable relationship is the opposite of the relationship estimated by UIP. Plus, the way for evaluating the slope estimates of the STR specification is as follows. One needs to add the linear part's slope estimate to the slope estimate of the nonlinear part. If for example the slope estimated in the linear part happens to be insignificant, then only the nonlinear part's slope should be evaluated since an insignificant estimate is not statistically different from zero. Those explanations made, the UIP holds for both in TAR and STR for AUD/USD. UIP holds only in the second regime of TAR or the nonlinear part of STR for NZD/USD. Conversely, it holds only in the first regime of TAR or the linear part of STR for NZD/JPY. For AUD/JPY, it holds only in the second regime of TAR but not STR. Therefore, TAR seems to be able to identify subperiods of UIP in all the cases while STR can do that in three out of four cases.

5 Conclusion

Exchange rate prediction is a problematic research area. There still does not exist a structural exchange rate model that can consistently produce better out-of-sample predictions than a driftless random walk model. In a recent study, Söylemez (2021) has tried a number of state-of-the-art supervised machine learning methods allowing bias-variance trade-off in order to control for the significant foreign exchange volatility observed in practice only to conclude that none of these models could surpass the out-of-sample forecasting performance of a simple random walk model.

This paper has drawn attention to two important points regarding the prediction problem of exchange rates. First of all, many of the structural exchange rate models inherently depend on the uncovered interest parity rule. However, the empirical evidence shows us that the uncovered interest parity rule does not hold in reality. Therefore, the first question of this paper was whether we could improve the predictions of the uncovered interest parity rule by converting it into a nonlinear form. The emphasis placed on nonlinearity was the second important point raised in this paper. Exchange rates should be modeled nonlinearly because they typically follow a meandering path in time. Although there does not exist any formal test for identifying the exact cause of nonlinearity in time series, the tendency of exchange rates to meander is still suggestive of some sort of regime switching pattern. Plus, an excerpt from a BIS report suggests a precise kind of a volatility dependent regime-switching behavior. That is why, we have estimated the uncovered interest parity rule using data from two important carry trade currencies (Australian dollar and New Zealand dollar) against the US dollar and the Japanese yen, in two different regime-switching forms, i.e. TAR and STR forms. Because of its popularity and availability as a standard measure of global market fear with an established track record of power to cause sentiment shifts in the markets, VIX volatility index has been employed as the switching variable between the regimes.

In the end, estimated results for the currency pairs studied in this paper have given just another hard proof of the strength of the random walk model as the best predictor of exchange rates, although—and quite paradoxically—random walk model cannot be used as a prediction model itself for the technical reasons explained in this paper. However, the results are nevertheless encouraging since TAR and STR could easily surpass the out-of-sample prediction performance of the linear model. This is suggestive of why we need to stick to nonlinear specifications for exchange rate predictions. One could question the novelty of this finding on the grounds that the empirical literature in the last decade has already embraced nonlinear specifications for the uncovered interest parity. Although the popularity of nonlinear modeling is a fact—at least for the uncovered interest parity rule—it is also a fact that we do not have any econometric model that can identify the type of nonlinearities in a time series. That is to say there are tests that help us to decide whether a time series is linear or nonlinear but once we conclude that the series is nonlinear, there does not exist any further test that can tell us what is the real cause of the observed nonlinearity is. In this paper, we have tried a volatility-dependent regime switching model with the inspiration from an excerpt from a BIS report. Since volatility-dependent regime switching behavior has consistently performed better than the linear specification for a very simple prediction model based only on the interest rate differentials as the sole explanatory variable, volatility-dependent regime switching behavior can be used as a 'technically easy to implement' and 'reasonable from the economic theory point of view' basis of converting all the other structural models discussed in this paper into their nonlinear counterparts as well for even better prediction results.

Table 1 Estimation results for the TAR models

				Estimate	Std. dev.	p-value
TAR model for AUD/USD						
Regime 1 (VIX < 21.4)						
Constant term				0.7221	0.0215	0.0000
Interest differential (US–Australia)				− 0.0315	0.0107	0.0034
Number of months in Regime 1				198 months		
Regime 2 (VIX ≥ 21.4)						
Constant term				0.6257	0.0244	0.0000
Interest differential (US–Australia)				− 0.0373	0.0090	0.0000
Number of months in Regime 1				103 months		
R-squared	0.3034	AIC	− 1.4434			
Adj. R-squared	0.2964	SBC	− 1.3941			
TAR model for NZD/USD						
Regime 1 (VIX < 19.7)						
Constant term				0.7048	0.0281	0.0000
Interest differential (US–New Zealand)				0.0098	0.0087	0.2613
Number of months in Regime 1				171 months		
Regime 2 (VIX ≥ 19.7)						
Constant term				0.5341	0.0154	0.0000
Interest differential (US–New Zealand)				− 0.0214	0.0048	0.0000
Number of months in Regime 2				130 months		
R-squared	0.1891	AIC	− 1.4326			
Adj. R-squared	0.1809	SBC	− 1.3833			
TAR model for AUD/JPY						
Regime 1 (VIX < 20.1)						
Constant term				89.0514	3.2823	0.0000
Interest differential (US–Australia)				0.3143	0.8135	0.6997
Number of months in Regime 1				115 months		
Regime 2 (VIX ≥ 20.1)						
Constant term				60.5303	5.2544	0.0000
Interest differential (US–Australia)				− 4.8619	0.9961	0.0000
Number of months in Regime 1				48 months		
R-squared	0.3297	AIC	3.4342			
Adj. R-squared	0.3170	SBC	3.3545			
TAR model for NZD/JPY						
Regime 1 (VIX < 16.8)						
Constant term				79.6453	2.8829	0.0000
Interest differential (US–New Zealand)				0.1419	0.5338	0.7908
Number of months in Regime 1				90 months		
Regime 2 (VIX ≥ 16.8)						
Constant term				57.4187	2.4218	0.0000
Interest differential (US–New Zealand)				− 2.8221	0.5417	0.0000
Number of months in Regime 2				73 months		

Table 1 (continued)

				Estimate	Std. dev.	p-value
R-squared	0.4484	AIC	3.5532			
Adj. R-squared	0.4380	SBC	3.6726			

6 Appendix-1: estimation outputs and out-of-sample comparisons

See Tables 1, 2, 3 and 4.

7 Appendix-2: evaluating the gamma (γ) scores of the STR models

As it has been shown by Eq. 11 the two regimes are connected to each other through a logistic function in our STR specification, which was as shown below:

$$G_{1t} = G(r_t, \gamma, c) = (1 + exp\{-\gamma(r_t - c)\})^{-1}$$

This logistic function assumes three factors, one of which is the gamma parameter (γ). Gamma parameter adjusts the speed of transition from one regime to the other. As gamma becomes larger, the speed of adjustment increases. This is presented in Fig. 2 below for four hypothetical gamma values (0, 0.5, 1, and 35), where r_t is assumed to be a random variable ranging from 0 to 40 (mimicking VIX) with $c = 20$ (mimicking the most common fear threshold of VIX). Please note that no regime switching occurs when $\gamma = 0$. As gamma becomes larger, transitions start to take place and when $\gamma = 35$ regime switches happen to be almost abrupt around c. While Fig. 2 is there to provide us with general insight about the influence of γ in STR specifications, Fig. 3 presents the cases for each exchange rate studied in this paper with their estimated gamma values. As presented in Table 2, the estimated gamma values range from 0.8454 to 47.6587 for the exchange rate series studied in this paper. The graphs below indicate the likely speed of complete regime switches around the estimated thresholds of VIX with these estimated gamma values in each currency pair.

Table 2 Estimation results for the STR models

				Estimate	Std. dev.	p-value
STR model for AUD/USD						
Linear part						
Constant term				0.7383	0.0143	0.0000
Interest differential (US–Australia)				− 0.0275	0.0047	0.0000
Nonlinear part						
Constant term				− 0.1390	0.0344	0.0001
Interest differential (US–Australia)				− 0.0136	0.0098	0.1662
Gamma (γ)				3.2654	1.5932	0.0048
Threshold (c)				22.7976	1.6325	0.0000
R-squared	0.3108	AIC	− 1.3385			
Adj. R-squared	0.3089	SBC	− 1.2758			
STR model for NZD/USD						
Linear part						
Constant term				0.6914	0.0141	0.0000
Interest differential (US–Australia)				0.0046	0.0047	0.3248
Nonlinear part						
Constant term				− 0.1452	0.0209	0.0000
Interest differential (US–Australia)				− 0.0216	0.0067	0.0014
Gamma (γ)				9.9588	6.4097	0.1211
Threshold (c)				20.1802	0.5917	0.0000
R-squared	0.1820	AIC	− 1.5667			
Adj. R-squared	0.1798	SBC	− 1.4065			
STR model for AUD/JPY						
Linear part						
Constant term				86.9152	6.9278	0.0000
Interest differential (US–Australia)				− 0.9987	0.7129	0.1629
Nonlinear part						
Constant term				− 36.4880	17.2130	0.0353
Interest differential (US–Australia)				− 2.4261	2.4419	0.3217
Gamma (γ)				0.8454	0.5542	0.1287
Threshold (c)				24.2541	3.7104	0.0000
R-squared	0.3885	AIC	3.8618			
Adj. R-squared	0.3854	SBC	3.9604			
STR model for NZD/JPY						
Linear part						
Constant term				70.3211	1.4359	0.0000
Interest differential (US–Australia)				− 1.2937	0.2968	0.0000
Nonlinear part						
Constant term				− 8.2922	6.2820	0.1888
Interest differential (US–Australia)				2.9521	1.7667	0.0967
Gamma (γ)				47.6587	2.3814	0.0000
Threshold (c)				20.3070	0.1938	0.0000

Table 2 (continued)

				Estimate	Std. dev.	p-value
R-squared	0.3586	AIC	4.2404			
Adj. R-squared	0.3546	SBC	4.3542			

Table 3 Linear estimation results

				Estimate	Std. dev.	p-value
Fama equation model for AUD/USD						
Linear part						
Constant term				0.6826	0.0115	0.0000
Interest differential (US–Australia)				− 0.0362	0.0039	0.0000
R-squared	0.2217	AIC	− 1.3457			
Adj. R-squared	0.2191	SBC	− 1.3211			
Fama equation model for NZD/USD						
Linear part						
Constant term				0.5836	0.0143	0.0000
Interest differential (US–New Zealand)				− 0.0188	0.0045	0.0000
R-squared	0.0562	AIC	− 1.4940			
Adj. R-squared	0.0530	SBC	− 1.4694			
Fama equation model for AUD/JPY						
Linear part						
Constant term				79.3082	1.2535	0.0000
Interest differential (US–Australia)				− 1.5407	0.3202	0.0000
R-squared	0.1042	AIC	7.4322			
Adj. R-squared	0.0997	SBC	7.5564			
Fama equation model for NZD/JPY						
Linear part						
Constant term				69.6158	1.1870	0.0000
Interest differential (US–New Zealand)				− 1.2368	0.2731	0.0000
R-squared	0.0934	AIC	8.0321			
Adj. R-squared	0.0887	SBC	8.0074			

Table 4 RMSE, MAE and MAPE values for the models

	RMSE	MAE	MAPE
Results for the AUD/USD exchange rate			
TAR model	0.0422	0.0298	0.0404
STR model	0.0421	0.0307	0.0426
Linear model	0.0536	0.0469	0.0634
Random walk	0.0162	0.0130	0.0179
Results for the NZD/USD exchange rate			
TAR model	0.0598	0.0456	0.0675
STR model	0.0536	0.0409	0.0600
Linear model	0.0953	0.0891	0.1283
Random walk	0.0165	0.0131	0.0192
Results for the AUD/JPY exchange rate			
TAR model	5.1870	4.4887	0.0588
STR model	5.6060	4.6643	0.0619
Linear model	5.5848	4.6081	0.0604
Random walk	4.4533	3.7141	0.0480
Results for the NZD/JPY exchange rate			
TAR model	4.7826	4.1070	0.0565
STR model	6.1618	4.7809	0.0643
Linear model	6.9074	5.6745	0.0754
Random walk	4.2874	3.5026	0.0479

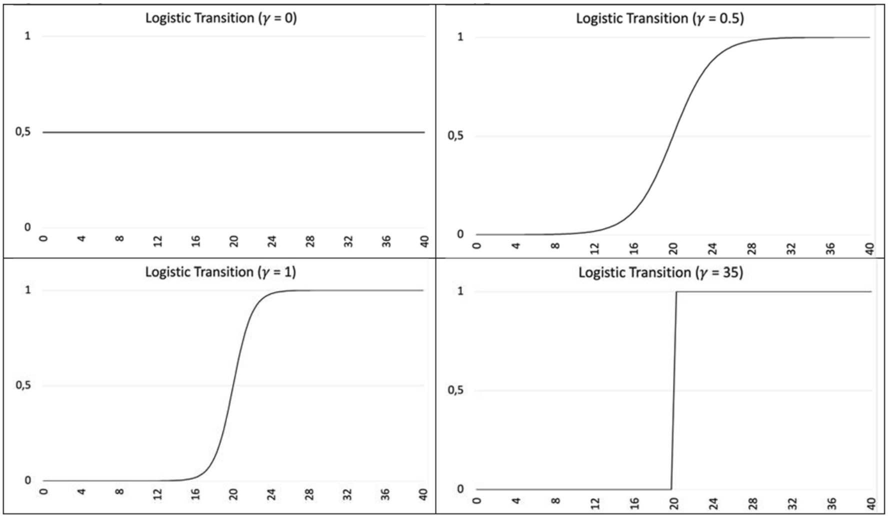

Fig. 2 Logistic transitions under four different hypothetical γ values

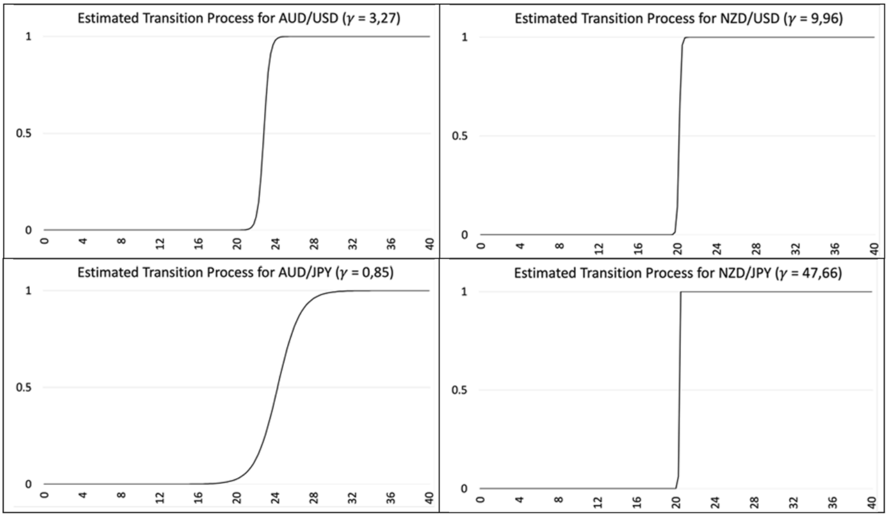

Fig. 3 Estimated speed of logistic transitions for the exchange rates

Funding The author has no relevant financial or non-financial interests to disclose.

Declarations

Conflict of interest I certify that there is no actual or potential conflict of interest in relation to this article.

References

Ahmad, Y., & Glosser, S. (2011). Searching for nonlinearities in real exchange rates. *Applied Economics, 43*(15), 1829–1845.

Bank for International Settlements. (2008). 78th annual report

Burnside, C. (2019). Exchange rates, interest parity, and the carry trade. *Oxford Research Encyclopedia of Economics and Finance*. https://doi.org/10.1093/acrefore/9780190625979.013.315. Retrieved 18 Apr 2021

Chinn, M. D., & Meredith, G. (2005). Testing uncovered interest parity at short and long horizons during the post-Bretton woods era. NBER Working Papers, 11077.

Fama, E. F. (1984). Forward and spot exchange rates. *Journal of Monetary Economics, 14*(3), 319–338.

Guyon, I. (1997). A scaling law for the validation-set training-set size ratio. AT&T Bell Laboratories. http://citeseerx.ist.psu.edu. Retrieved 19 Apr 2021

Hu, B., & McInish, T. (2013). Greed and fear in financial markets: The case of stock spam E-mails. *Journal of Behavioral Finance, 14*, 83–93. https://doi.org/10.1080/15427560.2013.761630

Johansson, M. (2001). TAR models and real exchange rates. Working papers 21, Lund University, Department of Economics.

Kilian, L., & Taylor, M. P. (2003). Why is it so difficult to beat the random walk forecast of exchange rates? *Journal of International Economics, 60*(1), 61–83.

Livanas, J. (2011). Are investors rational and does it matter? Determining the expected utility function for a group of investors. *Journal of Behavioral Finance, 12*, 53–67. https://doi.org/10.1080/15427560.2011.553003

López-Cabarcos, M. Á., Pérez-Pico, A. M., Vázquez-Rodríguez, P., & López-Pérez, M. L. (2020). Investor sentiment in the theoretical field of behavioural finance. *Economic Research-Ekonomska Istraživanja, 33*(1), 2101–2119. https://doi.org/10.1080/1331677X.2018.1559748

Lux, T. (1995). Herd behaviour, bubbles and crashes. *The Economic Journal, 105*(431), 881–896.

Meese, R. A., & Rogoff, K. (1983). Empirical exchange rate models of the seventies: Do they fit out of sample? *Journal of International Economics, 14*(1–2), 3–24.

Saadon, Y., & Sussman, N. (2018). Nominal exchange rate dynamics and monetary policy: Uncovered interest rate parity and purchasing power parity revisited. CEPR, Discussion Paper, 13235.

Söylemez, A. O. (2021). Foreign exchange rates: A research overview of the latest prediction techniques. London: Routledge.

Taylor, A. M., & Taylor, M. P. (2004). The purchasing power parity debate. *Journal of Economic Perspectives, 18*(4), 135–158.