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Do mathematical thinking processes influence visual estimation skills in students?

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ABSTRACT

In this study, the visual estimation skills of fourth grade elementary school students based on their mathematical thinking processes were investigated. The research questions focus on understanding how students' mathematical thinking processes influence their visual estimation skills. A total of 445 students participated in the study. The relational scanning model was used to determine the relationship between students' visual estimation skills and their mathematical thinking processes. The Mathematical Process Instrument and the Visual Estimation Skills test were used as data-gathering tools. The data obtained were analyzed using different statistical methods, including the chi-squared test, t-test, and single-factor analysis of variance. The findings contribute to our understanding of the complex relationship between mathematical thinking processes and visual estimation skills in students, addressing the research questions raised in the study. These results can inform educators and instructional designers when developing effective teaching materials and strategies that cater to students' diverse thinking processes.

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Introduction

Estimation is a crucial skill that is necessary for everyday life and useful in various situations in mathematics. In Turkish culture, for example, it is considered impolite to count the guests in a room by hand or to follow them with your eyes to provide enough servings of food. Instead, hosts offer an estimated number of servings and any lacking are added later. Many problems, situations, and issues require estimation. You need to estimate in order to answer questions quickly, such as how many cars are on the street, how many seagulls are flying over the beach, how many students are in the school, or what time it is (if you do not have a watch). Estimation allows you to provide a quick, approximate answer rather than waiting to calculate the exact value. Leutzinger et al. (1986) noted that it is not important to know the exact value and students should have the opportunity to generously estimate. Depending on these situations, students' information processing skills will be strengthened. In math classes, students can take risks when estimating or developing different strategies, becoming more flexible in managing these strategies. Trying new strategies poses no risk (Adams et al., 2005).

Estimation is a complex cognitive process that involves the use of different types of knowledge, such as number sense, spatial sense, and proportional reasoning (Siegler, 2016). Estimation is a decision-making process based on prior experience and theoretical knowledge (Segovia & Castro, 2009). It involves using trained responses without standard measurements (Aytekin & Toluk-Uçar, 2014). Estimation skills, identified as crucial for various types of work and daily activities, have been linked to several other mathematical proficiencies, such as spatial visualization, mental calculation, and measurement, according to data from the National Council of Teachers of Mathematics [NCTM] (2000). To account for different kinds of estimation tasks, scholars have divided estimation into two categories: computational and visual estimation (Brydges, 2013; Jones et al., 2012; Markovits & Hershkowitz, 1997). In the classroom, students tend to focus on mathematical calculations, which may lead to neglecting the importance of visual estimation. However, visual estimation is a valuable skill for daily life, helping individuals to develop predictive skills and a sense of quantity and to count through visual perception. Studies indicate that visual estimation activities enhance students' visual skills (Markovits & Hershkowitz, 1997). Thus, visual estimation aims to foster students' visual intuitions through experiences derived from visual estimation activities. Visual estimation, in particular, is a valuable skill that allows individuals to make approximate judgments about quantities and sizes based on visual cues (Brydges, 2013). In addition, visual estimation can help students develop their visual–spatial reasoning abilities, which are essential for success in many STEM fields (Wai et al., 2009).

Visual estimation can be particularly useful in the classroom, as it allows students to engage with mathematical

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concepts in a more intuitive and accessible way. By incorporating visual estimation activities into math lessons, teachers can help students develop their estimation skills and improve their ability to reason and make decisions based on visual information (Jones et al., 2012). Furthermore, visual estimation activities can be used to enhance students' understanding of mathematical concepts, such as measurement and proportionality (Markovits & Hershkowitz, 1997). Overall, visual estimation is an important skill that can benefit individuals in a wide range of contexts. By emphasizing the importance of visual estimation in the classroom and providing opportunities for students to practice this skill, educators can help students develop their cognitive abilities and prepare them for success in STEM fields and beyond.

It is predicted that students employ various strategies as in measurement estimation when making visual estimations and that students with differing cognitive structures utilize distinct methods when selecting these strategies. For example, some students draw upon past experiences to make predictions, while others identify and use different reference frames. Some students use familiar objects as a reference, while others group predicted situations and still others use counting, although this method is typically time-consuming. However, most studies on mathematics and cognitive structure treat individuals as a single homogeneous group, neglecting the possibility that the degree of visual estimation and mental calculation abilities may differ among people, indicating cognitive differences. Since the late 1970s, there has been growing interest in differentiating cognitive styles, particularly visual/verbal styles, in the field of mathematics education. While visualizers rely on visual representations, verbalizers employ analytical strategies. Numerous studies have investigated the nature, role, and importance of representations and visualization in various aspects of mathematical activity, including Gal and Linchevski (2010).

Preliminary studies conducted by Krutetskii (1976) and Presmeg (1986a) examined students' preferences for visual or nonvisual methods in problem-solving. Krutetskii (1976) highlighted the existence of different mathematical thinking processes that contribute to successful performance in mathematical activities, based on the relation between visual-pictorial and verbal-logical components. Krutetskii categorized students into three groups based on their mathematical thinking processes: geometric (visualizers), analytical (verbal), and harmonic (a combination of visual and analytical thinking). Analytic thinkers rely on a strong verbal-logical component and a weak visual-pictorial component when reasoning through problems. Geometric thinkers rely more heavily on visualization skills, which enable them to create images and manipulate representations of the problem in their minds. Harmonic thinkers, on the other hand, combine both visual-pictorial and verbal-logical components to solve problems. Bakker et al. (2022), who conducted a recent study with reference to Krutetski's work, drew attention to the importance of diversity in the use of thinking processes for high mathematics achievement in primary school.

There is a significant difference in the mathematical thinking processes of third and fourth grade students as they transition to the abstract thinking period (Pilten, 2008). However, among the sources examined, there have been no studies to date on the mathematical thinking processes at the primary school level. Most studies have focused on sixth grade, secondary school, and university level students (Çilingir-Altıner 2018). Nonetheless, the primary school period is critical for the development of various mathematical skills that lay the foundation for future classes. Therefore, it is considered an important period.

The acquisition of mathematical skills during primary school is crucial and, therefore, it is important for teachers to determine the approximate mathematical thinking processes of their students. By doing so, they can employ various representations during problem-solving to enhance their students' understanding. However, despite the importance of this, there have been no studies conducted on the determination of the mathematical thinking processes of elementary school students in this country or elsewhere. Additionally, visual estimation skills have been identified as a deficiency in the field of mathematics education, despite their importance in contributing to students' acquisition of mathematical skills in various disciplines such as science, physical therapy, dietetics, biology, geography, and the environment, not only in school but also outside.

As children require visual aids more than adults do, visual representations should be included when teaching various mathematical topics at the elementary school level (Seng & Chan, 2000). In the present study the importance of acquiring visual estimation skills in elementary school is emphasized since school is not only an environment that focuses on academic success but also prepares students for daily life and the future. It is crucial to understand the value of these skills, which make life easier. Although some individuals are better at predicting approximate numbers, it is not clear how visual estimation skills relate to mathematical thinking processes involving visual and analytical skills. Furthermore, some studies on students' mathematical thinking processes have examined the effect of gender (Csíkos et al., 2012; Lowrie & Kay, 2001; Van Garderen, 2006). However, due to the limited number of studies on visual estimation skills, the effect of gender cannot be clearly determined. Therefore, the research hypotheses in the present study are based on previous studies that investigate the relationship between students' estimation skills and their sense of numbers with their gender (Hanson & Hogan, 2000; LeFevre et al., 2013; Reys et al., 1991). Herein, we examined the visual estimation skills of fourth grade students according to their mathematical thinking processes. Hence, answers were sought to the following questions:

For fourth grade students:

- What are their mathematical thinking processes? Is there a significant difference between these processes?
- Do mathematical thinking processes differ according to gender?
- Do visual estimation skills differ according to gender?
- Do visual estimation skills differ according to mathematical thinking processes?

Method

We used the relational scanning model, which is a quantitative research method. Relational scanning models involve using different types of statistical analysis to measure and define the relationship between two or more variables (Creswell, 2014). The relational scanning model is useful in identifying the nature and strength of the relationship between variables and can provide valuable insights for researchers in understanding complex phenomena.

Participants

The study's participants were 445 fourth grade students from four different state schools who were chosen by convenient sampling. Of the participants, 219 were boys and 226 were girls, with ages ranging from 107 months (9 years, 1 month) to 129 months (10 years, 9 months). The mean age of the students was 117 months (9 years, 9 months). It is important to note that the gender distribution was relatively balanced, which may be useful when examining any potential gender differences in the study's variables. Additionally, the age range and mean age of the students appear appropriate for the study's focus on fourth grade students. In the study, when the socio-economic contexts of the participants regarding their schools or the region they come from are analyzed, it is seen that they come from the middle and upper socio-economic levels (according to the TÜİK [Turkish Statistical Institute]). Based on the TÜİK classification, middle and upper socio-economic levels generally indicate families with moderate to high income levels and higher levels of education. Therefore, it is likely that the children in our study come from families with more resources and access to educational opportunities. However, it is important to note that socio-economic status can vary within each level and that other factors, such as cultural and environmental factors, can also influence a child's development and academic performance.

Data collection tool

The Mathematical Processing Instrument (MPI) was used to determine the mathematical thinking processes of the students, and the Visual Estimation Skills Test was used to reveal their visual estimation skills.

Mathematical processing instrument

To measure the mathematical thinking processes of the students, the researchers used the MPI, which was originally developed by Suwarsono (1982) for sixth grade students and translated into Turkish by Hacıömeroğlu and Hacıömeroğlu (2013). The test consists of 30 questions and there is an answer key with different possible solutions for each problem. Students were given a score of "0" if they chose analytical solutions and a score of "2" if they chose visual solutions, based on the solutions evaluated according to the answer key and their own solutions. The processes conducted for the MPI are presented in Figure 1.

The exclusion of certain items from the tool was based on several factors, such as expert opinions, considerations of the fourth grade level, item analysis results, Lawshe analysis, and confirmatory factor analysis (CFA). The original test comprised 30 questions, but, for the present research, the number of questions was reduced to 20 by the math curriculum, considering expert opinions and the fourth grade level. To ensure the validity and reliability of the MPI, item analysis was conducted to examine the performance of each item. This analysis involves evaluating the difficulty level, discrimination power, and overall quality of the items. Based on the results of this analysis, items that did not meet the desired criteria or were found to be problematic were excluded from the final version of the tool. As a result of the item analysis, questions with an item difficulty index below .30 and an item discrimination index below .20 were identified, leaving 15 questions.

Lawshe analysis was performed to ensure the content validity of the test used in the study. This analysis is commonly used to assess the agreement among experts regarding the relevance and appropriateness of test items. In line with this analysis, the minimum values of content validity ratios according to the number of experts (5) at $p<.05$ significance level were taken as used by Veneziano and Hooper (1997) (as cited in Yurdugül, 2005). Table 1 shows the

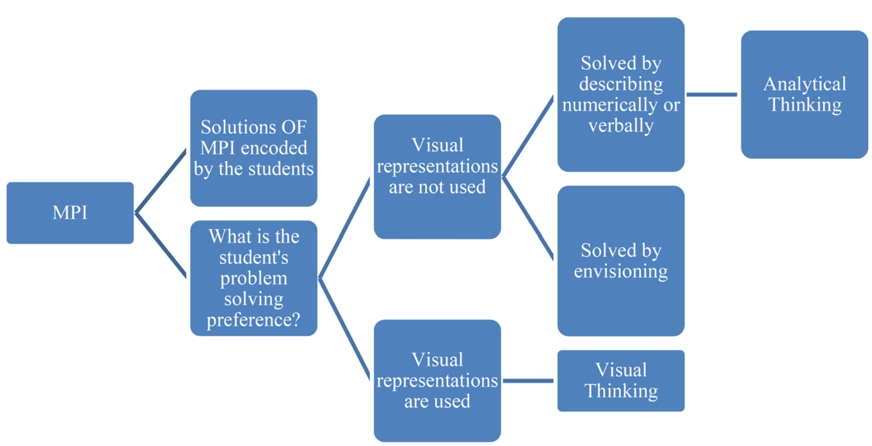


Figure 1. Evaluation scheme of MPI for researchers.

content validity ratios and content validity indices of the items in the test.

When Table 1 is analyzed, the items with content validity rate values of 0 and above were examined and the corrections suggested by the experts were made. Items with values below zero were discarded. As a result of these corrections, 11 questions were obtained in total.

According to Figure 2, linguistic and numerical corrections were made to items 1, 2, 4, 10, and 11. Items 12, 13, 14, and 15 were removed. Thus, a test consisting of 11 questions was obtained and the construct validity and reliability of the test were analyzed by performing the second pilot application. An example of a question removed from the test is as follows:

> *A man cuts 6 short sticks, each 2 fingers long, from a long wooden stick. He then realizes that there is a piece of the stick 1 finger long left. What is the original length of the stick?*

Table 1. Content validity ratio and content validity index values.

Items	Necessary	Necessary but with correction	Unnecessary	Content Validity Ratios
item 1	4	1	0	0.6
item 2	4	1	0	0.6
item 3	5	0	0	1
item 4	4	1	0	0.6
item 5	5	0	0	1
item 6	5	0	0	1
item 7	5	0	0	1
item 8	5	0	0	1
item 9	5	0	0	1
item 10	3	2	0	0.2
item 11	4	1	0	0.6
item 12	1	1	3	−0.6
item 13	0	2	3	−1
item 14	1	1	3	−0.6
item 15	1	1	3	−0.6
Number of Experts	5			
Content Validity Rate	0.99			
Content Validity Index	0.2			

An example of a question with both numerical and linguistic corrections can be given as follows:

> Original: *A man plants a tree at each end (starting and ending points) of a straight path. Then he plants trees at intervals of 5 meters each, starting from one end of a path 25 meters long. What is the number of trees planted on the path?*

> Organized numerically and linguistically: *Ali planted a tree and then planted another tree every 5 steps along the path. Since the length of the path is 15 steps, how many trees did Ali plant in total?*

The reliability of the test was analyzed and the KR-20 value of the test was found to be 0.78, with a Cronbach's alpha reliability coefficient of 0.78, which confirms the validity of this test. Furthermore, CFA was conducted to examine the structural validity of the test. CFA is a statistical technique used to assess the fit between the observed data and a hypothesized factor structure. It was aimed to determine whether the data collected from the test align with the proposed one-dimensional structure of the mathematical thinking processes, and the analysis showed that the fit indices of the data were in good agreement (χ^2 (240) = 73.821, $p < 0.001$; GFI = 0.95, CFI = 0.92, SRMR = 0.05, RMSEA = 0.05).

The students were given two points for visual solutions and zero points for analytical solutions, and these values were recorded in Excel, regardless of whether the problems were true or false. When deciding on whether a problem was solved by visual means or by analytical means, the researchers first examined the solution given by the students and then examined the answer given by the students to the answer key based on the solution offered by these students. The solutions provided by the students for all questions were scored by another expert. Cohen's kappa coefficient (κ) was calculated to examine the reliability between the scores given by the experts. For calculating this coefficient, 100 randomly selected (about 22% of the sample) students' answers to the math test and the answer keys used for these problems were investigated by both the researcher and a second researcher, and the reliability coefficient between the scores of both

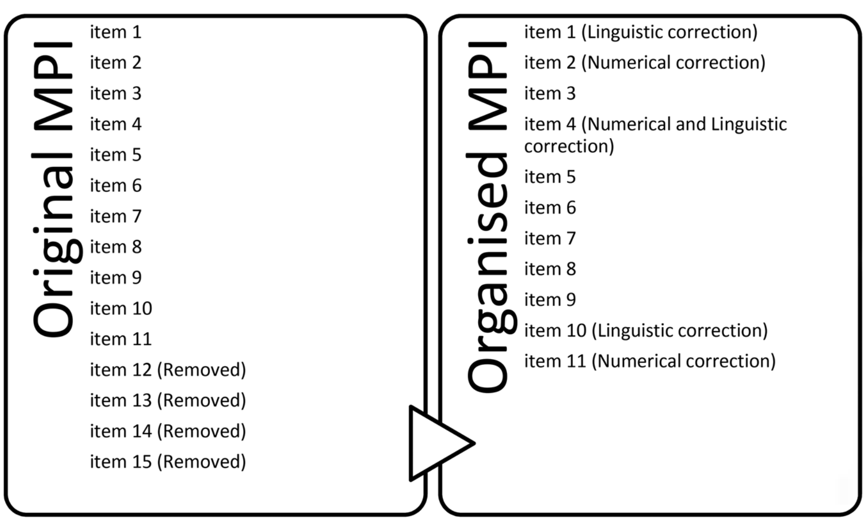


Figure 2. Modifications on the MPI test.

researchers was calculated. Cohen's kappa coefficients of 0.88 and above were statistically significant ($p < 0.001$), and the mean of these values was 0.97. As the values were greater than 0.81, it can be inferred that there is a very good fit between the scores and the scoring of the researcher can be trusted. However, the two researchers discussed conflicting points until a consensus was reached and, as a result, a common decision was reached to increase the reliability of the determination of the solutions for each problem.

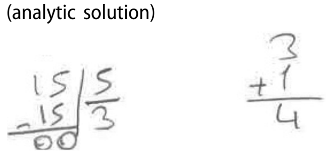
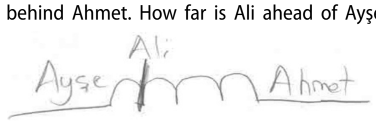
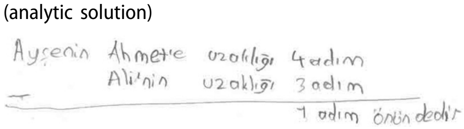
Examples of questions and answers to them are listed in Table 2.

Visual estimation skills test

Based on the work by various researchers such as Markovits and Hershkowitz (1997, 1993) and Ansari et al. (2007), a pool of 30 questions was initially developed for the visual estimation skills test. Half of the questions were related to shape estimation, including estimating the marked point on a number line, estimating angles and ratios, and estimating the quantity and size of shapes using reference and number sense. The other half of the questions were related to counting dots in numbers ranging from 1 to 20, which has been used in other studies. A PowerPoint presentation was prepared for each question, and a time limit was set for each slide based on expert opinions on the organization and presentation of the questions. The development of the PowerPoint slides followed a rigorous process aimed at ensuring the accuracy and appropriateness of the visual stimuli presented to the participants. To achieve this, visual prediction assessment and guidelines recommended by experts in the field of instructional design and visual arts were followed. The following are the main steps taken in the development of the slides:

1. Item Selection: Initially, a pool of possible questions based on concepts and skills related to visual estimation was created. These questions were derived from the established literature on visual estimation and mathematical thinking processes. A panel of mathematics education experts reviewed these items and selected the most appropriate ones for inclusion in the post-test.
2. Slide Selection Criteria: The final selection of PowerPoint slides was based on several criteria. These included clarity of the visual presentation, appropriateness of content for the target age group, avoidance of potential bias, and the extent to which the slides could effectively assess participants' visual prediction skills.
3. Evaluation of Duration: According to Healey et al. (1996), tasks that can be completed on large multi-element screens in 200 milliseconds or less were considered a prerequisite because eye movements require at least 200 milliseconds to start. Ansari et al. (2007) reported that using 250 milliseconds for children was appropriate for the age groups employed in the present study.
4. Pretest and Piloting: Prior to the actual administration of the test to the participants, a pretest and piloting phase was conducted. In the initial test, 30 questions were presented to 66 students on a 15-inch computer screen for 250 milliseconds. For each question, two points were given for a complete and correct answer, one point was given for a partially correct answer, and zero points were given for an incorrect answer. Fully accurate and partially accurate intervals were formed based on the answers collected from the students. After conducting item analysis, a

Table 2. Selected questions in MPI and answer key.

Selected Questions in MPI	Answer Key
At each of the two ends of a straight path a man planted a tree and then every 5 feet along the path he planted another tree. The length of the path is 15 feet. How many trees were planted?(visual solution)  (analytic solution) 	__ I wrote mathematical expression(s) to represent the problem. __ I imagined the path, and the man planting the trees. __ I solved the problem by drawing (or imagining) a diagram schematically representing the relative position of trees along the path. __ *I did not use any of the above methods* I attempted the problem in this way:
In an athletics race, Ahmet is four feet ahead of Ayşe and Ali is three feet behind Ahmet. How far is Ali ahead of Ayşe?(visual solution)  (analytic solution) 	__ I wrote mathematical eexpression(s) to represent the problem. __ I solved the problem by imagining Jim, Peter, and Tom running in an athletics race. __ I solved the problem by drawing (or imagining) a diagram schematically representing the relative position of Jim, Peter, and Tom in an athletics race. __ I did not use any of the above methods I attempted the problem in this way:

test of 20 questions was obtained, with a KR-20 value of 0.63.

5. Reliability Analyses of the Resulting Test: The final version of the test was again subjected to Lawshe analysis for expert review. Lawshe analysis was performed to ensure the validity of the test and none of the statistical questions were excluded. CFA was also used to determine the structural validity of the test. Most of the CFA and fit indices of the test were within the boundary values (χ^2 (244) = 190.182, p = 0.058; GFI = 0.93; AGFI = 0.91; CFI = 0.85; SRMR = 0.056; RMSEA = 0.027).

Some examples of the visual estimation skills test can be seen in Table 3.

Data collection process

Quantitative data collection tools were used to achieve the research objectives. The tools were developed and their structural compliance was tested, followed by validity and reliability studies. The necessary permissions were obtained from the Ministry of Education and the study groups were determined. The teachers were informed about the study's purpose, applications, and data collection processes to be applied in their classrooms. Students who did not wish to participate in the research were excluded. The MPI was administered to determine students' mathematical thinking processes and they were divided into groups accordingly. To determine visual estimation skills, a test was conducted with a PowerPoint presentation in a suitable environment (such as a classroom, activity class, or guide teacher's room) approved by the school administration. The participants were tested one by one and no distracting materials were present in the environment. The data were collected by showing PowerPoint slides for a mean of 250 milliseconds, which were previously prepared on a 15-inch computer brought by the researcher. The mean duration for each student to answer the questions was 6 min.

Data analysis

The data obtained to evaluate the test scores of the students were transferred to Excel and SPSS 24.00. The mathematical thinking processes of the students were determined in the present study using a technique for establishing the distribution interval, as employed by Taşova (2011), Sevimli (2013), Krutetskii (1976), and Galindo-Morales (1994). The distribution interval was calculated by finding the difference between the highest and lowest scores obtained from the test. Whether the students' mathematical thinking processes differ according to their gender was examined using chi-squared analysis. This analysis assesses whether there is a significant correlation between two categorical variables (Büyüköztürk, 2011) and was considered suitable for the present study because the mathematical thinking processes have three categories and the gender of the students has two categories. T-test analysis was performed for two variables to determine whether the visual estimation skills of the students varied with gender. Single-factor ANOVA was used to test the hypothesis that "The mathematical thinking processes of students significantly influence visual estimation skills".

Results

What are the students' mathematical thinking processes? Is there a meaningful difference between mathematical thinking processes?

The mathematical thinking processes of the students were determined using a technique for establishing the distribution interval. The resulting distribution interval is obtained by dividing the distribution interval by the desired number of groups, which in this case is three (analytic, harmonic, and visual). There were 11 questions on the MPI, and the resulting class interval was summed with the lowest score obtained from the MPI to determine the max-min limit of the analytical thinking processes subgroup. The maximum and minimum limits of the visual thinking process were calculated using the same system. For example, in the present study, students who chose analytical solutions were given "0" points, while those who chose visual solutions were given "2" points, based on the answer key and student solutions. The minimum score was 0 and the maximum score was 22. The distribution interval was calculated to be $22-0=22$ and the class interval was $22/3=7.3$. Although the exact limits were unclear and there was a possibility of other problems arising in other calculations, based on the mentioned calculation, it was estimated that those scoring between 0 and 7 were analytical, those scoring between 8 and 14 were harmonic, and those scoring between 15 and 22 were generally prone to the visual thinking processes. The frequency and percentages of the mathematical thinking processes (visual, analytical, and harmonic) of the students who participated in the study are given in Table 4.

According to Table 4, 27.4% of students preferred analytical thinking processes, 46.3% preferred harmonic thinking processes, and 26.3% preferred visual thinking processes. These findings suggest that students' problem-solving approaches vary, with harmonic thinking being the most preferred and visual thinking the least preferred representation.

Table 3. Examples of visual estimation skills test.

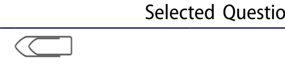
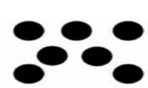
Estimation	Selected Questions in VES
estimation through the figure	 *How many paper clips ar there along the length of the line?*
estimation the number of points	 *How many dots are there?*

Table 4. Mathematical thinking processes of students.

Mathematical Thinking Process	*f*	%
Analytical	122	27.4
Harmonic	206	46.3
Visual	117	26.3
Total	445	100.0

Table 5 shows the results of the chi-squared test for a single sample based on the mathematical thinking processes.

Based on the results of the single sample chi-squared test presented in Table 5, out of 445 students, 122 showed a preference for analytical thinking processes, 206 for harmonic thinking processes, and 117 for visual thinking processes. It was observed that the least preferred thinking process representation for problem-solving was visual, while the most preferred representation was harmonic. The statistical analysis showed a significant difference between the students' preferred problem-solving representations (χ^2 (2, 445) = 33.712, $p < 0.05$).

Do students' mathematical thinking processes differ according to gender?

An analysis of the relationship between students' gender and their preferred mathematical thinking processes was conducted using a chi-squared test. The Pearson chi-squared test given in Table 6 was used to investigate the ratio of analytical, harmonic, and visual thinking processes for boys and girls.

According to Table 6, the p-value was 0.47, suggesting that there is no significant relationship between students' gender and their preference for visual, harmonic, and analytical thinking processes (χ^2 (2, 445)= 1.52, $p > 0.05$) in the chi-squared test with two variables. Although it appears that boys tend to think more analytically and girls are more inclined toward visual thinking, this difference is not statistically significant. Therefore, based on the results of this study, it can be concluded that mathematical thinking processes are independent of gender. These findings suggest that gender may not play a role in the development and use of mathematical thinking processes in students, emphasizing that both boys and girls can adopt diverse thinking approaches when solving mathematical problems.

Table 5. Chi-square test results for single sample according to mathematical thinking processes.

Mathematical Thinking Processes	Observed-n	Expected-n	Residual	χ^2	*s*	p
Analytical	122	148.3	−26.3	33.712	2	0.000
Harmonic	206		57.7			
Visual	117		−31.3			
Total	445					

Table 6. Chi-square test for two variables based on mathematical thinking processes and gender.

			Gender				
			Boy	Girl	χ^2	sd	p
Mathematical Thinking Process	Analytical	N	65	57	1.52	2	0.46
		%	53.3	46.7			
	Harmonic	N	101	105			
		%	49	51			
	Visual	N	53	64			
		%	43.3	54.7			

Do the visual estimation skills of the students differ according to gender?

To explore whether students' visual estimation skills differ based on gender, a t-test for independent samples was performed. The scores of the students obtained from the visual estimation skills test by gender are shown in Table 7.

It is clear that there was no significant difference in visual estimation skills between the genders (t[443] = 0.69, $p > 0.01$). Although the mean score obtained by the boys in the Visual Estimation Skills Test ($\bar{X}$ = 18.62) was higher than that obtained by the girls ($\bar{X}$ = 18.59), the difference was not significant. The effect size (η^2) was 0.00, indicating that gender has no practical significance in explaining the variability in visual estimation skills. Therefore, based on the results of the present study, it can be concluded that there is no significant difference in visual estimation skills between boys and girls, suggesting that gender does not play a significant role in shaping students' visual estimation abilities.

Do the visual estimation skills of the students differ according to their mathematical thinking processes?

Single-factor ANOVA was used to examine whether the visual estimation skills of the students differ according to their mathematical thinking processes. A descriptive analysis of visual estimation skills based on mathematical thinking processes is shown in Table 8.

As shown in Table 8, the mean score of students who exhibited a preference for analytical thinking processes was 16.31, for those with a preference for harmonic thinking processes was 18.27, and for those with a preference for visual thinking processes was 21.61. These results indicate that students who showed a preference for visual thinking processes had higher mean scores in the Visual Estimation Skills Test. The analysis revealed a significant difference in visual estimation skills based on thinking processes. To determine whether this difference was significant, one-way ANOVA was conducted, and the results are presented in Table 9.

The scores obtained from the students' visual estimation skills varied significantly based on their thinking processes (F (2, 442) = 40.51, $p < 0.05$). The results indicate that students who are prone to visual thinking processes are more likely to have higher scores ($\bar{X}$ = 21.61) compared to those who are prone to other thinking processes. Specifically, students who showed a preference for visual thinking processes obtained higher scores in the Visual Estimation Skills Test compared to those with analytical or harmonic thinking preferences. Scheffé's test further confirmed significant differences between visual thinking and both analytical and harmonic thinking. Moreover, there was a significant difference between analytical and harmonic thinking. The calculated eta-squared (η^2) value was 0.15, indicating that approximately 15% of the observed variances of the scores

Table 7. The t-test results of visual estimation skills test scores by gender.

Dependent variable	Gender	N	$\bar{X}$	*s*	sd	t	p	η^2
Visual estimation Skills	Boy	219	18.62	4.95	443	0.069	0.975	0.00
	Girl	226	18.59	5				

Table 8. Descriptive analysis.

	Mathematical Thinking Process	N	$\bar{X}$	s
Visual Estimation Skill Test	Analytical	122	16.31	4.54
	Harmonic	206	18.27	4.65
	Visual	117	21.61	4.58
	Total	445	18.61	4.99

Table 9. One way ANOVA test results.

Source of Variance	Sum of squares	sd	Average Squares	F	p	Significant difference
Intergroup	1695.724	2-442	847.862235	40.51	0.000	V*-H*
Intragroup						V-A*
	9296.239575		21.032216			H-A
Total	10991.964045					

*Visual: V; Harmonic: H; Analytical: A.

obtained from the visual estimation test depend on the mathematical thinking processes.

In conclusion, this study provides valuable insights into students' mathematical thinking processes and their impact on visual estimation skills. The results indicate that students' problem-solving approaches are diverse and not influenced by gender. However, there is a meaningful relationship between students' thinking processes and their performance in visual estimation tasks. Specifically, students with a preference for visual thinking demonstrate higher proficiency in visual estimation skills. These findings have implications for educators and instructional designers, emphasizing the importance of understanding individual differences in thinking styles and tailoring teaching methods to accommodate diverse problem-solving approaches.

Conclusion and recommendations

What are the students' mathematical thinking processes? Is there a meaningful difference between mathematical thinking processes?

In the present study, we examined the solution strategies used by students in solving verbal problems and identified their problem-solving preferences, with visual thinking being the least preferred and harmonic thinking being the most preferred. We investigated whether there was a significant difference in the mathematical thinking processes of the students and the results revealed significant differences. Fourth graders were found to prefer harmonic thinking in verbal problem-solving, which is consistent with previous studies by Taşova (2011) and Hacıömeroğlu and Hacıömeroğlu (2013, 2017).

According to Montenegro et al. (2018), primary school students often use analytical solutions and representations, but struggle with calculations. The researchers also noted that classroom activities can influence students' use of visual representations. In the third and fourth grades of primary school, there are significant differences in the thinking structures of students, as this is when the transition to the abstract period in thinking processes, i.e., problem-solving preferences, occurs (Pilten, 2008).

Lowrie and Clements (2001) suggested that problem-solving preferences vary from visual to analytic as students grow up. However, El Mouhayar and Jurdak (2016) found that analytical thinking was more dominant in the fourth and fifth grade groups, whereas visual thinking was more prevalent in upper grade levels (10th and 11th grades). The researchers emphasized the importance of using visual representations as a learning strategy, particularly in mathematics.

In the present study, the mathematical thinking processes of fourth grade students in verbal problem-solving were examined, with the least preferred thinking process being visual, followed by analytical thinking, and the most preferred being harmonic thinking. This finding is consistent with previous studies, which also indicate a transition from visual to analytic thinking as students grow up. Classroom activities and the use of visual representations can affect students' problem-solving preferences. It is important to use visual representations as a learning strategy for students, particularly in mathematics.

Do students' mathematical thinking processes differ according to gender?

It was found that gender does not have a statistically significant relation with mathematical thinking processes, suggesting that these processes are independent of gender. This is consistent with previous studies by Csíkos et al. (2012) in third grade students, Van Garderen (2006) in 11-12-year-old students, and Lowrie and Kay (2001) in 11-13-year-old students. The last study also found that the choice of problem-solving method and the difficulty of the problem had a greater effect on mathematical thinking processes than gender. Additionally, studies by Hacıömeroğlu and Hacıömeroğlu (2017), Hacıomeroglu et al. (2013), Swoyer (2012), and Haciomeroglu and Chicken (2012) have investigated the differences between visual and analytical thinking and found that gender is independent of students' visual or analytical preferences.

This situation can be explained by the fact that boys and girls attend mathematics courses together, have equal opportunities, undergo similar processes, and are equally affected by their teacher's problem-solving processes. Lowrie and Clements (2001) suggest that if class teachers can provide solution strategies in various visual and analytical forms and model them appropriately, their students can gain a powerful repertoire of solution methods. It is essential for teachers to understand the differences in problem-solving preferences and thinking processes among students, regardless of their gender, because the impact of teachers on the choice of solution method of students cannot be ignored. As verbal problem-solving requires an adequate level of reading skills, students should be at least in the third grade to solve such problems (Csíkos et al., 2012). Therefore, teachers should provide problems to students that enable them to use different solution strategies. According to Presmeg (1986b), teachers play an important role in improving the ability of students to represent verbal problems in various forms and developing their mathematical thinking. Therefore, it is crucial for teachers to provide a variety of solution strategies and activities that help students develop a strong repertoire of problem-solving methods, which can improve their performance in verbal problem-solving tasks. Additionally, teachers should be aware of the impact of their teaching practices and classroom activities on students' use of visual representations in problem-solving. Overall, this highlights the importance of teachers in fostering the development of students' mathematical thinking and problem-solving abilities, regardless of their gender.

Do the visual estimation skills of the students differ according to gender?

In the present study, there was no significant difference in the visual estimation skills of students based on their gender. Although the mean score of the boys was slightly higher than that of the girls, the difference was not statistically significant. This finding is consistent with the results of a study conducted by Kayhan-Altay (2010) on numerical sense, which also found no significant difference based on gender. Other studies on fraction estimation (Aytekin and Toluk-Uçar, 2014) and statistical estimation skills (Tekinkir, 2008) have also reported no effect of gender. These findings suggest that visual estimation and other related skills are not significantly influenced by gender and can be developed equally by both boys and girls with appropriate training and instruction.

The studies conducted by Reys et al. (1991), Lewis (1994), and Hanson and Hogan (2000) investigated the effect of gender on estimation skills across different age groups and found that gender did not have any significant effect on estimation skills for either boys or girls. The Trends in International Mathematics and Science Study conducted in 2003 also found no gender differences in estimation skills (Mullis et al., 2004). Yun-Hing (2007) also found no significant difference in sense of number skills between genders in the teaching of numerical estimation. However, LeFevre et al. (2013) reported that boys at the second, third, and fourth grade levels performed better than girls in the estimation of the number line. Additionally, Brydges (2013) found that age and gender were significant factors in the visual estimation of angles among eighth grade and university level students.

As a result, no gender-related differences were observed in the present study for students' visual estimation skills. To date, no research has been conducted on the effect of gender on students' visual estimation skills. Any differences found in previous studies can be attributed to factors other than gender. Therefore, it is important to avoid making assumptions about students' abilities based on their gender and instead focus on providing all students with opportunities to develop their skills in mathematics and estimation through effective teaching practices.

Do the visual estimation skills of the students differ according to their mathematical thinking processes?

According to the results of the present study, visual estimation skills are influenced by students' thinking processes, with those who are capable of visual thinking being more successful in estimation. This finding is consistent with previous research by Chrysostomou et al. (2013) and Patton and Santos (2012), in which spatial and visual thinking individuals were more successful in estimation. However, due to the limited number of studies on visual estimation skills, it is difficult to draw comparisons between this study and others. Nonetheless, the results highlight the importance of providing opportunities for all students to develop their visual thinking skills, which can lead to improvements in their estimation abilities.

As a result, it is assumed that the visual estimation skills of students thinking visually can be higher. The supposed reason for this is that children can easily compare quantities, predict distances, and determine which items belong to a cluster. They can learn about the environment and area, size and scale, and symmetry. Representations of size can be said to be effective in the development of the visual estimation skills of students. In doing so, students are asked to numerically predict information in the visuals that are parallel to each other or that are presented sequentially through visual methods. Additionally, students can develop strategies to interpret quantitative information presented in charts and to count money.

Conclusion

The finding that students who tend toward visual thinking had higher scores on the visual estimation skills test than those who tend toward analytical or harmonic thinking is important for educators and instructional designers. It suggests that students with a visual thinking style may benefit from visual aids and demonstrations in the learning process, and that incorporating more visual elements into instruction may help these students better understand and retain the material. This knowledge could be used to design more effective and inclusive teaching materials and methods, ultimately leading to better academic outcomes for students with different thinking styles. Additionally, understanding the relationship between thinking style and performance on visual estimation tasks may have implications for careers or fields that require strong visual-spatial skills, such as engineering or design.

The study's findings emphasize the importance of understanding individual differences in thinking styles and their relationship to learning outcomes. Educators can use this knowledge to design more personalized and effective instruction for students. For example, students with a tendency toward analytical thinking may benefit from more structured and logical approaches, while students with a tendency toward visual thinking may benefit from more creative and intuitive approaches. By recognizing and accommodating these differences, educators can help students develop their strengths and overcome their weaknesses, leading to better overall academic achievement. Moreover, understanding the relationship between thinking style and performance on visual estimation tasks can help students identify their strengths and weaknesses, and choose careers or fields that align with their strengths and interests. Overall, this research contributes to our understanding of the complex relationship between thinking style and academic achievement, highlighting the importance of individual differences in designing effective teaching and learning environments.

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References

Adams, L., Onslow, B., Edmunds, G., Chapple, N., & Waters, J. (2005). Children's development of range based estimation skills: Far more than guess work. *Proceedings of the Third International Conference on Education*, Honolulu, HA.

Ansari, D., Donlan, C., & Karmiloff-Smith, A. (2007). Typical and atypical development of visual estimation abilities. *Cortex; A Journal Devoted to the Study of the Nervous System and Behavior*, *43*(6), 758–768. https://doi.org/10.1016/s0010-9452(08)70504-5

Aytekin, C., & Toluk-Uçar, Z. (2014). Ortaokul öğrencilerinin kesirlerde tahmin becerilerinin incelenmesi [Investigation of middle school students' estimation ability with fractions]. *İlkögretim Online*, *13*(2), 546–563.

Bakker, M., Torbeyns, J., Verschaffel, L., & De Smedt, B. (2022). The mathematical, motivational, and cognitive characteristics of high mathematics achievers in primary school. *Journal of Educational Psychology*, *114*(5), 992–1004. https://doi.org/10.1037/edu0000678

Brydges, C. E. (2013). *Students' visual estimation of angles and their proficiency with angular measurement tools* [Doctoral Dissertation]. State University of New York at Fredonia.

Büyüköztürk, Ş. (2011). *Sosyal bilimler için veri analizi el kitabı [Data analysis handbook for social sciences]*, 14. Pagem Akademi.

Chrysostomou, M., Pitta-Pantazi, D., Tsingi, C., Cleanthous, E., & Christou, C. (2013). Examining number sense and algebraic reasoning through cognitive styles. *Educational Studies in Mathematics*, *83*(2), 205–223. https://doi.org/10.1007/s10649-012-9448-0

Çilingir-Altıner, E. (2018). İlkokul dördüncü sınıf öğrencilerinin matematiksel düşünme profillerine göre görsel tahmin ile uzamsal akıl yürütme becerilerinin ve problem çözme performanslarının incelenmesi. [Examination of 4th grade students' visual estimation - spatial reasoning skills and problem-solving performances based on mathematical thinking profiles] [Doctoral Dissertation] Marmara University, Institute of Education Sciences, İstanbul.

Creswell, J. W. (2014). *Research design: Qualitative, quantitative and mixed methods approaches* (4th ed.). Sage.

Csíkos, C., Szitányi, J., & Kelemen, R. (2012). The effects of using drawings in developing young children's mathematical word problem solving: A design experiment with third-grade Hungarian students. *Educational Studies in Mathematics*, *81*(1), 47–65. https://doi.org/10.1007/s10649-011-9360-z

El Mouhayar, R., & Jurdak, M. (2016). Variation of student numerical and figural reasoning approaches by pattern generalization type, strategy use and grade level, International. *International Journal of Mathematical Education in Science and Technology*, *47*(2), 197–215. https://doi.org/10.1080/0020739X.2015.1068391

Gal, H., & Linchevski, L. (2010). To see or not to see: Analyzing difficulties in geometry from the perspective of visual perception. *Educational Studies in Mathematics*, *74*(2), 163–183. https://doi.org/10.1007/s10649-010-9232-y

Galindo-Morales, E. (1994). *Visualization in the calculus class: Relationship between cognitive style, gender, and use of technology* [Doctoral dissertation]. The Ohio State University. ABD.

Haciomeroglu, E. S., & Chicken, E. (2012). Visual thinking and gender differences in high school calculus. *International Journal of Mathematical Education in Science and Technology*, *43*(3), 303–313. https://doi.org/10.1080/0020739X.2011.618550

Hacıomeroglu, E. S., Chicken, E., & Dixon, J. K. (2013). Relationships between gender, cognitive ability, preference, and calculus performance. *Mathematical Thinking and Learning*, *15*(3), 175–189. https://doi.org/10.1080/10986065.2013.794255

Hacıömeroğlu, G., & Hacıömeroğlu, E. S. (2013). Matematik İşlem Testini Türkçe'ye uyarlama çalışması ve öğretmen adaylarının matematik problemlerini çözme tercihleri [Turkish adaptation of the Mathematical Processing Instrument and pre-service teachers' problem solving preferences]. *Journal of Theoretical Educational Science*, *6*(2), 196–213.

Hacıömeroğlu, G., & Hacıömeroğlu, E. S. (2017). Cinsiyet, uzamsal beceri, mantıksal düşünme becerisi ve çözüm tercihleri arasındaki ilişkinin incelenmesi [Examining the relationship between gender, spatial ability, logical reasoning ability, and preferred mode of processing]. *Adıyaman Üniversitesi Eğitim Bilimleri Dergisi*, *7*(1), 116–131. https://doi.org/10.17984/adyuebd.310833

Hanson, S. A., & Hogan, T. P. (2000). Computationalestimation skill of college students. *Journal for Research in Mathematics Education*, *31*(4), 483–499. https://doi.org/10.2307/749654

Healey, C. G., Booth, K. S., & Enns, J. T. (1996). High-speed visual estimation using preattentive processing. *ACM Transactions on Computer-Human Interaction*, *3*(2), 107–135. https://doi.org/10.1145/230562.230563

Jones, M. G., Gardner, G. E., Taylor, A. R., Forrester, J. H., & Andre, T. (2012). Students' accuracy of measurement estimation: Context, units, and logical thinking. *School Science and Mathematics*, *112*(3), 171–178. https://doi.org/10.1111/j.1949-8594.2011.00130.x

Kayhan-Altay, M. (2010). *İlköğretim ikinci kademe öğrencilerinin sayı duyularının; sınıf düzeyine, cinsiyete ve sayı duyusu bileşenlerine göre incelenmesi [Investigation number sense test achievements of middle school students according to different variables]* [Doctoral Dissertation]. Hacettepe University.

Krutetskii, V. A. (1976). *The psychology of mathematical abilities in schoolchildren*. University of Chicago Press.

LeFevre, J. A., Jimenez Lira, C., Sowinski, C., Cankaya, O., Kamawar, D., & Skwarchuk, S. L. (2013). Charting the role of the number line in mathematical development. *Frontiers in Psychology*, *4*(641), 641. https://doi.org/10.3389/fpsyg.2013.00641

Leutzinger, L. P., Rathmell, E. C., & Urbatsch, T. D. (1986). Developing estimation skills in the primary grades. In H. L. Schoen & M. J. Zweng (Eds.), *Estimation and mental computation: 1986 yearbook* (pp. 82–92). National Council of Teachers of Mathematics.

Lewis, J. C. (1994). The effect of Context and Gender on Assessment of Estimation. *Paper presented at the Annual Meeting of the Orleans*, LA, April 5–7, 1994.

Lowrie, T., & Clements, M. K. (2001). Visual and nonvisual processes in Grade 6 students' mathematical problem solving. *Journal of Research in Childhood Education*, *16*(1), 77–93. https://doi.org/10.1080/02568540109594976

Lowrie, T., & Kay, R. (2001). Relationship between visual and nonvisual solution methods and difficulty in elementary mathematics. The Journal of Educational Research, 94(4), 248–255. https://doi.org/10.1080/00220670109598758

Markovits, Z., & Hershkowitz, R. (1993). Visual estimation of discrete quantities. *ZDM*, *93*(4), 137–140.

Markovits, Z., & Hershkowitz, R. (1997). Relative and absolute thinking in visual estimation processes. *Educational Studies in Mathematics*, *32*(1), 29–47. https://doi.org/10.1023/A:1002911812669

Montenegro, P., Costa, C., & Lopes, B. (2018). Transformations in the visual representation of a figural pattern. *Mathematical Thinking and Learning*, *20*(2), 91–107. https://doi.org/10.1080/10986065.2018.1441599

Mullis, I. V., Martin, M. O., Gonzalez, E. J., & Chrostowski, S. J. (2004). *TIMSS 2003 international mathematics report: Findings from IEA's trends in international mathematics and science study at the fourth and eighth grades*. TIMSS & PIRLS International Study Center. 140 Commonwealth Avenue, Chestnut Hill, MA 02467.

National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and standards for school Mathematics*. National Council of Teachers of Mathematics.

Patton, B., & Santos, E. D. (2012). Analyzing algebraic thinking using "guess my number" problems. *International Journal of Instruction*, *5*(1), 1–18.

Pilten, P. (2008). *Üstbiliş stratejileri öğretiminin ilköğretim beşinci sınıf öğrencilerinin matematiksel muhakeme becerilerine etkisi [The effect of metacognitive instruction on mathematical reasoning of fifth grade primary school students]* [Doctoral Dissertation]. Gazi University.

Presmeg, N. C. (1986a). Visualization in high school mathematics. *For the Learning of Mathematics*, *6*(3), 42–46.

Presmeg, N. C. (1986b). Visualisation and mathematical giftedness. *Educational Studies in Mathematics*, *17*(3), 297–311. https://doi.org/10.1007/BF00305075

Reys, B. J., Reys, R. E., & Penafiel, A. F. (1991). Estimation performance and strategy use of Mexican 5th and 8th grade student sample. *Educational Studies in Mathematics*, *22*(4), 353–375. https://doi.org/10.1007/BF00369295

Segovia, İ., & Castro, E. (2009). Computational and measurement estimation; curriculum foundations and research carried out at the University of Granada. *Electronic Journal of Research in Educational Psychology*, *17*(7), 499–536.

Seng, S., & Chan, B. (2000). Spatial ability and mathematical performance: Gender differences in an elementary school. *Nature Biotechnology*, *18*, 1087.

Sevimli, E. (2013). *Bilgisayar Cebiri Sistemi destekli öğretimin farklı düşünme yapısındaki öğrencilerin integral konusundaki temsil dönüşüm süreçlerine etkisi [The effect of a computer algebra system supported teaching on processes of representational transition in integral topics of students with different types of thinking]* [Doctoral Dissertation]. Marmara Üniversity.

Siegler, R. S. (2016). Magnitude knowledge: The common core of numerical development. *Developmental Science*, *19*(3), 341–361. https://doi.org/10.1111/desc.12395

Suwarsono, S. (1982). *Visual imagery in the mathematical thinking of seventh grade students* [Doctoral Dissertation]. Monash University.

Swoyer, L. (2012). AP student visual preferences for problem solving [Honors Theses]. The Florida State University Department of Statistics.

Taşova, H. İ. (2011). *Matematik öğretmen adaylarinin modelleme etkinlikleri ve performansi sürecinde düşünme ve görselleme becerilerinin incelenmesi [Investigating thinking and visualisation skills of pre-service mathematics teachers in modelling activities and performance]* [Master of Dissertation]. Marmara University.

Tekinkir, D. (2008). *İlköğretim 6. ve 8. sinif öğrencilerinin matematik alanindaki tahmin stratejilerini belirleme ve tahmin becerisi ile matematik başarisi arasindaki ilişki [To determine the estimate strategies in maths field for the primary school studens of 6th-8th grades and the relation between the estimate ability and success for maths]* [Doctoral Dissertation]. Dokuz Eylül University.

Van Garderen, D. (2006). Spatial visualization, visual imagery, and mathematical problem solving of students with varying abilities. *Journal of Learning Disabilities*, *39*(6), 496–506. https://doi.org/10.1177/00222194060390060201

Veneziano, L., & Hooper, J. (1997). A method for quantifying content validity of health-related questionnaires. American Journal of Health Behavior, 21(1), 67–70.

Wai, J., Lubinski, D., & Benbow, C. P. (2009). Spatial ability for STEM domains: Aligning over 50 years of cumulative psychological knowledge solidifies itsimportance. *Journal of Educational Psychology*, *101*(4), 817–835. https://doi.org/10.1037/a0016127

Yun-Hing, L. (2007). *The relationship between numerical estimation and number sense in students' learning of mathematics* [Master of Dissertation]. The University of Hong Kong. Retrieved August 13, 2018, from http://hdl.handle.net/10722/51257.

Yurdugül, H. (2005). Ölçek geliştirme çalışmalarında kapsam geçerliği için kapsam geçerlik indekslerinin kullanılması [The use of content validity indices for content validity in scale development studies]. *XIV. Ulusal Eğitim Bilimleri Kongresi*, *1*, 771–774.