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JORDAN TYPE MAPPINGS IN RINGS AND ALGEBRAS

Shakir Ali

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Abstract:

Let R be an associative ring. For any $x,\ y \in R$, as usual the symbols $x \circ y$ and $[x, y]$ will denote the anti-commutator $xy + yx$ and commutator $xy - yx$ and called Jordan product and Lie product, respectively. Recall that an additive map $D: R \to R$ is called a derivation if $D(xy) = D(x)y + xD(y)$ holds for all $x, y \in R$. Following [I. N. Herstein, Proc. Amer Math. Soc. 8 (1957), 1104-1110], an additive map $D: R \to R$ is called a Jordan derivation if $D(x^2) = D(x)x + xD(x)$ holds for all $x \in R$. Such a map, we call Jordan type. An additive map $D: R \to R$ is called a Jordan *-derivation if $D(x^2) = D(x)x^* + xD(x)$ holds for all $x \in R$, where R is a ring with involution '*'. For an element $a \in R$, it is easy to verify that the map $D: x \to xa - ax^*$ for all $x \in R$, is a Jordan *-derivation. Such D is called an inner Jordan *-derivation (viz.; [S. Ali and N. A. Dar, Comm. Algebra 49(4) (2021), 1422-1430] and [T. K. Lee and Y. Zhou, J. Algebra Appl. 13(4)(2014)] for details and recent results).

In this talk, I will review some recent results of myself and collaborations in certain class of rings and algebras. Moreover, some examples and counter examples will be discussed for questions raised naturally. Finally, we conclude our talk with some open problems.

Keywords: Prime ring, semiprime ring, derivation, Jordan derivation
General area of research: Mathematics
ICFAS2022-ID: 1006

OSCILLATIONS DRIVEN BY DIFFUSION IN A SCALAR HEAT EQUATION

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Abstract:

In this talk we will present a general mechanism whereby diffusion causes the onset of stable oscillations in a simple nonlinear scalar heat equation that can be interpreted as a single-input single-output feedback control system. We call it a parabolic oscillator. We show that the trivial solution solution looses its stability due to a Hopf bifurcation. The discussion includes a detailed spectral analysis of the non self-adjoint linearization, which takes the form of a rank 1 non-symmetric perturbation of the Laplacian.

Keywords: Scalar heat equation
General area of research: Mathematics
ICFAS2022-ID: 1009

ON FRACTIONAL INTEGRAL TRANSFORMS: EXTENSIONS AND APPLICATIONS

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Abstract:

Integral transforms have a long, rich history that extends over 200 years. They have ubiquitous applications in mathematics, statistics, physics, engineering, and economics. The introduction of the fractional Fourier transform in the early 1980's and the myriads of its applications in signal processing and optics in the 1990's, gave birth to a novel notion in mathematical analysis known as fractional integral transforms. In this talk we shall give an overview of fractional integral transforms, their recent extensions and applications.

Keywords: Fractional integral transforms
General area of research: Mathematics
ICFAS2022-ID: 1010

A SURVEY ON BI-ORTHOGONAL POLYNOMIALS AND FUNCTIONS

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Abstract:

The theory of orthogonal polynomials is well established and detailed, covering a wide field of interesting results, as in particular for solving certain differential equations. On the other side the concepts and the related formalism of the theory of bi-orthogonal polynomials is less developed and much more limited. By starting from the orthogonality properties satisfied from the ordinary and generalized Hermite polynomials is possible to derive a further family (known in literature) of these kind of polynomials which are bi-orthogonal with their adjoint. This aspect allows us to introduce functions recognized as bi-orthogonal and investigate generalizations of families of orthogonal polynomials.

Keywords: Orthogonal polynomials, bi-orthogonal functions, generating functions
General area of research: Mathematics
ICFAS2022-ID: 1013

ON NONLINEAR LIE (JORDAN)-TYPE DERIVATIONS OF ALGEBRAS

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Abstract:

Let R be a commutative ring with identity and A be an algebra over R. An R-linear mapping $\delta : A \to A$ is called a derivation if $\delta(x_1x_2) = \delta(x_1)x_2 + x_1\,\delta(x_2)$ holds for all $x_1, x_2 \in A$. Let $[x_1, x_2] = x_1x_2 - x_2x_1$ denote the commutator of elements $x_1, x_2 \in A$. An R-linear mapping $L : A \to A$ is said to be a Lie derivation (resp. Lie triple derivation) if $L([x_1, x_2]) = [L(x_1), x_2] + [x_1, L(x_2)]$ (resp. $L([[x_1, x_2], x_3]) = [[L(x_1), x_2], x_3] + [[x_1, L(x_2)], x_3] + [[x_1, x_2], L(x_3)]$) holds for all $x_1, x_2, x_3 \in A$. For any $x_1, x_2 \dots x_n \in A$, define $p_1(x_1) = x_1, p_2(x_1, x_2) = [x_1, x_2]$ and $p_n(x_1, x_2, \dots, x_n) = [p_{n-1}(x_1, x_2, \dots, x_{n-1}), x_n]$for all integers $n \geq 2$. An R-linear mapping $\delta : A \to A$ is called a Lie n-derivation if $\delta(p_n(x_1, x_2, \dots, x_n)) = p_n(\delta(x_1), x_2, \dots, x_n) + p_n(x_1, \delta(x_2), \dots, x_n) + \cdots + p_n(x_1, x_2, \dots, \delta(x_n))$ for all $x_1, x_2 \dots x_n \in A$. In particular, a Lie 2-derivation is called a Lie derivation and a Lie 3-derivation is said to be a Lie triple derivation. Lie 2-derivations, Lie 3-derivations and Lie n-derivations are collectively referred to as Lie-type derivations. Analogously, the notion of Jordan-type derivations can be defined. In the year 2000 , Cheung initiated the study of linear maps on triangular algebras. He described Lie derivations, commuting maps and automorphisms of triangular algebras. Motivated by work of Cheung, linear as well nonlinear mappings on val;'ious algebras have been studied by many authors. Yu and Zhang [Linear Algebra Appl., 432 (2010), 2953-2960] initiated the study of nonlinear Lie derivations of triangular algebras. Ji et al. [Linear Multilinear Algebra, 60(10) (2012), 1155-1164) obtained a similar conclusion for nonlinear Lie triple derivations . Ashraf and Jabeen [Comm. Algebra, 45 (2017), 4380-4395] extended the above results for nonlinear generalized Lie triple derivations of triangular algebra and proved that under certain as-sumptions every nonlinear generalized Lie triple derivation on triangular algebras is the sum of an additive generalized derivation and a mapping from the triangular algebra into its center that vanishes on all second commutators . In the present talk, we shall present an up-to-date account of the wprk done in this direction. In fact, characterization and structure of these mappings on various algebras will be presented.

Keywords: Commutative ring, derivation, Lie- derivation, Lie triple derivation
General area of research: Mathematics
ICFAS2022-ID: 1108

ALGORITHMS FOR SOLVING A PARTIALLY PERIODIC OPTIMAL CONTROL PROBLEM WITH INITIAL CONTROL ACTIONS (CONTINUOUS AND DISCRETE CASES)

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Abstract:

The paper considers a partially periodic optimal control problem, where the control parameter is included in the initial condition. Here both continuous and discrete optimal control problems are analyzed. In both cases, the corresponding Euler-Lagrange equations are obtained, with the help of which an algorithm is developed for finding the optimal program trajectory and control. The results are illustrated by continuous and discrete examples, when the motion is described by a time-averaged hyperbolic equation at a sufficiently long well depth.

As is known, in order to find the optimal solution for the gas-lift operation of oil wells, a mathematical model of the gas-lift process is constructed in [1, 2], which is described by a system of linear partial differential equations. To simplify, the problem of optimizing the equation in partial derivatives using averaging over time [3] or over the depth of the well [4] is reduced to a system of ordinary differential equations, on the basis of which the optimization problem is posed. Note that in [3], the resulting nonlinear differential equation is used to develop an algorithm for calculating the hydraulic resistance of tubing. In [5], on the basis of averaged equations, an optimization problem with a periodic boundary condition and boundary control in gas-lift wells is considered. However, it should be clarified that in this problem the control actually enters not into the boundary conditions, but into the initial conditions. On the other hand, the periodicity condition connects solutions not at the ends of the segment, but the middle and end points. Therefore, we consider not a problem with a periodic boundary condition, but the so-called problem with a partially periodic boundary condition.

Note that [3, 4] the motion in the gas-lift process is described either by differential or by finite-difference equations, where such a description complicates the development of a "homogeneous" algorithm. And this creates difficulties in obtaining a solution that requires sufficient accuracy [2].

In this paper, we first study a continuous problem of optimization , where the motion of an object on a segment is described by various differential equations on intervals and, respectively, and at a point the solution satisfies to finite-difference equations. In addition, the middle (l) and end (2l) points are connected by a periodic condition. Further, the paper considers the problem of periodic optimal control , discrete on part . The paper investigates this problem and proposes an algorithm for solving this problem. To do this, both in the continuous and discrete cases, using the continuous or discrete Euler-Lagrange equations [6], solutions to these problems are given. Then, for each case, numerical examples are considered that arise during the control of the gas-lift process, the results of the solutions of which are illustrated. Thus, the study and solution of such problems is an urgent problem. On a specific example, the gas lift optimization problem is solved by a numerical method.

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Keywords: Optimal control problem
General area of research: Mathematics
ICFAS2022-ID: 1123

THE NONLOCAL ABSTRACT SCHRÖDINGER EQUATIONS AND APPLICATIONS

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Abstract:

This talk, that I would like to state today, devoted to the existence, uniqueness and L^p-regularity properties of Cauchy problem for nonlocal Schrodinger equations. The existence and regularity properties of solutions of Cauchy problem for Schrödinger equations (SE) studied e.g in [1, 4] and the references therein. The construction of general solutions of nonlocal SE were studied e.g. in [5-7]. Also, the existence and uniqueness of solutions of Cauchy problem for abstract SE were investigated in In [8, 9]. Here, the Cauchy problem for linear and nonlinear nonlocal Schrödinger equations are studied. The equation involves a convolution integral operators with a general kernel operator functions whose Fourier transform are operatör functions defined in a Banach space E together with some growth conditions. By assuming enough smoothness on the initial data and the operator functions, the local and global existence and uniqueness of solutions are established. We can obtain a di¤erent classes of nonlocal Schrödinger equations by choosing the space E and linear operators, which occur in a wide variety of physical systems The aim here, is to study the existence, uniqueness and regularity properties of solution of the initial value problem (IVP) for nonlocal nonlinear Schrödinger equat¬on (NSE),

$$i\partial_t + a\Delta u + A * u = B * f(u), \qquad t \in (0, T), \qquad x \in \mathbb{R}^n$$
$$u(x, 0) = \varphi(x) \text{ for a.e. } x \in \mathbb{R}^n$$

where $A = A(x), B = B(x)$ are linear and nonlinear operator functions in a Hilbert space H, respectively, a is a complex number, $T \in (0, \infty], f(u)$ is a given nonlinear function and $\varphi(x)$ is a given E-valued functions. The method of proofs here, naturally differs from those used in previous works. Indeed, since the problem includes an abstract operator in the leading part and the problem is considered in E-valued L^p -spaces, we need some extra mathematics tools for deriving considered conclusions. For this reason, in the proof we use modern analysis tools like the following: (1) operator-valued Fourier multiplier theorems in abstract L^p spaces; (2) Embedding and trace theorems in Banach space valued Sobolev-Lions and Besov-Lions spaces; (3) Theory of semigroups of linear operators in Banach spaces; (4) Perturbation theory of operators;(5) Interpolation of Banach Spaces, and etc.

Keywords: Schrödinger equations
General area of research: Mathematics
ICFAS2022-ID: 1124

HYPER-LEONARDO POLYNOMIALS

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Abstract:

In this paper, we define hyper-Leonardo polynomials and present some of their properties such as the recurrence relations, summation formulas and generating function. We also investigate some combinatorial properties of hyper-Leonardo polynomials and give the relation to the Leonardo Pisano polynomials.

Keywords: Leonardo sequences: Leonardo Pisano polynomials.
General area of research: Mathematics
ICFAS2022-ID: 1040

1. INTRODUCTION

The integer sequences have been the subject of many studies in science and technology. The most famous integer sequence is called Fibonacci sequence and defined by the following recurrence relation ($n \geq 1$) [7]

$$F_{n+1} = F_n + F_{n-1} \text{ with } F_0 = 0, F_1 = 1. \tag{1.1}$$

Leonardo sequence, which has similar properties to the Fibonacci sequence, is defined by Catarino and Borges [4], as follows:

$$Le_n = Le_{n-1} + Le_{n-2} + 1 \qquad (n \geq 2) \tag{1.2}$$

with the initial conditions $Le_0 = Le_1 = 1$. In recent years, there have been many studies on Leonardo numbers [1-3,8-11]. Although it is seen that the name "Leonardo numbers" is widely used in the literature, Kürüz et al. [8] preferred to call them "Leonardo Pisano numbers" and defined Leonardo Pisano polynomials by the formula

$$Le_n(x) = \begin{cases} 1, & n = 0,1 \\ x + 2, & n = 2 \\ 2xLe_{n-1}(x) - Le_{n-3}(x), & n \geq 3. \end{cases} \tag{1.3}$$

The generating function for the Leonardo Pisano polynomials is [8]

$$g_{Le_n(x)}(\lambda) = \frac{1 + (1 - 2x)\lambda + (2 - x)\lambda^2}{1 - 2x\lambda + \lambda^3}. \tag{1.4}$$

Hyper-Leonardo numbers $Le_n^{(r)}$, are defined as a generalization of the Leonardo numbers, by the formula

$$Le_n^{(r)} = \sum_{s=0}^{n} Le_s^{(r-1)} \quad \text{with} \quad Le_n^{(0)} = Le_n, \quad Le_0^{(r)} = Le_0 \quad \text{and} \quad Le_1^{(r)} = r+1, \tag{1.5}$$

where r is a positive integer and Le_n is the nth Leonardo number [9]. The hyper-Leonardo numbers have the following recurrence relation for $n \geq 1$ and $r \geq 1$:

$$Le_n^{(r)} = Le_{n-1}^{(r)} + Le_n^{(r-1)} \tag{1.6}$$

and generating function [9]:

$$g(r) = \sum_{n=0}^{\infty} Le_n^{(r)} t^n = \frac{1-t+t^2}{(1-2t+t^3)(1-t)^r}. \tag{1.7}$$

The aim of this paper is to introduce hyper-Leonardo polynomials and to investigate some algebraic and combinatorial properties of these polynomials such as the recurrence relation, summation formulas and generating function.

2. MAIN RESULTS

Definition 2.1. Hyper-Leonardo polynomials are defined as

$$Le_n^{(r)}(x) = \sum_{s=0}^{n} Le_s^{(r-1)}(x) \text{ with } Le_n^{(0)}(x) = Le_n(x), \quad Le_0^{(r)}(x) = 1, \quad Le_1^{(r)}(x) = r+1 \tag{2.1}$$

where $Le_n(x)$ is the n-th Leonardo Pisano polynomial.

The first few hyper-Leonardo polynomials are

$$\begin{aligned} Le_2^{(1)}(x) &= x+4, \\ Le_3^{(1)}(x) &= 2x^2+5x+3, \\ Le_4^{(1)}(x) &= 4x^3+10x^2+3x+2, \\ Le_5^{(1)}(x) &= 8x^4+20x^3+6x^2 \end{aligned}$$

and

$$\begin{aligned} Le_2^{(2)}(x) &= x+7, \\ Le_3^{(2)}(x) &= 2x^2+6x+10, \\ Le_4^{(2)}(x) &= 4x^3+12x^2+9x+12, \\ Le_5^{(2)}(x) &= 8x^4+24x^3+18x^2+9x+12. \end{aligned}$$

Note that, for $x = 1$, hyper-Leonardo polynomials give the hyper-Leonardo numbers.

Definition 2.1 yields that hyper-Leonardo polynomials have the following recurrence relation for $r \geq 1$ and $n \geq 1$:

$$Le_n^{(r)}(x) = Le_{n-1}^{(r)}(x) + Le_n^{(r-1)}(x). \tag{2.2}$$

Theorem 2.1. The generating function for the hyper-Leonardo polynomials is as follows:

$$g(r)=\sum_{n=0}^{\infty}Le_n^{(r)}(x)t^n=\frac{1+(1-2x)t+(2-x)t^2}{(1-2xt+t^3)(1-t)^r}. \tag{2.3}$$

Proof. We use the mathematical induction method on r. Since

$$g(0)=\sum_{n=0}^{\infty}Le_n^{(0)}(x)t^n=\frac{1+(1-2x)t+(2-x)t^2}{(1-2xt+t^3)(1-t)^0}=\sum_{n=0}^{\infty}Le_n(x)t^n,$$

the result is true for $r=0$. Now, assume the result is true for r and show that the result is true for $r+1$. Considering the Cauchy product, we have

$$\begin{aligned}g(r+1)=\sum_{n=0}^{\infty}Le_n^{(r+1)}(x)t^n &=\sum_{n=0}^{\infty}\left(\sum_{s=0}^{n}Le_n^{(r+1)}\right)t^n\\ &=\left(\sum_{i=0}^{\infty}Le_i^{(r)}(x)t^i\right)\sum_{j=0}^{\infty}t^j\\ &=\frac{1+(1-2x)t+(2-x)t^2}{(1-2xt+t^3)(1-t)^{r+1}}.\end{aligned}$$

Theorem 2.2. If $n\geq 1$ and $r\geq 1$, then the following summation formula gives the relation between the hyper-Leonardo polynomials and Leonardo Pisano polynomials:

$$Le_n^{(r)}(x)=\sum_{s=0}^{n}\binom{n+r-s-1}{r-1}Le_s(x). \tag{2.4}$$

Proof. For two real initial sequences (a_n) and (a^n), the entries of symmetric infinite matrix, a_n^r has the following recurrence relation [5]:

$$a_n^r=\sum_{i=1}^{r}\binom{n+r-i-1}{n-1}a_0^i+\sum_{s=1}^{n}\binom{n+r-s-1}{r-1}a_s^0. \tag{2.5}$$

By using the values $a_n^r=Le_n^{(r)}(x)$, equation (2.5) is of the form:

$$Le_n^{(r)}(x)=\sum_{i=1}^{r}\binom{n+r-i-1}{n-1}Le_0^{(i)}(x)+\sum_{s=1}^{n}\binom{n+r-s-1}{r-1}Le_s^{(0)}(x).$$

Considering the initial conditions in Definition 2.1, we have

$$\begin{aligned} Le_n^{(r)} &= \sum_{i=1}^{r} \binom{n+r-i-1}{n-1} + \sum_{s=1}^{n} \binom{n+r-s-1}{r-1} Le_s(x) \\ &= \sum_{i=0}^{r-1} \binom{n+r-i-2}{n-1} + \sum_{s=0}^{n-1} \binom{n+r-s-2}{r-1} Le_{s+1}(x) \\ &= \sum_{k=0}^{r-1} \binom{n+k-1}{n-1} + \sum_{b=0}^{n-1} \binom{r+b-1}{r-1} Le_{n-b}(x), \end{aligned}$$

where $k = r - i - 1$ and $b = n - s - 1$. If we use the following property of the combination in [6]:

$$\sum_{t=a}^{c} \binom{t}{a} = \binom{c+1}{a+1}, \tag{2.6}$$

we have

$$\begin{aligned} Le_n^{(r)}(x) &= \binom{n+r-1}{n} + \sum_{b=0}^{n-1} \binom{r+b-1}{r-1} Le_{n-b}(x) \\ &= \sum_{b=0}^{n} \binom{r+b-1}{r-1} Le_{n-b}(x), \\ &= \sum_{s=0}^{n} \binom{n+r-s-1}{r-1} Le_s(x). \end{aligned}$$

Theorem 2.3. For n ≥ 3 and r ≥ 1, the following recurrence relation is valid:

$$\begin{aligned} Le_n^{(r)} = 2xLe_{n-1}^{(r)}(x) - Le_{n-3}^{(r)}(x) + \binom{n+r-1}{r-1} - \binom{n+r-2}{r-1}(2x-1) \\ - \binom{n+r-3}{r-1}(x-2). \end{aligned} \tag{2.7}$$

Proof. Considering Theorem 2.2 and equation (1.3), we have

$$
\begin{aligned}
Le_n^{(r)}(x) &= \sum_{s=0}^{n} \binom{n+r-s-1}{r-1} Le_s(x) \\
&= \sum_{s=0}^{n} \binom{n+r-s-1}{r-1} \left(2xLe_{s-1}(x) - Le_{s-3}(x)\right) \\
&= 2x \sum_{s=-1}^{n-1} \binom{n+r-(s+1)-1}{r-1} Le_s(x) - \sum_{s=-3}^{n-3} \binom{n+r-(s+3)-1}{r-1} Le_s(x) \\
&= 2x \left(\sum_{s=0}^{n-1} \binom{(n-1)+r-s-1}{r-1} Le_s(x) + \binom{n+r-1}{r-1} Le_{-1}(x) \right) \\
&- \left(\sum_{s=0}^{n-3} \binom{(n-3)+r-s-1}{r-1} Le_s(x) + \binom{n+r-3}{r-1} Le_{-1}(x) + \binom{n+r-2}{r-1} Le_{-2}(x) \right. \\
&\qquad \left. + \binom{n+r-1}{r-1} Le_{-3}(x) \right) \\
&= 2xLe_{n-1}^{(r)}(x) - Le_{n-3}^{(r)}(x) + \binom{n+r-1}{r-1} - \binom{n+r-2}{r-1}(2x-1) - \binom{n+r-3}{r-1}(x-2).
\end{aligned}
$$

Theorem 2.4. If n ≥ 1 and r ≥ 1, then there is the summation formula for the hyper-Leonardo polynomials:

$$
\sum_{s=0}^{r} Le_n^{(s)}(x) = Le_{n+1}^{(r)}(x) + (1-2x)Le_n(x) + Le_{n-2}(x). \tag{2.8}
$$

Proof. By using Theorem 2.2 and equation (2.6), we get

$$
\begin{aligned}
\sum_{s=1}^{r} Le_n^{(s)}(x) &= \sum_{s=1}^{r} \left(\sum_{t=0}^{n} \binom{n+s-t-1}{s-1} Le_t(x) \right) \\
&= \sum_{t=0}^{n} \left(Le_t(x) \sum_{s=1}^{r} \binom{n+s-t-1}{s-1} \right) \\
&= \sum_{t=0}^{n} \binom{n+r-t}{r-1} Le_t(x) \\
&= \sum_{t=0}^{n+1} \binom{n+r-t}{r-1} Le_t(x) - Le_{n+1}(x).
\end{aligned}
$$

Then, we have

$$
\begin{aligned}
\sum_{s=0}^{r} Le_n^{(s)}(x) &= Le_{n+1}^{(r)}(x) - Le_{n+1}(x) + Le_n(x) \\
&= Le_{n+1}^{(r)}(x) + (1-2x)Le_n(x) + Le_{n-2}(x).
\end{aligned}
$$

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GENERALIZED SPACE-TIME AUTOREGRESSIVE AND ONLINE LEARNING IN TIME SERIES MODELING

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Abstract:

Big data and streaming data technology are progressing faster than ever. In this paper, two developments of time series modeling using AR (autoregressive) are addressed, Online AR for streaming data and Generalized Space-Time Autoregressive (GSTAR) for static space-time data. Online AR is developed using regret minimization technique from popular online convex optimization solvers. On the other hand, GSTAR is built by capturing the location information using weight matrices and relating observations from different locations with varying parameters.

Online AR is shown to have the performance asymptotically approaches the best AR model in hindsight and can adapt to changes in underlying parameters with fast training and prediction time. GSTAR can also model the inter-location influence to improve performance compared to the separated AR implementation for each location. While the performance of Online AR is not always better than AR (depends on data), its training time is much faster than AR and GSTAR.

Keywords: Generalized space-time autoregressive, online learning, regret, time series, streaming data
General area of research: Mathematics
ICFAS2022-ID: 1071

1. INTRODUCTION

In the GSTAR model, the observation for each location is expressed as a linear combination of the previous observations in various locations weighted by weight matrices. Unlike STAR (Cliff & Ord, 1975), GSTAR lets its parameters be different for each location, hence the name. Also recently, the era of streaming data becomes mainstream and online learning is relevant once again. By regret minimization technique, online learning is combined with time series modeling. This results in the adaptability of time series models to the change of properties of data. In this paper, some recent advancements in time series modeling will be reviewed, the application of the autoregressive model in online settings, and its generalization to GSTAR. Along with the basic AR, these models are applied to real data and their pros and cons are analyzed.

2. METHOD

For time order p and spatial order $\lambda_k, k = 1, \dots, p$, GSTAR$\left(p;\ \lambda_1, \lambda_2, \dots, \lambda_p\right)$ can be represented as

$$\boldsymbol{Y}_t = \sum_{k=1}^{p} \sum_{\ell=0}^{\lambda_k} \boldsymbol{\Phi}_{k\ell} \mathbf{W}^{(\ell)} \boldsymbol{Y}_{t-k} + \boldsymbol{\varepsilon}_t,$$

where $\boldsymbol{Y}_t$, $\boldsymbol{\Phi}_{k\ell}$, $\mathbf{W}^{(\ell)}$, and $\boldsymbol{\varepsilon}_t$ are observation vector, parameters, weight matrices, and residuals respectively, $\mathbf{W}^{(0)} = \mathbf{I}_N$, and $\boldsymbol{\varepsilon}_t \sim N(\mathbf{0}, \sigma^2 \mathbf{I}_N)$. By rearranging vector $\boldsymbol{Y}_t$ vertically, GSTAR can be written in an elegant linear form $\boldsymbol{Y} = \mathbf{X}\boldsymbol{\Phi} + \boldsymbol{\varepsilon}$ for some matrix $\mathbf{X}$. The method of GSTAR modeling includes model identification using STACF and STPACF, parameter estimation by implementing OLS estimation on the linear form, and diagnostic checking for stationary and residual tests.

Let $\mathcal{P} \subset \mathbb{R}^d$ be a non-empty closed convex set and f_t be a cost function (squared error). For each time t, $\hat{\boldsymbol{x}}_t \in \mathcal{P}$ is drawn by some algorithm $\mathcal{A}$ and regret in online learning is defined by

$$R_T(\mathcal{A}) = \sum_{t=1}^{T} f_t(\boldsymbol{x}_t, \hat{\boldsymbol{x}}_t) - \min_{x \in \mathcal{P}} \sum_{t=1}^{T} f_t(\boldsymbol{x}_t, \boldsymbol{x}).$$

Two online algorithms $\mathcal{A}$ are developed: AR-OGD and AR-ONS, such that $R_T(\mathcal{A})/T$ approaches zero as T approaches infinity, with the assumption every observation and diameter of $\mathcal{P}$ is bounded.

3. EXPERIMENTAL RESULTS

AR, AR-OGD, AR-ONS, and GSTAR are applied to the wind dataset (Haslett and Raftery, 1989), with two results considered: average error and time needed to train the model and do prediction.

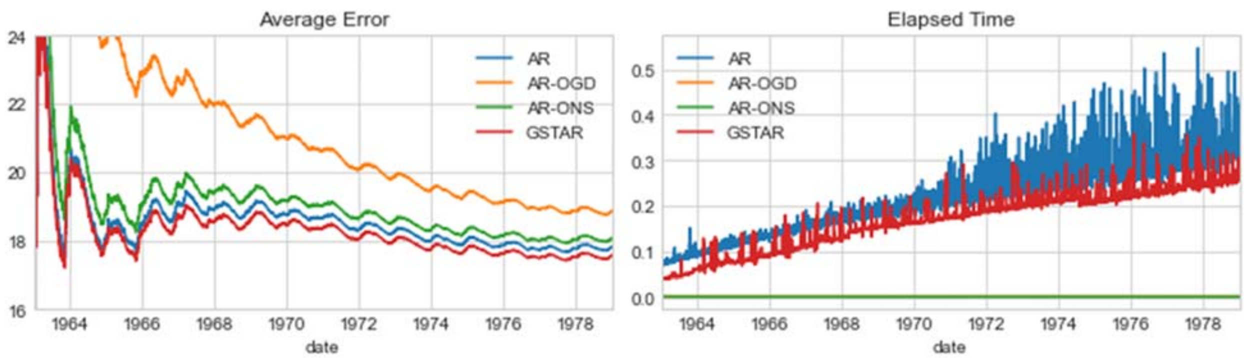

Fig 1. Experimental results

As expected, GSTAR can capture the information of locations and use it as leverage compared to the separated AR. The performances of AR-OGD and AR-ONS are not better than AR, indicating there's only a small disturbance in the nature of data which can be captured as the changes in model parameters. It's also been found that Online AR is much faster. Relative to AR-ONS, the time needed for AR-OGD is 1.2 ×, GSTAR is 40 × to 255 ×, and AR is 75 × to 316 ×.

4. CONCLUSIONS

For static data, GSTAR gives a performance boost to separated AR since it uses location information. For streaming data, AR-OGD and AR-ONS are much faster with the time needed doesn't depend on the number of observations that have been streamed, and are better if there is a significant disturbance of the data that can be represented as the changes of model parameters.

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ANALYSIS OF (G/M/c):(FIFO/c/∞) – ERLANG B QUEUEING SYSTEM ON BED AVAILABILITY FOR COVID-19 PATIENTS

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Abstract:

When the service capacity is unable to serve but customers keep arriving, it creates a queue that makes customers have to wait. Apart from waiting, customers can be rejected from the system (balking). The data used as customer arrivals is the increase in positive cases of COVID-19 patients in Arcamanik Sub City Area (SCA) of Bandung, Indonesia, from September 17th, 2020 to February 4th, 2021. Arcamanik SCA is often included in one of the highest positive cases of COVID-19 in Bandung City. The server used is the number of beds in the Hermina Arcamanik Hospital. The method used is a discrete event simulation, with a queuing model (G / M / c):(FIFO / c / ∞)- Erlang B. The more servers, the chances of customers being rejected will converge to zero. The minimum required bed is 32 beds, so patients who come are not rejected. Refused patients can visit general hospital as an alternative, namely RSHS. The proportion of beds for patients from Arcamanik SCA is 0,16 times the excess empty beds. For patients to receive treatment at the two hospitals, the number of cases COVID-19 must not exceed 62 cases

Keywords: COVID-19, queueing model, numerical simulation, probability distribution, bed availability.
General area of research: Mathematics
ICFAS2022-ID: 1072

1. INTRODUCTION

Waiting is an activity that is often encountered in daily life, one of which is in hospital services. Of course, the waiting time is not expected to be too long. In this final project, the observed event was the increase in cases of COVID-19 patients at Arcamanik SCA, in a period of approximately 20 weeks, from September 2020 to February 2021. The data was obtained from the Bandung City COVID-19 Information Center website. Arcamanik SCA, particularly Arcamanik and Antapani Subdistricts, is an area that is often included in the 10 sub-districts with the highest positive cases of COVID-19 in Bandung City. The positive COVID-19 patients at Arcamanik SCA generally want to receive treatment at the nearest hospital in Arcamanik SCA, namely Hermina Arcamanik Hospital. However, when compared, it turns out that the number of positive cases of COVID-19 at Arcamanik SCA is more than the data on the number of empty beds at the Hermina Arcamanik Hospital obtained from the Bandung City Government.

2. METHOD

Queuing model (M/M/c):(FIFO/c/∞) or commonly called Erlang B, occurs if the arrivals are unlimited but the service facilities are limited. The Erlang B model has a system condition with random and independent arrivals, an infinite number of customer arrivals, and a constant average arrival rate. The time between arrivals follows a Poisson distribution and the time between services has an exponential distribution. The number of facilities or services is limited and is a complete file (full availability). Not all customers who come can be served, when customers come when all the facilities are serving, then the customer cannot be served and is eliminated or rejected. Customer refusal in the Erlang B queuing model is called the Erlang loss system. Meanwhile, these lost customers are said to have been blocked [1].

There are several ways to study a system, one of them by way of simulation. Simulation is a technique for imitating processes that occur in a system with the help of computers and using several assumptions so that they can be studied scientifically [2].

The research stage used in this research is a descriptive method, through a literature study on queuing theory. Most of the literature studies used are studying journals. Apart from journals, it is also supported by reading various other literature such as books, the internet, and articles. The stages of the research carried out are as follows:

a. Learn the basic theory of (M/M/c):(FIFO/c/∞) queuing systems or Erlang B queuing models, Pareto and Exponential distributions, and simulation methods.
b. Determine the distribution of time data between the addition of positive cases of COVID-19 patients at Arcamanik SCA.
c. Implementing a simulation of the queuing model on positive patient data for COVID-19 at Arcamanik SCA, using the R Studio software. The simulation results of the queuing model system were analyzed.

3. EXPERIMENTAL RESULTS

Simulations are performed for the queuing model (G/M/c):(FIFO/c/). The service time in this simulation is the length of time the patient has been hospitalized, namely the duration of time from the patient coming to leaving the hospital. The server used is an empty bed (TT) at Hermina Arcamanik Hospital. In this model, it is assumed that no customers are waiting in line or customers waiting to get a bed. Another assumption used is that simulations are carried out for systems or hospitals that work for at least 150 days. In addition, it is considered that every COVID-19 positive patient at Arcamanik SCA is treated at the hospital.

The results of the simulation have been processed into several performance measures of the queuing system. Measures of system performance include customers served (L_s), rejected customers, W_s, which is the average waiting time in the system in minutes, and E_B, which is the probability that customers are rejected in the queuing system. At the graphs in Figure 1, as the number of beds (TT) increases, the number of rejected patients will decrease and converge to zero. So it can be concluded that, for all COVID-19 patients at Arcamanik SCA to get a bed for isolation, with a cure rate of $\frac{1}{14}$, $\frac{1}{21}$, and $\frac{1}{28}$, then the minimum empty bed for at least 150 days needs to be provided by Hermina Hospital. Arcamanik, respectively, is 20, 26, and 32 beds.

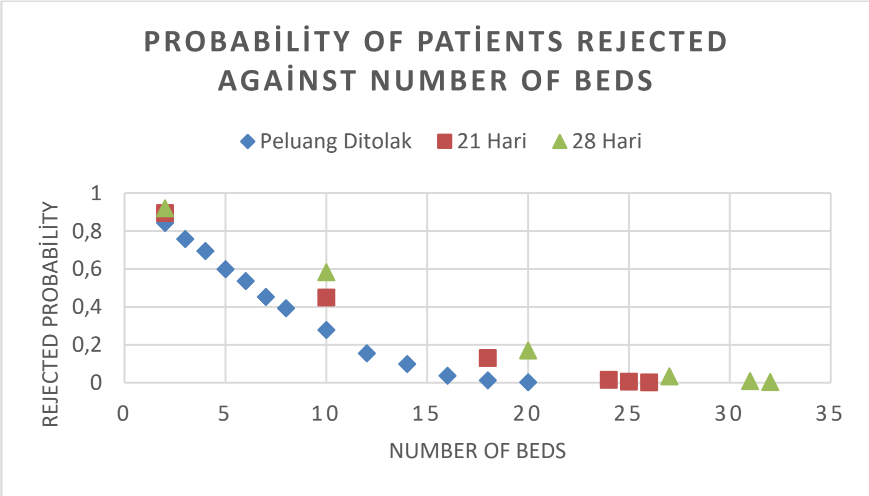

Figure 1 The customer odds graph is rejected for Arcamanik SCA with cure rates of 14, 21, and 28 days and decreasing towards zero.

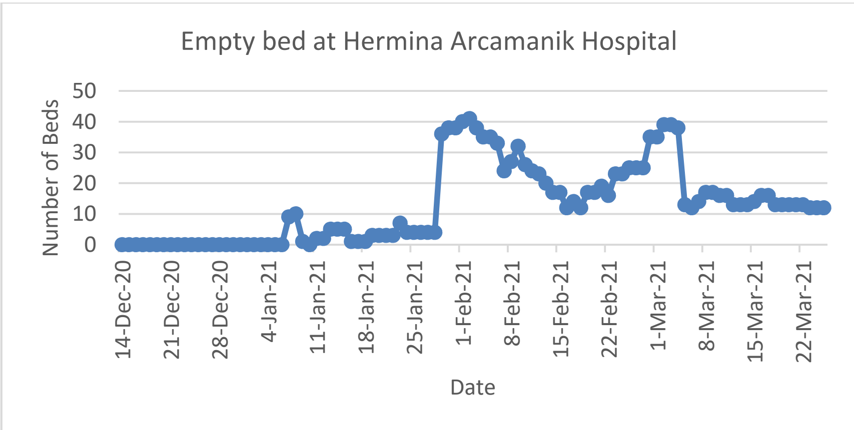

Figure 2 The customer opportunity graph is rejected for Arcamanik SCA with cure rates of 14, 21, and 28 days. Beds increased dramatically around January 29, 2021. The data fluctuated in February and March.

It can be seen in the graph in Figure 2 that there is a high spike in beds around January 29th, 2021. There were several times that the vacant beds did not reach the minimum number of vacant beds for Arcamanik SCA. So there are patients who come but have to leave without getting services from Hermina Arcamanik Hospital. The patient must go to another hospital that still has available vacant beds. Assume Hasan Sadikin Hospital (RSHS) as an alternative hospital. RSHS is located in Bojonagara SCA, so assume that vacant beds at RSHS are prioritized for COVID-19 patients who are in Bojonagara SCA. Then look for the minimum bed needed by Bojonagara SCA using the simulation method.

4. CONCLUSIONS

Some conslusions be drawn from this research are:

a. From the simulation results of the queuing system (M/M/c):(FIFO/c/∞) and (G/M/c):(FIFO/c/∞) or Erlang B, it is found that the more servers, the more customers and the probability of the customer being rejected will decrease and converge towards zero. This means that all customers will get service from the system. But the chances of idle servers increase.
b. For all COVID-19 patients at Arcamanik SCA to receive treatment, a minimum of empty beds that need to be provided for at least 150 days, for Hermina Arcamanik Hospital with a recovery rate of 14, 21, and 28 days respectively are 20, 26, and 32 sleep.
c. If a COVID-19 patient is rejected by Hermina Arcamanik Hospital, the alternative hospital to visit is RSHS. The proportion of beds in the RSHS for Arcamanik SCA COVID-19 patients, for at least 150 days, is 0.16 times the number of empty beds in the RSHS. The probability of the patient being admitted to RSHS is 0.63 to one. This means that COVID-19 patients who come from Arcamanik SCA, have a big enough opportunity to get treatment at RSHS.
d. The number of cases of increasing COVID-19 patients, so that all patients can get treatment at the Hermina Arcamanik Hospital is a maximum of 35 cases. If the additional cases are between 36 and 62, then the patient is rejected from the Hermina Arcamanik Hospital, but can still get treatment at the RSHS. For the increase in COVID-19 patients that exceeds 62 cases, these patients will not receive treatment at the two hospitals.

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DISCRETE APPROXIMATIONS FOR ELLEPTIC BOUNDARY VALUE PROBLEMS

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Abstract:

We study some discrete boundary value problems for discrete elliptic pseudo-differential equations in a half-space. These statements are related with a special periodic factorization of an elliptic symbol and a number of boundary conditions depends on an index of periodic factorization. This approach was earlier used by authors for studying special types of discrete convolution equations. Here we consider more general equations and functional spaces.

Keywords: Digital pseudo-differential operator, periodic symbol, factorization, discrete boundary value problem
General area of research: Mathematics
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1. We consider the following boundary value problem

$$\begin{cases} (Au)(x) = 0, \quad x \in \mathbb{R}^m_+ \\ (B_j u)(x) \,|x_{m=0} = b_j(x'), \quad x' \in \mathbb{R}^{m-1}, \end{cases} \tag{1}$$

$j = 0, 1, \dots, n-1$, where A is a pseudo-differential operator [1] with the symbol $\tilde{A}(\xi)$ satisfying the
condition

$$c_1(1+|\xi|)^\alpha \le |\tilde{A}(\xi)| \le c_2(1+|\xi|)^\alpha,$$

B_j, $j = 0, 1, \dots, n-1$, are also pseudo-differential operators with symbols $\tilde{B}_j(\xi)$ satisfying similar
condition

$$c_3(1+|\xi|)^{\beta_j} \le |\tilde{B}_j(\xi)| \le c_4(1+|\xi|)^{\beta_j}.$$

We seek a solution in Sobolev—Slobodetskii space $H^s(\mathbb{R}^m_+)$, $s \in \mathbb{R}$, where $\mathbb{R}^m_+ = \{x \in \mathbb{R}^m : x = (x', x_m), x_m > 0\}$, $b_j \in H^{s-\beta_j-\frac{1}{2}}(\mathbb{R}^{m-1}), j = 0, 1, \dots, n-1$. We assume also that index of factorization [1] æ of the symbol $\tilde{A}(\xi)$ satisfies the condition æ $- s = n +$

$\delta, n \in \mathbb{N}, |\delta| < 1/2$. Then the problem (1) has unique solution [1] under certain additional condition

$$\text{ess inf}\,|\det\big(S(\xi')\big)_{k,j=0}^{n-1}| > 0, \;\; \xi' \in \mathbb{R}^{m-1},$$

(2)

where

$$S_{jk}(\xi') = \int_{-\infty}^{\infty} \tilde{A}_{+}^{-1}(\xi)\tilde{B}_{j}(\xi', \xi_m)\xi_m^k \, d\xi_m, \quad \tilde{A}(\xi) = \tilde{A}_{+}(\xi)\cdot\tilde{A}_{-}(\xi).$$

2. To construct approximating discrete boundary value we use a concept of a digital pseudo-differential operator [2].

Let $u_d(\tilde{x})$ be a function of a discrete variable $\tilde{x} \in h\mathbb{Z}^m, h > 0$. The discrete Fourier transform F_d of the function u_d is called the following series

$$(F_d u_d)(\xi) \equiv \tilde{u}(\xi) \equiv \sum_{\tilde{x} \in h\mathbb{Z}^m} e^{i\tilde{x}\cdot\xi} u_d(\tilde{x}) h^m, \; \xi \in \hbar\mathbb{T}^m, \; \hbar \equiv h^{-1},$$

if the series converges. Using divided differences and their Fourier transforms we introduce discrete functional spaces. Discrete Sobolev-Slobodetskii space $H^s(h\mathbb{Z}^m)$, $s \in \mathbb{R}$, consists of functions for which the following norm

$$\|u_d\|_s = \left(\int_{\hbar\mathbb{T}^m} (1 + |\hat{\zeta}^2|)^s |\tilde{u}_d(\xi)|^2 \, d\xi\right)^{1/2}$$

is finite,

$$\hat{\zeta}^2 \equiv \hbar^2 \sum_{k=1}^{m} \left(e^{ih\xi_k} - 1\right)^2, \;\; \hat{\zeta}_k = \hbar\left(e^{ih\xi_k} - 1\right)^2, k = 1, 2, \dots, m..$$

Let $h\mathbb{Z}_+^m$ be a discrete half-space. The discrete space $H^s(h\mathbb{Z}_+^m)$ consists of functions from $H^s(h\mathbb{Z}^m)$ for which their supports belong to $\overline{D_d}$. A norm in the space $H^s(h\mathbb{Z}_+^m)$ is induced by the norm of $H^s(h\mathbb{Z}^m)$.

Let $\tilde{A}_d(\xi)$ be a measurable periodic function with basic cube of periods $\hbar\mathbb{T}^m$. The function $\tilde{A}_d(\xi)$ is called a symbol of digital pseudo-differential operator A_d, which is defined by the formula

$$(A_d u_d)(\tilde{x}) = \frac{1}{(2\pi)^m} \sum_{\tilde{y} \in h\mathbb{Z}^m} \int_{\hbar\mathbb{T}^m} e^{ih\cdot(\tilde{x}-\tilde{y})} \tilde{A}_d(\xi)\tilde{u}(\xi) d\xi, \; \tilde{x} \in h\mathbb{Z}^m.$$

Using such digital operators we introduce discrete boundary value problem

$$\begin{cases} (A_d u_d)(\tilde{x}) = 0, \quad h\mathbb{Z}_+^m \\ (B_{d,j} u_d)(\tilde{x})\big|\tilde{x}_{m=0} = b_{d,j}(\tilde{x}'), \quad h\mathbb{Z}^{m-1}, \end{cases} \tag{3}$$

$j = 0, 1, \dots, n-1$, where digital pseudo-differential operators A_d, $B_{d,j}$ and boundary functions $b_{d,j}$ are chosen in a special way. It gives a possibility to prove the unique solvability of discrete boundary value problem in a discrete half-space in corresponding discrete Sobolev—Slobodetskii space and to obtain a comparison for discrete and continuous solutions.

Theorem. Let æ be index of factorization of the symbol $\widetilde{A}(\xi)$ such that $æ - s = n + \delta, n \in \mathbb{N}$,
$|\delta| < 1/2$, 1/2 < $s < \beta_j < s + \delta - 1, s > \frac{m+2}{2} + (\delta - \beta_j), j = 0, 1, \ldots, n-1$, and the condition (2)holds.
A comparison between discrete and continuous solution of problems (3) and (1) respectively is given by the estimate

$$|\, u_d(\tilde{x}) - u(\tilde{x})| \leq \text{const}\, h \sum_{j=0}^{n-1} \|b_j\|_{\beta_j},$$

for enough small h, where const does not depend on h.

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MAXIMAL VARIETY LEIBNIZIAN STRINGS FOR LARGE N

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Abstract:

We made further calculations on the maximum variety of Leibnizian strings of length in between 21 and 37. Previously, the maximum varieties of Leibnizian strings of length up to 20 have been calculated [3]. Here we calculated the number of distinct maximal variety strings (modulo symmetries) of length in between 6 and 35. We will make available these maximal variety Leibnizian strings in Ref. [4] because it is not convenient to list them all here.

Keywords: Leibniz, Maximal Variety, Character Strings
General area of research: Physics
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1. INTRODUCTION

30 years ago, in 1992, Barbour and Smolin [2] considered a toy model universe that is represented as a cyclic character string of two letters, which we label as '`X`'and '`-`'. Then they put a criterion for a string to be *Leibnizian* and defined the concept of *variety* for character strings. In short Leibnizian strings are those that the *view* (defined through its neighbors) of each character is different from all the others. For more discussion of the idea, one may consult to [1].

Dündar [3] created a dynamical model of such a toy universe as the one that hops between strings of maximal variety and calculated the maximum variety of Leibnizian strings of lengths N in between 6 and 20 using a Haskell code that is available in Ref. [4]. Here we present the maximum variety of Leibnizian strings of length N in between 21 and 37 using an optimized C code (more details in the next section) which will be available in Ref. [4]. We also present the number of distinct (modulo symmetries) maximal variety strings for N in between 6 and 35. These are new contributions to the literature.

2. METHOD

The values of maximum variety for Leibnizian strings of length in between 6 and 20 have been calculated in Ref. [3] through a GPL3 licensed Haskell code `sv.hs` available in Ref. [4] which was written specifically for that paper. Though this code was sufficient for illustrative purposes, it was not optimized. In order to make calculation for large N (larger than 20) we have written an optimized C code (`svParallel.c`) that fully takes advantage of CPU parallelization. The C code will be available in Ref. [4] under the GPL3 license.

Since we are dealing with strings that can be composed of only two letters, we could represent the string as an `unsigned long int` variable in C. This type can hold upto 64 bits and since we have dealt with N which is at most 37, the use of `unsigned long int` type has been sufficient for our purposes.

We have done the calculations on the National Center for High Performance Computing of Turkey (UHeM) and used about 81,000 core-hours. The C code we have written utilizes OpenMP® [5], so we could use only one node (128 cores) of the cluster. In the future, one may also try to use e.g. Open MPI [6] to eliminate the obstacle of running the code on a single node. This will allow the calculation of properties of Leibnizian strings of various lengths up to 64 if strings are represented by the `unsigned long int` type. However since we do not have enough resources, in this work we could work up to N = 37 for calculating the maximum variety of Leibnizian strings, and up to N = 35 for listing the Leibnizian maximal variety strings. The maximal variety Leibnizian strsings for N in between 6 and 35 will be available in Ref. [4].

3. EXPERIMENTAL RESULTS

In this section we list the maximum variety of Leibnizian strings of lengths between 6 and 37. We also list the number strings of maximal variety (length between 6 and 35) modulo symmetries. The symmetries of strings consist of cyclic rotations and mirror reflection of character strings. The full list of maximal variety strings will be available in GitHub repository [4] named “string-variety”.

N (String Length)	Maximum Variety	N (String Length)	Maximum Variety
6	4	21	10
7	5	22	31/3
8	6	23	31/3
9	17/3	24	32/3
10	37/6	25	11
11	20/3	26	34/3
12	8	27	23/2
13	23/3	28	12
14	49/6	29	12
15	49/6	30	37/3
16	9	31	38/3
17	26/3	32	13
18	55/6	33	40/3
19	29/3	34	40/3
20	59/6	35	41/3

		36	14
		37	43/3

Table 1. In Ref. [3] the maximum variety values of Leibnizian strings of lengths in between 6 and 20 have been presented. Here we give exact results for Leibnizian strings of lengths in between 21 and 37.

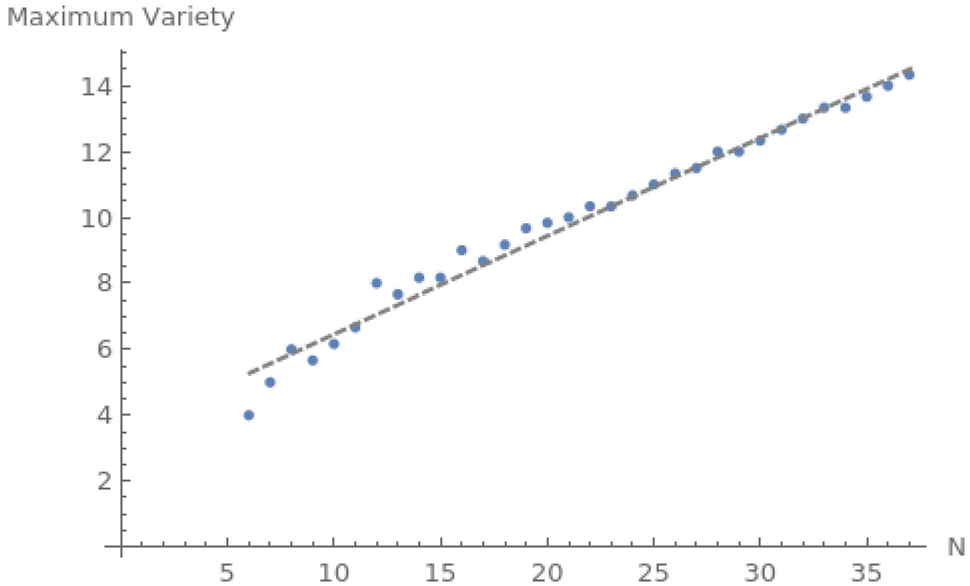

Figure 1. Here we plotted the length of Leibnizian strings vs. maximum variety using the data we list in Table 1. The dashed gray line represents a linear fit which has been found in the following form: 3.52621 + 0.297715 N. As it is seen on the figure, we can qualitatively say that there is a more regular linear trend in maximum variety as N increases.

N (String Length)	#Maximum Variety Strings	N (String Length)	#Maximum Variety Strings
6	1	21	1
7	1	22	13
8	1	23	48
9	2	24	18
10	1	25	18
11	2	26	20
12	2	27	12
13	2	28	14
14	2	29	72
15	2	30	7

16	1	31	70
17	3	32	58
18	2	33	48
19	1	34	377
20	2	35	264

Table 2. The number of distinct Leibnizian maximal variety strings (modulo symmetries) for N in between 6 and 35.

4. CONCLUSIONS

In Ref. [3] maximum variety of Leibnizian strings of length in between 6 and 20 were presented as well as a dynamical toy model of a toy model universe that consist of cyclic strings that are composed of two letters. Here we calculated maximum variety of Leibnizian strings between 21 and 37 and provided the number of distinct maximal variety strings (modulo symmetries). Since we do not have enough space here, we will present the list of maximal variety strings in Ref. [4].

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SOLUTION PROPERTIES OF A FRACTIONAL ACUTE AND CHRONIC HEPATITIS B

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Abstract:

Hepatitis B disease is an infectious disease caused by the hepatitis B virus. The acute and chronic hepatitis B model was coined by Khan et al. (2017) with the aiming of the control the spread of hepatitis B disease in a public. This work focuses on fractional acute and chronic hepatitis B model from the point of Caputo-Fabrizio derivative and analyze the features of this fractional model.

Keywords: Caputo-Fabrizio derivative, existence and uniqueness, acute and chronic hepatitis B model
General area of research: Mathematics
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1. INTRODUCTION

Hepatitis literally means inflammation of the liver. While some types of hepatitis go away on its own without serious problems, others can be long-lasting (chronic) and create a cicatrix tissue called scarring of the liver, causing cirrhosis, loss of liver function, and in some cases, liver cancer. Hepatitis B is a serious disease that affects the liver and is caused by the Hepatitis B virus. Hepatitis B virus is transmitted through close contact of blood, sexually transmitted and bodily fluids (saliva). Hepatitis B can be passed from mother to baby during birth. 400 million people in the world and 3 million people in Turkey carry the hepatitis B virus. If untreated, there is a risk of developing cirrhosis and liver cancer. In this case, liver transplantation may be considered. While hepatitis B carriers do not experience any problems causing the virus during their lifetime, they have a 10 percent risk of developing the disease. For this reason, it is important to have blood checks in certain periods (for more details [1], [2], [3], [4], [5], [6] and references therein).
In more recent times, fractional calculus (FC) plays an important role to characterize many problems in physics, biology, medicine, control theory and many more, because fractional calculus is more multiple compared to classical derivative on account of memory and hereditary equipments. Caputo [7], Liouville-Caputo [8], as well as Caputo and Fabrizio [9] bring forward many notations about fractional order operators and these conceptions have been implemented quite adequate when reflecting many real-world problems [10], [11], [12], [13], [14], [15], [16]. Motivated with these studies, we give existence and uniqueness criteria

for acute and chronic hepatitis B model with the newly-introduced Caputo-Fabrizio (CF) derivative.
In this study, we handle a classical acute and chronic hepatitis B model established by the scientists [17]. In this model the total population $T(t)$ are divided into four different groups: Susceptible people $S(t)$, not infective but possess the probability to get the illness; infected $I_1(t)$ stand for individuals who have infective with acute hepatitis, $I_2(t)$ display people who are infected with chronic hepatitis and $R(t)$ show person who recuperate after the infection with a life-time immunity. [17] establishes the model involving classical derivative:

$$\begin{aligned} \frac{dS(t)}{dt} &= b-\alpha S(t)I_2(t)-(\mu_0+v)S(t), \\ \frac{dI_1(t)}{dt} &= \alpha S(t)I_2(t)-(\mu_0+\beta+\gamma_1)I_1(t), \\ \frac{dI_2(t)}{dt} &= \beta I_1(t)-(\mu_0+\mu_1+\gamma_2)I_2(t), \\ \frac{dR(t)}{dt} &= \gamma_1 I_1(t)+\gamma_2 I_2(t)+vS(t)-\mu_0 R(t), \end{aligned} \tag{1}$$

where v presents hepatitis B vaccination ratio, b is the birth ratio, γ_1 is the recovery ratio from acute level to recovered, α is the transfer rate from susceptible to infected with acute hepatitis B, β is the transfer ratio from acute level to infected with chronic hepatitis, μ_0 is the natural death ratio, γ_2 is the recovery ratio from chronic level to recovered part and μ_1 is the death ratio because of hepatitis B.

2. BASIC PRELIMINARIES

This section includes the basic new concepts of CF derivative.
Definition 1. [9] Let $\eta\in[0,1]$, $a<b$ and $f\in H^1(a,b)$ the Caputo-Fabrizio derivative is defined as:

$$D_t^\eta f(t)=\frac{M(\eta)}{1-\eta}\int_a^t f'(x)\exp\left(-\eta\frac{t-x}{1-\eta}\right)dx, \tag{2}$$

where $M(\eta)$ is a normalization function verifying $M(0)=M(1)=1$. If $f\notin H^1(a,b)$, this derivative can be rewrite of the following:

$$D_t^\eta f(t)=\frac{\eta M(\eta)}{1-\eta}\int_a^t (f(t)-f(x))\exp\left(-\eta\frac{t-x}{1-\eta}\right)dx. \tag{3}$$

Remark. If $\zeta=\frac{1-\eta}{\eta}\in[0,\infty)$, $\eta=\frac{1}{1+\zeta}\in[0,1]$ then the Eq. (3) is given by

$$D_t^\eta f(t)=\frac{N(\zeta)}{\zeta}\int_a^t f'(x)\exp\left(-\frac{t-x}{\zeta}\right)dx,$$

where $N(0)=N(\infty)=1$.
The associated integral of this derivative was introduced by Nieto and Losada [18]:
Definition 2. [18] Let f be a function and $0<\eta<1$. The fractional integral of order η is defined by:

$$I_t^{\eta} f(t) = \frac{2(1-\eta)}{(2-\eta)M(\eta)} f(t) + \frac{2\eta}{(2-\eta)M(\eta)} \int_0^t f(s)\,ds, \quad t \geq 0. \tag{4}$$

Withal, we can yield the below result:

$$\frac{2(1-\eta)}{(2-\eta)M(\eta)} + \frac{2\eta}{(2-\eta)M(\eta)} = 1,$$

then $M(\eta) = \frac{2}{2-\eta}$ and $0 < \eta < 1$.

Benefiting from the above results, another form of the new Caputo derivative with order $0 < \eta < 1$ given by [18]:

$$D_t^{\eta} f(t) = \frac{1}{1-\eta} \int_a^t f'(x) \exp\left(-\eta \frac{t-x}{1-\eta} \right) dx. \tag{5}$$

3. A FRACTIONAL ACUTE AND CHRONIC HEPATITIS B MODEL

In this part, we extend the acute and chronic hepatitis B model in [17] benefiting from CF derivative:

$$\begin{aligned}
{}_0^{CF}D_t^{\eta} S(t) &= b - \alpha S(t) I_2(t) - (\mu_0 + v) S(t), \\
{}_0^{CF}D_t^{\eta} I_1(t) &= \alpha S(t) I_2(t) - (\mu_0 + \beta + \gamma_1) I_1(t), \\
{}_0^{CF}D_t^{\eta} I_2(t) &= \beta I_1(t) - (\mu_0 + \mu_1 + \gamma_2) I_2(t), \\
{}_0^{CF}D_t^{\eta} R(t) &= \gamma_1 I_1(t) + \gamma_2 I_2(t) + vS(t) - \mu_0 R(t).
\end{aligned} \tag{6}$$

with the initial conditions $S(0) = S_0$, $I_1(0) = I_{10}$, $I_2(0) = I_{20}$, $R(0) = R_0$. Implementing the integral operator to the system (6), we find

$$\begin{aligned}
S(t) - h_1(t) &= \frac{2(1-\eta)}{(2-\eta)M(\eta)} \{ b - \alpha S(t) I_2(t) - (\mu_0 + v) S(t) \} \\
&\quad + \frac{2\eta}{(2-\eta)M(\eta)} \int_0^t \{ b - \alpha S(s) I_2(s) - (\mu_0 + v) S(s) \} ds, \\
I_1(t) - h_2(t) &= \frac{2(1-\eta)}{(2-\eta)M(\eta)} \{ \alpha S(t) I_2(t) - (\mu_0 + \beta + \gamma_1) I_1(t) \} \\
&\quad + \frac{2\eta}{(2-\eta)M(\eta)} \int_0^t \{ \alpha S(s) I_2(s) - (\mu_0 + \beta + \gamma_1) I_1(s) \} ds, \\
I_2(t) - h_3(t) &= \frac{2(1-\eta)}{(2-\eta)M(\eta)} \{ \beta I_1(t) - (\mu_0 + \mu_1 + \gamma_2) I_2(t) \} \\
&\quad + \frac{2\eta}{(2-\eta)M(\eta)} \int_0^t \{ \beta I_1(s) - (\mu_0 + \mu_1 + \gamma_2) I_2(s) \} ds, \\
R(t) - h_4(t) &= \frac{2(1-\eta)}{(2-\eta)M(\eta)} \{ \gamma_1 I_1(t) + \gamma_2 I_2(t) + vS(t) - \mu_0 R(t) \} \\
&\quad + \frac{2\eta}{(2-\eta)M(\eta)} \int_0^t \{ \gamma_1 I_1(s) + \gamma_2 I_2(s) + vS(s) - \mu_0 R(s) \} ds.
\end{aligned} \tag{7}$$

Using iterative, above equation can be rearrange as

$$S_0(t)=h_1(t), I_{10}(t)=h_2(t), I_{20}(t)=h_3(t), R_0(t)=h_4(t)$$

and

$$\begin{aligned}
S_{(n+1)}(t) &= \frac{2(1-\eta)}{(2-\eta)M(\eta)}\left\{b-\alpha S_n(t)I_{2n}(t)-(\mu_0+v)S_n(t)\right\} \\
&+\frac{2\eta}{(2-\eta)M(\eta)}\int_0^t\left\{b-\alpha S_n(s)I_{2n}(s)-(\mu_0+v)S_n(s)\right\}ds, \\
T_{1(n+1)}(t) &= \frac{2(1-\eta)}{(2-\eta)M(\eta)}\left\{\alpha S(t)I_{2n}(t)-(\mu_0+\beta+\gamma_1)I_{1n}(t)\right\} \\
&+\frac{2\eta}{(2-\eta)M(\eta)}\int_0^t\left\{\alpha S(s)I_{2n}(s)-(\mu_0+\beta+\gamma_1)I_{1n}(s)\right\}ds, \\
I_{2(n+1)}(t) &= \frac{2(1-\eta)}{(2-\eta)M(\eta)}\left\{\beta I_{1n}(t)-(\mu_0+\mu_1+\gamma_2)I_{2n}(t)\right\} \\
&+\frac{2\eta}{(2-\eta)M(\eta)}\int_0^t\left\{\beta I_{1n}(s)-(\mu_0+\mu_1+\gamma_2)I_{2n}(s)\right\}ds, \\
R_{(n+1)}(t) &= \frac{2(1-\eta)}{(2-\eta)M(\eta)}\left\{\gamma_1 I_{1n}(t)+\gamma_2 I_{2n}(t)+vS(t)-\mu_0 R_n(t)\right\} \\
&+\frac{2\eta}{(2-\eta)M(\eta)}\int_0^t\left\{\gamma_1 I_{1n}(s)+\gamma_2 I_{2n}(s)+vS_n(s)-\mu_0 R_n(s)\right\}ds. \qquad (8)
\end{aligned}$$

By taking limit as n approaches to infinity, we can find the exact solution.

4. EXISTENCE OF SOLUTION BY PICARD_LINDELOF APPROACH

In order to present the existence of a unique solution, we define

$$\begin{aligned}
m_1(t,S) &= b-\alpha S(t)I_2(t)-(\mu_0+v)S(t), \\
m_2(t,I_1) &= \alpha S(t)I_2(t)-(\mu_0+\beta+\gamma_1)I_1(t), \\
m_3(t,I_2) &= \beta I_1(t)-(\mu_0+\mu_1+\gamma_2)I_2(t), \\
m_4(t,R) &= \gamma_1 I_1(t)+\gamma_2 I_2(t)+vS(t)-\mu_0 R(t),
\end{aligned} \qquad (9)$$

where $m_1(t,S)$, $m_2(t,I_1)$, $m_3(t,I_3)$, $m_4(t,R)$ are contraction related to S, I_1, I_2, R. Let

$$\begin{aligned}
M_1 &= \sup_{C[a,b_1]}\left\|m_1(t,S)\right\|, \\
M_2 &= \sup_{C[a,b_2]}\left\|m_2(t,I_1)\right\|, \\
M_3 &= \sup_{C[a,b_3]}\left\|m_3(t,I_2)\right\|, \\
M_4 &= \sup_{C[a,b_4]}\left\|m_4(t,R)\right\|,
\end{aligned}$$

where

$$\begin{aligned} C[a,b_1] &= [t-a,t+a]\times[x-b_1,x-b_1]=E\times D_1, \\ C[a,b_2] &= [t-a,t+a]\times[x-b_2,x-b_2]=E\times D_2, \\ C[a,b_3] &= [t-a,t+a]\times[x-b_3,x-b_3]=E\times D_3, \\ C[a,b_4] &= [t-a,t+a]\times[x-b_4,x-b_4]=E\times D_4. \end{aligned}$$

We apply Banach-fixed point theorem using metric on $C[a,b_i]$, $(i=1,2,3,4)$ with the norm:

$$\|f(t)\| = \sup_{t\in[t-a,t+a]} |f(t)|.$$

Benefiting from Picard's operator, we have

$$\Psi : C(E,D_1,D_2,D_3,D_4) \to C(E,D_1,D_2,D_3,D_4)$$

defined as follows:

$$\Psi X(t) = X_0(t) + \Delta(t,X(t))\frac{2(1-\eta)}{(2-\eta)M(\eta)} + \frac{2\eta}{(2-\eta)M(\eta)}\int_0^t \Delta(s,X(s))ds, \tag{10}$$

where $X(t)=\{S(t),I_1(t),I_2(t),R(t)\}$, $X_0(t)=\{S(0),I_1(0),I_2(0),R(0)\}$ and $\Delta(t,X(t))=\{m_1(t,S(t)),m_2(t,I_1(t)),m_3(t,I_2(t)),m_4(t,R(t))\}$. Now, we suppose that the solution of fractional acute and chronic hepatitis B model are bounded within a time period,

$$\|X(t)\| \le \{b_1,b_2,b_3,b_4\}. \tag{11}$$

We give

$$\begin{aligned} \|X(t)-X_0(t)\| &= \left\| \Delta(t,X(t))\frac{2(1-\eta)}{(2-\eta)M(\eta)} + \frac{2\eta}{(2-\eta)M(\eta)}\int_0^t \Delta(s,X(s))ds \right\| \\ &\le \frac{2(1-\eta)}{(2-\eta)M(\eta)}\|\Delta(t,X(t))\| + \frac{2\eta}{(2-\eta)M(\eta)}\int_0^t \|\Delta(s,X(s))\|ds \qquad (12) \\ &\le \left(\frac{2(1-\eta)}{(2-\eta)M(\eta)} + \frac{2\eta t_0}{(2-\eta)M(\eta)}\right)\max\{b_1,b_2,b_3,b_4\}. \end{aligned}$$

Let $M=\max\{M_1,M_2,M_3,M_4\}$ and $B=\max\{b_1,b_2,b_3,b_4\}$. So, we get $\|X(t)-X_0(t)\| \le BM < \theta$. Also, we find the following

$$\|\Psi X_1 - \Psi X_2\| = \sup_{t\in E}|X_1 - X_2|. \tag{13}$$

Then, we obtain

$$\begin{aligned}
\|\Psi X_1 - \Psi X_2\| &= \left\| \left\{ \Delta(t, X_1(t)) - \Delta(t, X_2(t)) \right\} \frac{2(1-\eta)}{(2-\eta)M(\eta)} \right. \\
&\quad \left. + \frac{2\eta}{(2-\eta)M(\eta)} \int_0^t \left\{ \Delta(s, X_1(s)) - \Delta(s, X_2(s)) \right\} \right\| \\
&\leq \frac{2(1-\eta)}{(2-\eta)M(\eta)} \left\| \Delta(t, X_1(t)) - \Delta(t, X_2(t)) \right\| \\
&\quad + \frac{2(1-\eta)}{(2-\eta)M(\eta)} \int_0^t \left\| \Delta(s, X_1(s)) - \Delta(s, X_2(s)) \right\| ds \\
&\leq \frac{2(1-\eta)}{(2-\eta)M(\eta)} \delta \left\| X_1(t) - X_2(t) \right\| \\
&\quad + \frac{2\eta\delta}{(2-\eta)M(\eta)} \int_0^t \left\| X_1(s) - X_2(s) \right\| ds \\
&\leq \frac{2(1-\eta)\delta}{(2-\eta)M(\eta)} + \frac{2\eta\delta t_0}{(2-\eta)M(\eta)} \left\| X_1(t) - X_2(t) \right\| \\
&\leq a\gamma \left\| X_1(t) - X_2(t) \right\|
\end{aligned} \tag{14}$$

with $\gamma < 1$. Since X is contraction, we find $a\gamma < 1$. So Ψ is also contraction. Finally, the model (6) has a unique solution.

5. CONCLUSIONS

Hepatitis B disease is an infectious disease affecting the liver. It can cause both acute and chronic infection. While it is an acute infection that the hepatitis B virus creates the symptoms of the disease after it is infected, then heals and there is no trace of the virus in the body, the long-term presence of the virus by settling in the liver cells and multiplying is a chronic infection. Therefore, lots of researchers from many subject area want to understand such an important disease [19], [20], [21]. Here, we show the existence of solutions for an acute and chronic hepatitis B model equipped with CF derivative taking into Picard-Lindelof approach consideration.

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DESIGNING AND MANUFACTURING THE SCISSOR PLATFORM PROTOTYPE USING NUMERICAL ANALYSIS

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Abstract:

In this study, prototype production of the scissor platform, an important transport mechanism frequently used in the sector, was carried out. The load-carrying capacity of the designed prototype platform and the maximum deformations that will occur on the platform were determined in the numerical analysis. Numerical analyzes were carried out in the Ansys finite element program. However, before that, the three-dimensional view of the platform was designed in the Solidworks program. The maximum load that the platform can carry depending on the stress values formed on the platform was determined utilizing the parametric study. In addition, the reaction forces of all contact points creating the scissor platform under all applied loads and the connection states were observed by the structural analysis. In particular, it is known that the contact surfaces are the areas in which the scissor platform is most easily damaged under loading conditions. Therefore, it is crucial to determine the contact reaction forces to decide whether or not damage will occur in these areas.

Keywords: Scissor platform, Numerical analysis, Parametric study, Loading capacity
General area of research: Mechanical Engineering
ICFAS2022-ID: 1100

1. INTRODUCTION

Nowadays, it can be said that the scissor platforms are used in many sectors. It is frequently preferred, especially in works that require load carrying. The scissor platform movement mechanism varies according to the place it will use, the height, or the load it will carry [1]. Although the platform has different movement mechanisms, the working principle is similar for all application areas. Its working principle is to raise and lower the load placed on it at a certain distance [2].
In this study, firstly, scissor platform systems with different motion mechanism systems were investigated [3-4]. A prototype suitable for the general working principle and capable of moving a specific load capacity has been determined among these systems. The dimensions of the prototype have been resized, taking into account the actual mechanism values. In addition, the maximum load to be carried by the prototype, the stress, and deformations that will occur on it were determined by finite element analysis. In literature studies, similar studies in which both production and finite element analyses were carried out together are not encountered yet. Therefore, it is thought that this study will contribute to the literature.

2. MANUFACTURING METHOD

The manufacturing process of the prototype first started with the design of the lower unit part. The lower unit consists of 3 steel L profiles joined by electrode welding. The foot mechanism, which will enable the movement of the prototype, is placed on the lower unit using electrode welding. The connection points of the legs are connected by using a nut-bolt connection for the mechanism to move. The thin sheet metal plate to carry the load of the platform is joined on foot with the help of electrode welding. It should be noted that the platform's stability is ensured with the use of a clamp in the joining of the sheet metal plate with the foot. In addition, two thin sheet metal plates with a thickness of 2 mm are placed on top of the lower unit. M8 long gear is placed in the middle of these plates. As the gear compresses the plates, the plates move the foot upwards. Similarly, when the gears are released, the plates break the contact with the feet and move the mechanism down.

An image recorded during the production phase of the platform and the final version of the prototype mechanism are shown in Figure 1.

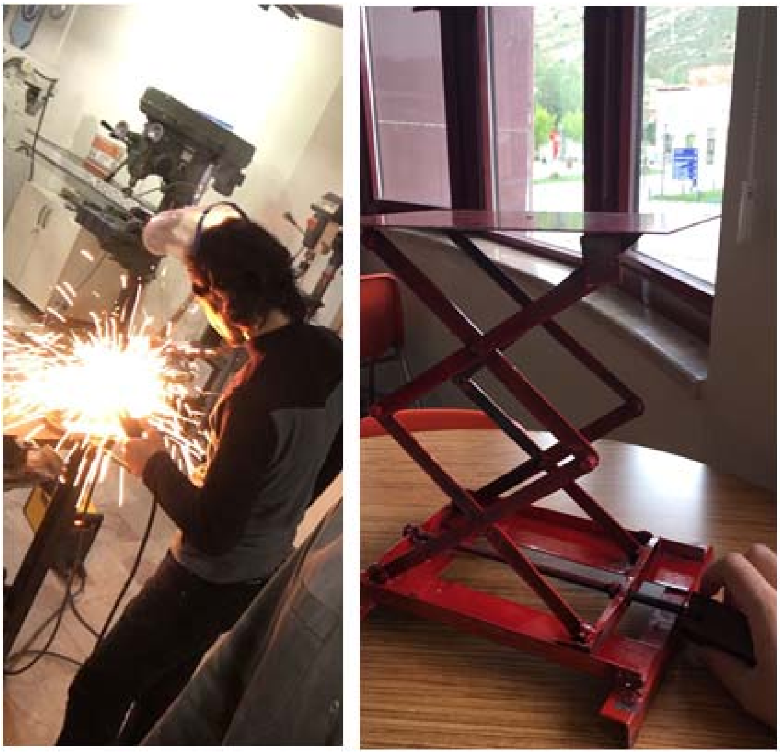

Fig.1. Manufacturing process and prototype mechanism

3. NUMERICAL ANALYSIS

Numerical analysis of the scissor platform mechanism was carried out in the finite element program within the scope of the study. Finite element analysis was performed using the Ansys program. In the study, firstly, the three-dimensional drawing of the platform mechanism was created using the Solidworks program. Then, the analysis was initiated by transferring the three-dimensional design to the Ansys program. The analysis consists of four steps. These steps include the contact zone, the mesh, the application of boundary conditions, and the solution, respectively.

3.1 Contact analysis of scissor platform

Two different contact elements are assigned to the contact points of the scissor platform mechanism in the analysis. During the analysis, "*bonded*" contact, a linear contact, was assigned to the welded points in the lower and upper parts. The reason for giving the relevant contact to these points is that normal or tangential deformation is not observed at the connection points under loading conditions. However, at the foot connection points, "*no separation*" contact is used, which allows tangential deformation under loading conditions, considering the up-down movement of the foot. It should be stated that the "*overconstrained*" situation was not observed that would disrupt the linear contact behavior at all points.

3.2 Mesh analysis of scissor platform

The platform mechanism is divided into finite elements by applying the meshing process before the boundary conditions. Two different elements (triangular and hexagonal) geometries are utilized for the entire platform mechanism. In the separation into finite elements, it is crucial to consider mesh quality and aspect ratio of the mesh geometry. Especially at the connection points, the element size has been tightened using the program's "*re-mesh*" command since calculating contact forces accurately. Finite element analysis is completed with 142855 elements and 524233 nodes. In Figure 2, the view of the scissor platform mechanism divided into finite elements is given.

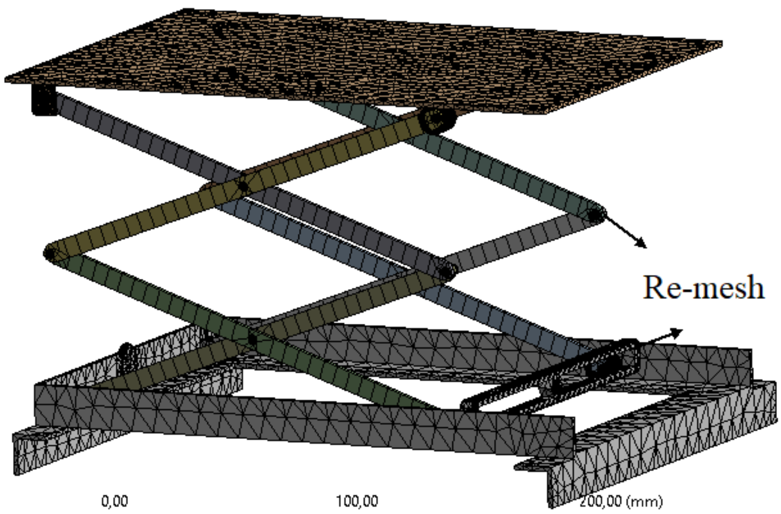

Fig2. Finite element view of the scissor platform

3.3 Boundary conditions of finite element analysis

In the finite element analysis, boundary conditions are determined before the solution process. In the boundary conditions, fixed support was defined as the support, and a point mass was selected as the loading. The reason for choosing the fixed support as the support is that the moment reaction occurs on the supports under the loading conditions. Loading conditions were determined by the parametric study, considering the stress results on the platform. Moreover, the parametric study is explained in detail in the solution section.

3.4 Solution of finite element analysis

In the solution part of the analysis, the platform's maximum deformation and equivalent stress distributions under loading conditions were investigated. Before interpreting the results, the maximum load that the scissor platform mechanism can carry was determined by a parametric study. In the parametric study, firstly, the point mass value applied from the upper part of the platform was chosen as the input value. Then, equivalent stress is selected as the output parameter. Subsequently, different masses were selected, and the stress values caused by each mass were calculated. The maximum mass value that can be applied using the mass-stress graph was determined by considering the yield stress of the scissors platform. The chart is shown in Figure 3.

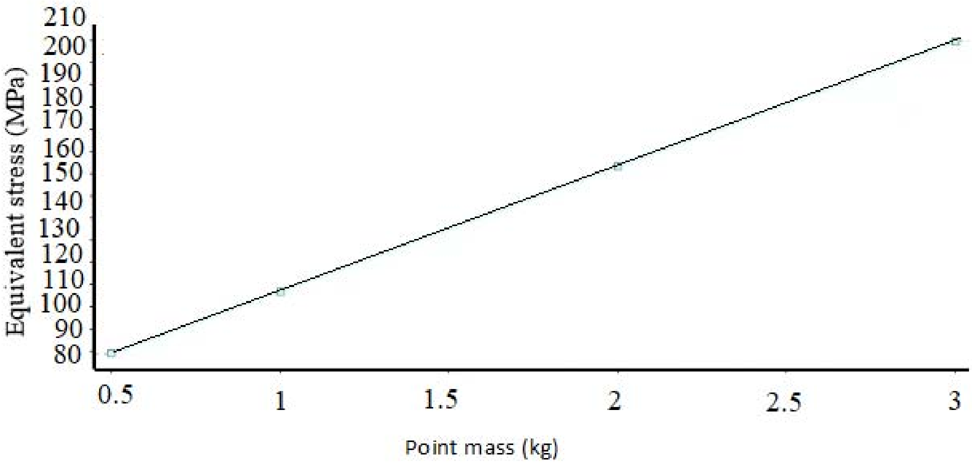

Fig3. Point mass versus equivalent stress

In figure 3, it is observed that as the mass value increases, the stress value also increases due to the linear elastic behavior. The maximum mass value was determined as 3 kg, considering the yield strength of the structural steel material [5] from which the platform mechanism is made.

The stress distribution on the platform mechanism was also determined in the solution section (see figure 4). In the stress distribution, the maximum stress occurred when the platform was connected to the lower unit. The region determined at the end of the analysis was controlled under the loading condition in the prototype mechanism, and the point at which the stress was maximum was radiused using hand grinding equipment.

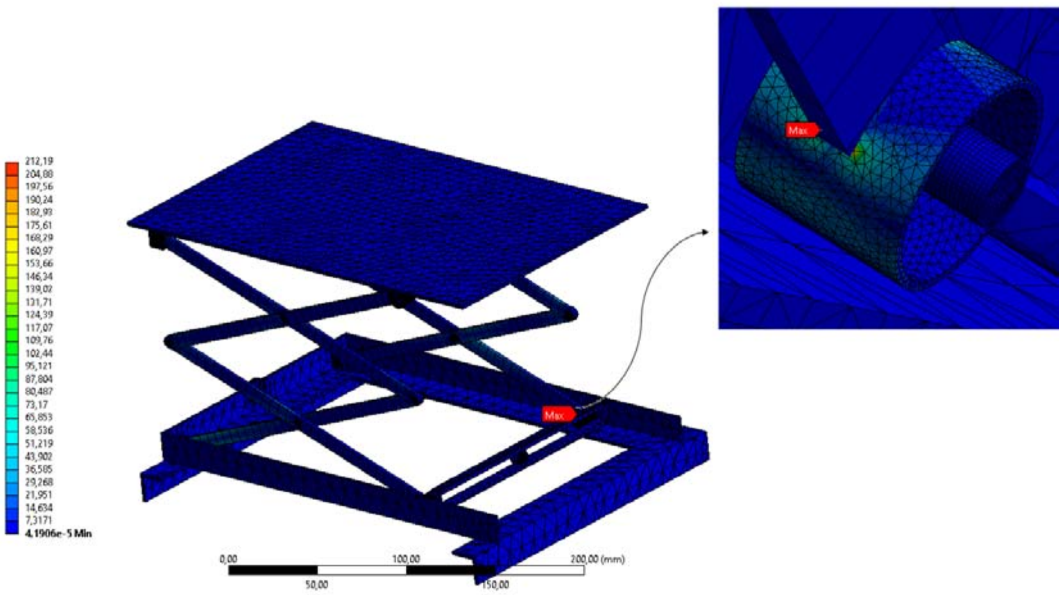

Fig4. Stress distribution of scissor platform prototype in finite element analysis

4. CONCLUSIONS

In this study, the prototype design of the scissor platform mechanism, which is used in many areas in the sector, was implemented using numerical analysis. Analysis conditions, including different loading and boundary conditions, were tested on the produced prototype. Consequently, considering the analysis results, the platform's maximum load to be carried was 3 kg. Furthermore, the critical region of the platform was determined under the loading condition, and the stress concentration in that region was removed by the hand grinding process.

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A SURVEY OF RECENT DEVELOPMENTS ON SPECTRAL PROPERTIES OF THE NON-SELFADJOINT DIRAC OPERATORS

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Abstract:

The fundamental advancements in the spectral theory of Dirac operators defined on infinite interval over the last few years are discussed in this review. We provide a comprehensive overview on the continuous and discrete analogues of the non-selfadjoint singular Dirac operator and present the some recent results. Eigenparameter dependent boundary values have been a popular research question since it has many application areas from quantum physics to engineering. However, in recent years impulsive boundary value problems have taken prominent attention from various authors. Beside summarizing the fundamental concepts regarding the Dirac operator, we discuss the effect of different boundary conditions on the structure of the spectrum of the Dirac operators. At the conclusion part of the study, we will present some set of research topics which may include open problems for this specific investigation area. Hence, this short survey might help the readers to better understand the recent studies on Dirac operators and lay the groundwork for future research questions.

Keywords: Non-selfadjoint operators, dirac operator, spectral theory
General area of research: Mathematics
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1. INTRODUCTION

Paul Adrien Maurice Dirac was a founder of quantum physics and the inventor of many of the field's later developments [17,25]. Dirac is considered one of the greatest physicists of all time, with Newton, Maxwell, Einstein, and others. He was born on August 8, 1902, in Bristol, England, and died in Tallahasee, Florida, on October 20, 1984. In 1933, Dirac and Erwin Schrödinger were jointly awarded the Nobel Prize in Physics. The most well-known of Dirac's accomplishments is his prediction of antimatter's existence, which is viewed as an unavoidable aspect of the Dirac equation.

After brief information about Dirac's life, let's move on to the main topic of this article. Dirac operators has a wide range of application areas ranging from quantum physics to functional analysis [1-34]. To be specific in our study, we will elaborate on the spectral theory of the non-selfadjoint Dirac operators on infinite interval. In particular, we concentrate on Dirac operators of the form

$$B\frac{dy}{dx}+P(x)y=\lambda y, y(x)=\begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}, \tag{1.1}$$

where

$$B=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

$$P(x)=\begin{pmatrix} p_{11}(x) & p_{12}(x) \\ p_{21}(x) & p_{22}(x) \end{pmatrix},$$

$$p_{12}(x)=p_{21}(x),$$

$p_{ik}(x),\ i,k=1,2$ are complex valued functions defined on the interval $x\in[0,\infty)$ and λ is a eigenparameter. The system (1.1) is equivalent to the system of two consistent ordinary first order differential equations

$$\begin{cases} y_2'+p_{11}(x)y_1+p_{12}(x)y_2=\lambda y_1, \\ -y_1'+p_{21}(x)y_1+p_{22}(x)y_2=\lambda y_2. \end{cases} \tag{1.2}$$

For $p_{12}(x)=p_{21}(x)=0$ and $p_{11}(x)=V(x)+m,\ p_{22}(x)=V(x)-m,\ V(x)$ is the potential function and m is the mass of the particle. The system of first order differential equations (1.2) is called a one dimensional stationary Dirac system in relativistic quantum theory [26,27]. The representation (1.1) is called canonical Dirac system.
Another representation for the Dirac system is

$$\begin{cases} i\dfrac{du_1(x,\lambda)}{dx}+q_1(x)u_2(x,\lambda)=\lambda u_1(x,\lambda), \\ -i\dfrac{du_2(x,\lambda)}{dx}+q_2(x)u_1(x,\lambda)=\lambda u_2(x,\lambda), \end{cases} \tag{1.3}$$

for which the functions in the Hilbert space where $U\in L_2(0,\infty;\mathbb{C}_2)$, such that $L_2(0,\infty;\mathbb{C}_2):=\left\{f:f(x)=\begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix},\int_0^\infty\left(|f_1(x)|^2+|f_2(x)|^2\right)dx<\infty\right\}.$
Under the transformations

$$y_1(x,\lambda)=\frac{1}{2}\left[u_1(x,\lambda)+u_2(x,\lambda)\right],$$

$$y_2(x,\lambda)=\frac{1}{2i}\left[u_2(x,\lambda)-u_1(x,\lambda)\right],$$

and for the special choices of

$$P(x)=\begin{pmatrix} \alpha(x) & \beta(x) \\ \beta(x) & -\alpha(x) \end{pmatrix},$$

$$\alpha(x)=\frac{1}{2}\left[q_1(x)+q_2(x)\right],$$

$$\beta(x)=\frac{i}{2}\left[q_1(x)-q_2(x)\right],$$

it can be seen that the all representations (1.1)-(1.3) are equivalent to each other.
Akin and Bairamov [3] investigated the spectral properties of the system (1.3) for

$$|q_i(x)|\le Ce^{\varepsilon\sqrt{x}},\ \varepsilon>0,\quad i=1,2, \tag{1.4}$$

and the boundary condition

$$u_2(0,\lambda)-hu_1(0,\lambda)=0. \tag{1.5}$$

In particular, they proved that the operator defined by help of expressions from (1.3) to (1.5) has a finite number of eigenvalues and spectral singularities. They benefited from the uniqueness theorem of Beurling to prove these spectral quantitative properties.
Later, the spectral singularities of the system (1.3) and eigenparamer dependent boundary condition has been considered by Karaman [18]. Also, spectral singularities of the operator under the integral boundary condition has been taken into investigation by Yardimci and et. all [33].
Spectral expansion theorems for non-seladjoint Dirac operator represented by (1.3) has been main topic of the doctorate thesis of Kir [19].
Recently, spectral singularities and bound states of Dirac operator of with a point interaction at origin has been treated by Bairamov and et all [13].

2. DISCRETE DIRAC OPERATORS

Let us define the regions

$$P_0 = \{z : z = x + iy, y > 0, 0 \le x < 4\pi\},$$

and

$$P = P_0 \cup [0, 4\pi).$$

Define also the Hilbert space of vector sequences $l_2(C, C^2)$ with the inner product

$$\langle y, u \rangle = \sum_{n\in\mathbb{N}} \left(y_n^{(1)} \overline{u_n^{(1)}} + y_n^{(2)} \overline{u_n^{(2)}} \right).$$

The sequences $\{p_n\}_{n\in N}$ and $\{q_n\}_{n\in N}$ are complex valued. We define discrete Dirac operator with the difference expression [11]

$$(L_0 y)_n = \begin{pmatrix} \Delta y_n^{(2)} + p_n y_n^{(1)} \\ -\Delta y_{n-1}^{(1)} + q_n y_n^{(2)} \end{pmatrix}, \ n \in N.$$

The system of equations $(L_0 y)_n = \lambda y_n$ can also be rewritten in the form

$$\begin{cases} y_{n+1}^{(2)} - y_n^{(2)} + p_n y_n^{(1)} = \lambda y_n^{(1)} \\ -y_n^{(1)} + y_{n-1}^{(1)} + q_n y_n^{(2)} = \lambda y_n^{(2)} \end{cases}, \ n \in N. \tag{2.1}$$

Let us assume that the sequences $\{p_n\}_{n\in N}$ and $\{q_n\}_{n\in N}$ hold the relation

$$\sum_{n=1}^{\infty} n\left(|p_n| + |q_n|\right) < \infty. \tag{2.2}$$

Note that the system (2.1) is discrete analogue of the Dirac system (1.1)-(1.3) [9-15].
Bairamov and Celebi was first to investigate certain spectral properties of the non-selfadjoint discrete Dirac operators.
Now, we will present some well-known results regarding the discrete Dirac operators.

Teorem 2.1. [11] Let us aasume $\lambda = 2\sin\frac{z}{2}$ and $z \in \overline{C}_+$. Under the condition (2.2), the discrete Dirac system (2.1) has unique solution for

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$K_{nm} = \begin{pmatrix} K_{nm}^{11} & K_{nm}^{12} \\ K_{nm}^{21} & K_{nm}^{22} \end{pmatrix},$$

$n \in N$, $m = 1, 2, \ldots$ such that

$$f_0^{(1)}(z)=e^{i\frac{z}{2}}\left[1+\sum_{m=1}^{\infty}K_{0m}^{11}e^{imz}\right]-i\sum_{m=1}^{\infty}K_{0m}^{12}e^{imz} \tag{2.3}$$

and for $n=1,2,3,...$

$$f_n(z)=\begin{pmatrix} f_n^{(1)}(z) \\ f_n^{(2)}(z) \end{pmatrix}_{n\in N}=\left(\left(I+\sum_{m=1}^{\infty}K_{nm}e^{imz}\right)\begin{pmatrix} e^{i\frac{z}{2}} \\ -i \end{pmatrix}e^{inz}\right)_{n\in N}. \tag{2.4}$$

$f_n(z)$ is analytic on C_+ and continuous on $\overline{C}_+$. Note that $\left[\left|\frac{m}{2}\right|\right]$, denotes the integer part of $\frac{m}{2}$ and for $i,j=1,2$ and arbitrary positive constant $C>0$, the following inequality satisfies

$$\left|K_{nm}^{ij}\right|\leq C\sum_{k=n+\left[\left|\frac{m}{2}\right|\right]}^{\infty}\left(\left|p_k\right|+\left|q_k\right|\right). \tag{2.5}$$

The sequences of K_{nm}^{ij} can be written in a unique way in terms of $\{p_n\}_{n\in N}$ and $\{q_n\}_{n\in N}$.

Lemma 2.2. [11] Let $i,j=1,2$ and $\forall m\in R$. Then, $\left\{K_{nm}^{ij}\right\}_{n\in N}\in l_1(C)$.

Lemma 2.3. [11] The solution $f_n(z)$ given by (2.3) and (2.4) of the discrete Dirac system given by (2.1) hold the following asymptotic relations

$$\begin{pmatrix} f_n^{(1)}(z) \\ f_n^{(2)}(z) \end{pmatrix}_{n\in N}=\left[I+o(1)\right]\begin{pmatrix} e^{i\frac{z}{2}} \\ -i \end{pmatrix}e^{inz},\ z\in\overline{C}_+,\ n\to\infty,$$

and

$$\begin{pmatrix} f_n^{(1)}(z) \\ f_n^{(2)}(z) \end{pmatrix}_{n\in N}=\left[I+o(1)\right]\begin{pmatrix} e^{i\frac{z}{2}} \\ -i \end{pmatrix}e^{inz},\ n\in N,\ z\in\overline{C}_+,\ \operatorname{Im}z\to\infty.$$

Definition 2.4. [11] Let two arbitrary solutions of the difference system (2.1) is given by $y=\begin{pmatrix} y_n^{(1)}(\lambda) \\ y_n^{(2)}(\lambda) \end{pmatrix}_{n\in N}$ and $u=\begin{pmatrix} u_n^{(1)}(\lambda) \\ u_n^{(2)}(\lambda) \end{pmatrix}_{n\in N}$. Then, the Wronskian is defined by the equation

$$W[y,u]=y_n^{(1)}(\lambda)u_{n+1}^{(2)}(\lambda)-y_{n+1}^{(2)}(\lambda)u_n^{(1)}(\lambda). \tag{2.6}$$

Lemma 2.5. [11] The Wronskian defined by (3.6) is independent from the variable n.

The spectral properties of the non-selfadjoint operator L defined in the Hilbert space of sequences $l_2(C,C^2)$ and the general boundary condition

$$\sum_{n=0}^{\infty}h_ny_n=0 \tag{2.7}$$

has been considered by Yokus and Coskun [15] in 2020. Note that the condition (2.7) is discrete analogue of the integral boundary condition [33]. In that paper, not only the conditions imposed on the potential, but also the structure of the complex valued sequence h_n effects the quantitative properties of the spectrum of the discrete Dirac operator.

Non-selfadjoint discrete Dirac operator for eigenparameter dependent boundary condition

$$(\gamma_0+\gamma_1\lambda)y_1^{(2)}+(\beta_0+\beta_1\lambda)y_0^{(1)}=0,$$

have been major research topic of the papers [12].

Bairamov and Coskun generalized the results to $n\in R$ [9,10]. Note that $n\in R$ case includes $n\in R$ as a special case and corresponds to whole line problems in continuous cases; in other words, $x\in$ □ case.

Discrete Dirac operator with eigenparameter dependent boundary condition at generalized polynomial form

$$\sum_{k=0}^{p}\left(y_1^{(2)}\gamma_k + y_0^{(1)}\beta_k\right)\lambda^k = 0,$$

studied by Koprubasi and Mohapatra [23,24]. It is obvious that this form of general boundary condition includes the boundary conditions which are having the first and quadratic power of the eigenparameters. In such problems, Jost function and asymptotic properties of the Jost function strictly depend on the eigenparameter dependent boundary conditions.

Note also that in these studies, authors considered the spectral parameter under the trigonometric transformation

$$\lambda = 2\sin\frac{z}{2}.$$

However , in the recent study of Koprubasi [22], hyperbolic type transformation case

$$\lambda = 2\sinh\frac{z}{2}$$

has been adopted to the discrete Dirac operators along with transmission condition. With the help of this transformation, analicty region of the Jost solution has shifted from upper half-plane to the left half-plane.

It is also important to notice that spectral problems stimulate each other and push the researches to investigate for different cases. For instance, if a specific boundary condition has been solved for Sturm-Liouville type operator, one may ask the question of can we apply this boundary condition for other type of operators. Clearly, different boundary conditions model different physical models. Hence, to ask such questions provide a meaningful basis in applications of these problems in real world phenomenas.

In the studies of [5], the spectral properties of the Dirac operator for matrix coefficient case has been investigated.

3. CONCLUSIONS

To conculude, in this review we basicly considered the recent literature regarding spectral properties of the the non-selfadjoint Dirac operators investigated by the methods and theorems of Naimark, Pavlov, Lyance and others.

In particular, we presented a short survey on the papers which Naimark's and Pavlov's conditions for the finiteness of the eigenvalues and spectral singularities have been implemented. As a consequence of wide branch of application of Dirac operators, this problems remain a popular research area. Furthermore, there are a number of unresolved issues that need to be addressed. For instance, quantum and time scale calculus analogues of Dirac operators under various boundary conditions still remains an open research area. Clearly, the Hilbert spaces change in continuous and discrete cases. In a similar manner, quantum calculus and time scales cases will bring a completely a new type of inner product space. The representations of the fundamental solutions depend strictly on the definition of the Hilbert space. As a result, these type of problems might be more challenging and bring new application areas in quantum physics. Interested readers in this area should not restrict on themselves only to the singular non-selfadjoint cases. New developments in scattering and inverse scattering problems for self-adjoint operators may create inspiration for researchers, too.

It is also worth to emphasize that there are plenty of papers in the literature solved by this specific methods which we could not mention in this survey. Further, based on the notions referred in this study, interested researchers may solve the problems which may be seen as the continuation of these papers. For example, one may investigate the principal functions of the operator corresponding to the spectral singularities and eigenvalues. Also, spectral expansion problems still remain a productive research area.

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A GENERALISATION OF 12 SOFT DECISION-MAKING METHODS UTILISED IN FPFS-MATRICES SPACE TO THE IFPIFS-MATRICES SPACE AND THEIR APPLICATION IN DECISION MAKING

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Abstract:

Lately, the concept of intuitionistic fuzzy parameterized intuitionistic fuzzy soft matrices (*ifpifs*-matrices) has become a popular mathematical tool thanks to the skill of modelling the decision-making problems where the parameters and alternatives have intuitionistic fuzzy values. For this reason, the present study generalises 12 soft decision-making (SDM) methods provided in the study entitled "Configurations of SDM Methods Proposed between 1999 and 2012: A Follow-Up Study" to *ifpifs*-matrices space. It then analyses the generalised SDM methods by benefiting the five test cases to determine the SDM methods producing a valid ranking order in all test cases. Besides, to rank the performances of seven well-known noise removal filters, it applies the successful SDM methods in these test cases to a performance-based value assignment (PVA) problem. Finally, this study discusses the generalisations of the SDM methods to superstructures of *ifpifs*-matrices and the need for further research.

Keywords: Fuzzy sets, intuitionistic fuzzy sets, soft sets, *ifpifs*-matrices, soft decision making
General area of research: Mathematics
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1. INTRODUCTION

The concept of intuitionistic fuzzy sets [1] has been propounded as a generalisation of fuzzy sets [2]. Soft sets [3] have been then proposed to eliminate the lack of a parameterization tool in these concepts. Moreover, as hybrid versions of aforesaid concepts, fuzzy parameterized fuzzy soft sets (*fpfs*-sets) [4] and intuitionistic fuzzy parameterized intuitionistic fuzzy soft sets (*ifpifs*-sets) [5] have been introduced to model the decision-making problems, in which the parameters and alternatives contain fuzzy and intuitionistic fuzzy values, respectively. Later, the concepts of fuzzy parameterized fuzzy soft matrices (*fpfs*-matrices) [6] and intuitionistic fuzzy parameterized intuitionistic fuzzy soft matrices (*ifpifs*-matrices) [7] have been defined to improve these two concepts' modelling abilities in such problems having a large number of data, and efficacious soft decision-making (SDM) methods [6,7] have been suggested. Recently, [8] and [9] have generalised 24 SDM methods in [6,10-13] employing a single *fpfs*-matrix and 36 SDM methods in [12-21] employing multiple *fpfs*-matrices, respectively, to the *ifpifs*-matrices space. Besides, they have proposed five test cases to analyse the success of these SDM methods in the ranking orders of the alternatives in a decision-making problem. Inspired by these pioneering studies, the present study focuses on the generalisations of the SDM methods provided in [22] to *ifpifs*-matrices space. Moreover, the contributions of this study can be listed as follows:

- The current study makes a theoretical contribution to the literature by generalising 7 SDM methods employing two *fpfs*-matrices and 5 SDM methods using three *fpfs*-matrices to operable in *ifpifs*-matrices space.
- This study determines successful ones among the generalised SDM methods by utilising the five test cases.
- Unlike many SDM methods applied to fictitious decision-making problems, the study applies successful SDM methods to a real-life decision-making problem concerning evaluating the performance of the known noise removal filters.

Section 2 presents the definitions of some basic concepts to be employed in the study's following sections. Section 3 introduces the algorithm steps of the generalised SDM methods, namely iRM07a(C), iRM07o(C), iKGW09(C), iKWW11/2(λ_1,λ_2,R), iNS11, iBMM12, iBMM12/2, iBMM12/3, iCD12/3, iCD12/4, iFLC12, and iQYZ12(C). Section 4 analyses the success of these SDM methods according to five test cases. Section 5 applies the successful SDM methods in all test cases to a performance-based value assignment (PVA) problem to compare the performances of the well-known noise removal filters in image denoising. The last section discusses the need for further research.

2. PRELIMINARIES

This section presents some notions, i.e., intuitionistic fuzzy sets [1], *ifpifs*-sets [5], *ifpifs*-matrices [7], and linear ordering relation [23].

Definition 2.1. [1] Let E be a non-empty set and μ and ν be two functions from E to $[0,1]$ such that $0 \leq \mu(x) + \nu(x) \leq 1$, for all $x \in E$. Then, the set $\left\{\left(x, \mu(x), \nu(x)\right) : x \in E\right\}$ is called an intuitionistic fuzzy set over E.
Here, for all $x \in E$, $\mu(x)$ and $\nu(x)$ are called the membership and non-membership degrees, respectively, and $\pi(x) = 1 - \mu(x) - \nu(x)$ is called the indeterminacy degree of the element $x \in E$.

In the present study, the set of all the intuitionistic fuzzy sets over E is denoted by $IF(E)$ and $f \in IF(E)$. Besides, for brevity, the notation ${}^{\mu(x)}_{\nu(x)}x$ is used instead of $\left(x, \mu(x), \nu(x)\right)$. Hence, an intuitionistic fuzzy set f over E is denoted by $f := \left\{{}^{\mu(x)}_{\nu(x)}x : x \in E\right\}$. Moreover, we do not display the elements ${}^{0}_{1}x$ in an intuitionistic fuzzy set.

Definition 2.2. [5] Let $f \in IF(E)$ and α be a function from f to $IF(U)$. Then, the set $\left\{\left({}^{\mu(x)}_{\nu(x)}x, \alpha\left({}^{\mu(x)}_{\nu(x)}x\right)\right) : x \in E\right\}$, being the graphic of α, is called an intuitionistic fuzzy parameterized intuitionistic fuzzy soft set (*ifpifs*-set) parameterized via E over U (or briefly over U).

From now on, the set of all the *ifpifs*-sets over U is denoted by $IFPIFS_E(U)$. In $IFPIFS_E(U)$, since the graph(α) and α generate each other uniquely, the notations are interchangeable. Therefore, if it causes no confusion, we denote an *ifpifs*-set graph(α) by α. Moreover, we do not display the elements $({}^{0}_{1}x, 0_U)$ in an *ifpifs*-set. Here, $0_U := \{{}^{0}_{1}u : u \in U\}$.

Example 2.1. Let $E = \{x_1, x_2, x_3, x_4\}$ and $U = \{u_1, u_2, u_3, u_4\}$. Then,

$$\alpha = \left\{\left({}^{0.3}_{0.4}x_1, \left\{{}^{0.2}_{0.6}u_1, {}^{0.3}_{0.5}u_2, {}^{0.4}_{0.4}u_3, {}^{1}_{0}u_4\right\}\right), \left({}^{0.7}_{0.1}x_3, \left\{{}^{0.4}_{0}u_2, {}^{1}_{0}u_3, {}^{0.4}_{0.5}u_4\right\}\right), \left({}^{0.2}_{0.3}x_4, \left\{{}^{0.4}_{0.6}u_1, {}^{0.8}_{0.1}u_3, {}^{0.5}_{0.3}u_4\right\}\right)\right\}$$

is an *ifpifs*-set over U.

Definition 2.3. [7] Let $\alpha \in IFPIFS_E(U)$. Then, $[a_{ij}]$ is called *ifpifs*-matrix of α and is defined by

$$[a_{ij}] := \begin{bmatrix} a_{01} & a_{02} & a_{03} & \cdots & a_{0n} & \cdots \\ a_{11} & a_{12} & a_{13} & \cdots & a_{1n} & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots \end{bmatrix}$$

such that for $i \in \{0,1,2,\cdots\}$ and $j \in \{1,2,\cdots\}$,

$$a_{ij} := \begin{cases} {}^{\mu(x_j)}_{\nu(x_j)}, & i = 0 \\ \alpha\left({}^{\mu(x_j)}_{\nu(x_j)}x_j\right)(u_i), & i \neq 0 \end{cases}$$

or briefly $a_{ij} := {}^{\mu_{ij}}_{\nu_{ij}}$. Here, if $|U| = m - 1$ and $|E| = n$, then $[a_{ij}]$ is an $m \times n$ *ifpifs*-matrix.

In this paper, if it causes no confusion, the membership and non-membership functions of $[a_{ij}]$, i.e., μ_{ij} and ν_{ij}, will be represented by μ^a_{ij} and ν^a_{ij}, respectively. Moreover, the set of all the *ifpifs*-matrices parameterized via E over U is denoted by $IFPIFS_E[U]$ and $[a_{ij}], [b_{ij}], [c_{ij}] \in IFPIFS_E[U]$.

Example 2.2. The *ifpifs*-matrix of α provided in Example 2.1 is as follows:

$$[a_{ij}] = \begin{bmatrix} {}^{0.3}_{0.4} & {}^{0}_{1} & {}^{0.7}_{0.1} & {}^{0.2}_{0.3} \\ {}^{0.2}_{0.6} & {}^{0}_{1} & {}^{0}_{1} & {}^{0.4}_{0.6} \\ {}^{0.3}_{0.5} & {}^{0}_{1} & {}^{0.4}_{0} & {}^{0}_{1} \\ {}^{0.4}_{0.4} & {}^{0}_{1} & {}^{1}_{0} & {}^{0.8}_{0.1} \\ {}^{1}_{0} & {}^{0}_{1} & {}^{0.4}_{0.5} & {}^{0.5}_{0.3} \end{bmatrix}$$

Definition 2.4. [22] Let $(j_1, j_2, \cdots, j_n), (k_1, k_2, \cdots, k_n) \in \mathbb{N}^n$. Then, the relation "$\leq$" is called linear ordering relation and is defined by

$$(j_1, j_2, \cdots, j_n) \leq (k_1, k_2, \cdots, k_n) \Leftrightarrow [j_1 < k_1 \vee (j_1 = k_1 \wedge j_2 < k_2) \vee \cdots \vee (j_1 = k_1 \wedge j_2 = k_2 \wedge \cdots \wedge j_{n-1} = k_{n-1} \wedge j_n \leq k_n)]$$

Proposition 2.1. [23] Let $IFV([0,1])$ be the set of all the intuitionistic fuzzy values and ${}^{\mu_1}_{\nu_1}, {}^{\mu_2}_{\nu_2} \in IFV([0,1])$. Then, the relation "$\lesseqgtr$" defined by

$$ {}^{\mu_1}_{\nu_1} \lesssim {}^{\mu_2}_{\nu_2} \Leftrightarrow \left[s_1\binom{\mu_1}{\nu_1} < s_1\binom{\mu_2}{\nu_2} \vee \left(s_1\binom{\mu_1}{\nu_1} = s_1\binom{\mu_2}{\nu_2} \wedge s_2\binom{\mu_1}{\nu_1} \leq s_2\binom{\mu_2}{\nu_2}\right)\right]$$

is a linear ordering relation over $IFV([0,1])$. Here, $s_1\binom{\mu_1}{\nu_1} := \mu_1 - \nu_1$ and $s_2\binom{\mu_1}{\nu_1} := \mu_1 + \nu_1$. Moreover, $s_1\binom{\mu_1}{\nu_1}$ and $s_2\binom{\mu_1}{\nu_1}$ are called score value and accuracy value of intuitionistic fuzzy value $\frac{\mu_1}{\nu_1}$, respectively.

3. GENERALISATIONS OF THE SDM METHODS

This section focuses on generalising 12 SDM methods [22] employed in *fpfs*-matrices space [6] to *ifpifs*-matrices space [7]. Henceforth, $I_n^* := \{0,1,2,\cdots,n\}$ and $I_n := \{1,2,\cdots,n\}$.

The SDM methods iRM07a(C) and iRM07o(C) are generalisations of RM07a [22] and RM07o [22], respectively. In the generalised methods, the notations "a" and "o" signify AND-product and OR-product of *ifpifs*-matrices, respectively. Moreover, C is a certain set of indices related to sets of parameters.

Algorithm 1. iRM07a(C)

Step 1. Construct three *ifpifs*-matrices $[a_{ij}]_{m\times n_1}$, $[b_{ik}]_{m\times n_2}$, and $[c_{il}]_{m\times n_3}$

Step 2. Determine a cartesian set C of indices such that $C \subseteq I_{n_1} \times I_{n_2} \times I_{n_3}$

Step 3. Obtain increasing sequence (r_t) consisting of all the elements of C such that $r_t := (u_t, v_t, w_t)$

Step 4. Obtain $[d_{it}]_{m\times|C|}$ defined by $d_{it} := \frac{\mu_{it}^d}{\nu_{it}^d}$ such that $i \in I_{m-1}^*$,

$$\mu_{it}^d := \min_{r_t\in C}\{\mu_{iu_t}^a, \mu_{iv_t}^b, \mu_{iw_t}^c\} \quad \text{and} \quad \nu_{it}^d := \max_{r_t\in C}\{\nu_{iu_t}^a, \nu_{iv_t}^b, \nu_{iw_t}^c\}$$

Here, $|C|$ denotes the cardinality of C.

Step 5. Apply iMBR01 [8] to $[d_{it}]$

Here, if isMBR01 [8] or iMBR01/2 [8] are applied to $[d_{it}]$ in the last step of iRM07a(C), then the SDM method can be denoted by isMBR01-based iRM07a(C) or iMBR01/2-based iRM07a(C), respectively.

Algorithm 2. iRM07o(C)

Step 1. Construct three *ifpifs*-matrices $[a_{ij}]_{m\times n_1}$, $[b_{ik}]_{m\times n_2}$, and $[c_{il}]_{m\times n_3}$

Step 2. Determine a cartesian set C of indices such that $C \subseteq I_{n_1} \times I_{n_2} \times I_{n_3}$

Step 3. Obtain increasing sequence (r_t) consisting of all the elements of C such that $r_t := (u_t, v_t, w_t)$

Step 4. Obtain $[d_{it}]_{m\times|C|}$ defined by $d_{it} := \frac{\mu_{it}^d}{\nu_{it}^d}$ such that $i \in I_{m-1}^*$,

$$\mu_{it}^d := \max_{r_t\in C}\{\mu_{iu_t}^a, \mu_{iv_t}^b, \mu_{iw_t}^c\} \quad \text{and} \quad \nu_{it}^d := \min_{r_t\in C}\{\nu_{iu_t}^a, \nu_{iv_t}^b, \nu_{iw_t}^c\}$$

Here, $|C|$ denotes the cardinality of C.

Step 5. Apply iMBR01 [8] to $[d_{it}]$

Here, if isMBR01 [8] or iMBR01/2 [8] are applied to $[d_{it}]$ in the last step of iRM07o(C), then the SDM method can be denoted by isMBR01-based iRM07o(C) or iMBR01/2-based iRM07o(C), respectively.

The SDM method iKGW09(C) is a generalisation of KGW09 [22]. In the generalised method, C is a certain set of indices related to sets of parameters.

Algorithm 3. iKGW09(C)

Step 1. Apply iRM07a(C) except its Step 5

Step 2. Apply iM11 [8] to $[d_{it}]$

The SDM method iKWW11/2(λ_1, λ_2, R) is a generalisation of KWW11/2(w, z) [22]. Since the score matrices in the second step of iKWW11/2(λ_1, λ_2, R) have intuitionistic fuzzy values, the third step of KWW11/2(w, z) is ignored herein. In the generalised method, λ_1 and λ_2 represent the fuzzy values. Besides, R denotes a set of indices concerning parameters.

Algorithm 4. iKWW11/2(λ_1, λ_2, R)

Step 1. Construct two *ifpifs*-matrices $[a_{ij}]_{m\times n}$ and $[b_{ij}]_{m\times n}$

Step 2. Apply iMRB02(R) [8] to $[a_{ij}]$ and $[b_{ij}]$ and obtain the score matrices $[s_{i1}^1]_{(m-1)\times 1}$ and $[s_{i1}^2]_{(m-1)\times 1}$, respectively

Step 3. Obtain $[c_{i1}^1]_{(m-1)\times 1}$, $[c_{i1}^2]_{(m-1)\times 1}$, $[d_{i1}^1]_{(m-1)\times 1}$, and $[d_{i1}^2]_{(m-1)\times 1}$ defined by

$$c_{i1}^1 \coloneqq \max_{k\in I_{m-1}} \{\mu_{k1}^{s^1}\} - \mu_{i1}^{s^1} \quad \text{and} \quad c_{i1}^2 \coloneqq \nu_{i1}^{s^1} - \min_{k\in I_{m-1}} \{\nu_{k1}^{s^1}\}$$

and

$$d_{i1}^1 \coloneqq \max_{k\in I_{m-1}} \{\mu_{k1}^{s^2}\} - \mu_{i1}^{s^2} \quad \text{and} \quad d_{i1}^2 \coloneqq \nu_{i1}^{s^2} - \min_{k\in I_{m-1}} \{\nu_{k1}^{s^2}\}$$

such that $i \in I_{m-1}$

Step 4. For $\lambda_1 \in [0,1]$, obtain $[e_{i1}^1]_{(m-1)\times 1}$, $[e_{i1}^2]_{(m-1)\times 1}$, $[f_{i1}^1]_{(m-1)\times 1}$, and $[f_{i1}^2]_{(m-1)\times 1}$ defined by

$$e_{i1}^1 \coloneqq \begin{cases} \dfrac{\min\limits_{k\in I_{m-1}} \{\min\{c_{k1}^1, d_{k1}^1\}\} + \lambda_1 \max\limits_{k\in I_{m-1}} \{\max\{c_{k1}^1, d_{k1}^1\}\}}{c_{i1}^1 + \lambda_1 \max\limits_{k\in I_{m-1}} \{\max\{c_{k1}^1, d_{k1}^1\}\}}, & c_{i1}^1 + \lambda_1 \max\limits_{k\in I_{m-1}} \{\max\{c_{k1}^1, d_{k1}^1\}\} \neq 0 \\ 1, & c_{i1}^1 + \lambda_1 \max\limits_{k\in I_{m-1}} \{\max\{c_{k1}^1, d_{k1}^1\}\} = 0 \end{cases}$$

$$e_{i1}^2 \coloneqq \begin{cases} \dfrac{\min\limits_{k\in I_{m-1}} \{\min\{c_{k1}^2, d_{k1}^2\}\} + \lambda_1 \max\limits_{k\in I_{m-1}} \{\max\{c_{k1}^2, d_{k1}^2\}\}}{c_{i1}^2 + \lambda_1 \max\limits_{k\in I_{m-1}} \{\max\{c_{k1}^2, d_{k1}^2\}\}}, & c_{i1}^2 + \lambda_1 \max\limits_{k\in I_{m-1}} \{\max\{c_{k1}^2, d_{k1}^2\}\} \neq 0 \\ 1, & c_{i1}^2 + \lambda_1 \max\limits_{k\in I_{m-1}} \{\max\{c_{k1}^2, d_{k1}^2\}\} = 0 \end{cases}$$

$$f_{i1}^1 \coloneqq \begin{cases} \dfrac{\min\limits_{k\in I_{m-1}} \{\min\{c_{k1}^1, d_{k1}^1\}\} + \lambda_1 \max\limits_{k\in I_{m-1}} \{\max\{c_{k1}^1, d_{k1}^1\}\}}{d_{i1}^1 + \lambda_1 \max\limits_{k\in I_{m-1}} \{\max\{c_{k1}^1, d_{k1}^1\}\}}, & d_{i1}^1 + \lambda_1 \max\limits_{k\in I_{m-1}} \{\max\{c_{k1}^1, d_{k1}^1\}\} \neq 0 \\ 1, & d_{i1}^1 + \lambda_1 \max\limits_{k\in I_{m-1}} \{\max\{c_{k1}^1, d_{k1}^1\}\} = 0 \end{cases}$$

and

$$f_{i1}^2 \coloneqq \begin{cases} \dfrac{\min\limits_{k\in I_{m-1}} \{\min\{c_{k1}^2, d_{k1}^2\}\} + \lambda_1 \max\limits_{k\in I_{m-1}} \{\max\{c_{k1}^2, d_{k1}^2\}\}}{d_{i1}^2 + \lambda_1 \max\limits_{k\in I_{m-1}} \{\max\{c_{k1}^2, d_{k1}^2\}\}}, & d_{i1}^2 + \lambda_1 \max\limits_{k\in I_{m-1}} \{\max\{c_{k1}^2, d_{k1}^2\}\} \neq 0 \\ 1, & d_{i1}^2 + \lambda_1 \max\limits_{k\in I_{m-1}} \{\max\{c_{k1}^2, d_{k1}^2\}\} = 0 \end{cases}$$

such that $i \in I_{m-1}$

Step 5. For $\lambda_2 \in [0,1]$, obtain $[g_{i1}^1]_{(m-1)\times 1}$ and $[g_{i1}^2]_{(m-1)\times 1}$ defined by

$$g_{i1}^1 \coloneqq \lambda_2 e_{i1}^1 + (1-\lambda_2) f_{i1}^1 \quad \text{and} \quad g_{i1}^2 \coloneqq \lambda_2 e_{i1}^2 + (1-\lambda_2) f_{i1}^2$$

such that $i \in I_{m-1}$

Step 6. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ defined by $s_{i1} \coloneqq {}^{\mu_{i1}^s}_{\nu_{i1}^s}$ such that $i \in I_{m-1}$,

$$\mu_{i1}^s \coloneqq \begin{cases} \dfrac{g_{i1}^1 + \left|\min\limits_{k\in I_{m-1}} \{g_{k1}^1\}\right|}{\max\limits_{k\in I_{m-1}} \{g_{k1}^1 + |g_{k1}^2|\} + \left|\min\limits_{k\in I_{m-1}} \{g_{k1}^1\}\right|}, & \max\limits_{k\in I_{m-1}} \{g_{k1}^1 + |g_{k1}^2|\} + \left|\min\limits_{k\in I_{m-1}} \{g_{k1}^1\}\right| \neq 0 \\ 1, & \max\limits_{k\in I_{m-1}} \{g_{k1}^1 + |g_{k1}^2|\} + \left|\min\limits_{k\in I_{m-1}} \{g_{k1}^1\}\right| = 0 \end{cases}$$

and

$$\nu_{i1}^s \coloneqq \begin{cases} 1 - \dfrac{g_{i1}^1 + |g_{i1}^2| + \left|\min\limits_{k\in I_{m-1}} \{g_{k1}^1\}\right|}{\max\limits_{k\in I_{m-1}} \{g_{k1}^1 + |g_{k1}^2|\} + \left|\min\limits_{k\in I_{m-1}} \{g_{k1}^1\}\right|}, & \max\limits_{k\in I_{m-1}} \{g_{k1}^1 + |g_{k1}^2|\} + \left|\min\limits_{k\in I_{m-1}} \{g_{k1}^1\}\right| \neq 0 \\ 0, & \max\limits_{k\in I_{m-1}} \{g_{k1}^1 + |g_{k1}^2|\} + \left|\min\limits_{k\in I_{m-1}} \{g_{k1}^1\}\right| = 0 \end{cases}$$

Step 7. Obtain the decision set $\left\{{}^{\mu_{k1}^s}_{\nu_{k1}^s} u_k : u_k \in U\right\}$

The SDM method iNS11 is a generalisation of NS11 [22].

Algorithm 5. iNS11

Step 1. Construct two *ifpifs*-matrices $[a_{ij}]_{m\times n}$ and $[b_{ij}]_{m\times n}$

Step 2. Obtain $[c_{ij}]_{m\times n}$ and $[d_{ij}]_{m\times n}$ defined by $c_{ij} \coloneqq {}^{\mu_{ij}^c}_{\nu_{ij}^c}$ and $d_{ij} \coloneqq {}^{\mu_{ij}^d}_{\nu_{ij}^d}$ such that $i \in I_{m-1}^*$, $j \in I_n$,

$$\mu_{ij}^c \coloneqq \nu_{ij}^a \quad \text{and} \quad \nu_{ij}^c \coloneqq \mu_{ij}^a$$

and

$$\mu_{ij}^d \coloneqq \nu_{ij}^b \quad \text{and} \quad \nu_{ij}^d \coloneqq \mu_{ij}^b$$

Step 3. Obtain $[e_{ij}]_{m\times n}$ defined by $e_{ij} \coloneqq {}^{\mu_{ij}^e}_{\nu_{ij}^e}$ such that $i \in I_{m-1}^*$, $j \in I_n$,

$$\mu_{ij}^e \coloneqq \max\{\mu_{ij}^a, \mu_{ij}^b\} \quad \text{and} \quad \nu_{ij}^e \coloneqq \min\{\nu_{ij}^a, \nu_{ij}^b\}$$

Step 4. Obtain $[f_{ij}]_{m\times n}$ defined by $f_{ij} \coloneqq {}^{\mu_{ij}^f}_{\nu_{ij}^f}$ such that $i \in I_{m-1}^*$, $j \in I_n$,

$$\mu_{ij}^f \coloneqq \max\{\mu_{ij}^c, \mu_{ij}^d\} \quad \text{and} \quad \nu_{ij}^f \coloneqq \min\{\nu_{ij}^c, \nu_{ij}^d\}$$

Step 5. Obtain $[g_{ij}^1]_{(m-1)\times n}$ and $[g_{ij}^2]_{(m-1)\times n}$ defined by

$$g_{ij}^1 \coloneqq (\mu_{0j}^e - \mu_{0j}^f)(\mu_{ij}^e - \mu_{ij}^f) \quad \text{and} \quad g_{ij}^2 \coloneqq (\nu_{0j}^e - \nu_{0j}^f)(\nu_{ij}^e - \nu_{ij}^f)$$

such that $i \in I_{m-1}$ and $j \in I_n$

Step 6. Obtain $[h_{i1}^1]_{(m-1)\times 1}$ and $[h_{i1}^2]_{(m-1)\times 1}$ defined by

$$h_{i1}^1 \coloneqq \sum_{j=1}^{n} g_{ij}^1 \quad \text{and} \quad h_{i1}^2 \coloneqq \sum_{j=1}^{n} g_{ij}^2$$

such that $i \in I_{m-1}$

Step 7. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ defined by $s_{i1} := \frac{\mu_{i1}^s}{\nu_{i1}^s}$ such that $i \in I_{m-1}$,

$$\mu_{i1}^s := \begin{cases} \dfrac{h_{i1}^1 + \left|\min\limits_{k\in I_{m-1}} \{h_{k1}^1\}\right|}{\max\limits_{k\in I_{m-1}} \{h_{k1}^1 + |h_{k1}^2|\} + \left|\min\limits_{k\in I_{m-1}} \{h_{k1}^1\}\right|}, & \max\limits_{k\in I_{m-1}} \{h_{k1}^1 + |h_{k1}^2|\} + \left|\min\limits_{k\in I_{m-1}} \{h_{k1}^1\}\right| \neq 0 \\ 1, & \max\limits_{k\in I_{m-1}} \{h_{k1}^1 + |h_{k1}^2|\} + \left|\min\limits_{k\in I_{m-1}} \{h_{k1}^1\}\right| = 0 \end{cases}$$

and

$$\nu_{i1}^s := \begin{cases} 1 - \dfrac{h_{i1}^1 + |h_{i1}^2| + \left|\min\limits_{k\in I_{m-1}} \{h_{k1}^1\}\right|}{\max\limits_{k\in I_{m-1}} \{h_{k1}^1 + |h_{k1}^2|\} + \left|\min\limits_{k\in I_{m-1}} \{h_{k1}^1\}\right|}, & \max\limits_{k\in I_{m-1}} \{h_{k1}^1 + |h_{k1}^2|\} + \left|\min\limits_{k\in I_{m-1}} \{h_{k1}^1\}\right| \neq 0 \\ 0, & \max\limits_{k\in I_{m-1}} \{h_{k1}^1 + |h_{k1}^2|\} + \left|\min\limits_{k\in I_{m-1}} \{h_{k1}^1\}\right| = 0 \end{cases}$$

Step 8. Obtain the decision set $\left\{{}^{\mu_{k1}^s}_{\nu_{k1}^s}u_k : u_k \in U\right\}$

The SDM method iBMM12 is a generalisation of BMM12 [22].

Algorithm 6. iBMM12

Step 1. Construct two *ifpifs*-matrices $[a_{ij}]_{m\times n}$ and $[b_{ij}]_{m\times n}$

Step 2. Obtain $[c_{kj}]_{n\times n}$ and $[d_{kj}]_{n\times n}$ defined by $c_{kj} := \frac{\mu_{kj}^c}{\nu_{kj}^c}$ and $d_{kj} := \frac{\mu_{kj}^d}{\nu_{kj}^d}$ such that $k, j \in I_n$,

$$\mu_{kj}^c := \min\{\mu_{0j}^a, \mu_{0k}^b\} \quad \text{and} \quad \nu_{kj}^c := \max\{\nu_{0j}^a, \nu_{0k}^b\}$$

and

$$\mu_{kj}^d := \mu_{jk}^c \quad \text{and} \quad \nu_{kj}^d := \nu_{jk}^c$$

Step 3. Obtain $[e_{ij}^1]_{(m-1)\times n}$, $[e_{ij}^2]_{(m-1)\times n}$, $[f_{ij}^1]_{(m-1)\times n}$, and $[f_{ij}^2]_{(m-1)\times n}$ defined by

$$e_{ij}^1 := \sum_{k=1}^{n} \min\{\mu_{ik}^a, \mu_{kj}^d\} \quad \text{and} \quad e_{ij}^2 := \sum_{k=1}^{n} \max\{\nu_{ik}^a, \nu_{kj}^d\}$$

and

$$f_{ij}^1 := \sum_{k=1}^{n} \min\{\mu_{ik}^b, \mu_{kj}^c\} \quad \text{and} \quad f_{ij}^2 := \sum_{k=1}^{n} \max\{\nu_{ik}^b, \nu_{kj}^c\}$$

such that $i \in I_{m-1}$ and $j \in I_n$

Step 4. Obtain $[g_{ij}^1]_{(m-1)\times n}$, $[g_{ij}^2]_{(m-1)\times n}$, $[h_{ij}^1]_{(m-1)\times n}$, and $[h_{ij}^2]_{(m-1)\times n}$ defined by

$$g_{ij}^1 := \begin{cases} \dfrac{e_{ij}^1}{\sum_{k=1}^{m-1}\sum_{l=1}^{n} e_{kl}^1}, & \sum\limits_{k=1}^{m-1}\sum\limits_{l=1}^{n} e_{kl}^1 \neq 0 \\ 0, & \sum\limits_{k=1}^{m-1}\sum\limits_{l=1}^{n} e_{kl}^1 = 0 \end{cases} \quad \text{and} \quad g_{ij}^2 := \begin{cases} \dfrac{e_{ij}^2}{\sum_{k=1}^{m-1}\sum_{l=1}^{n} e_{kl}^2}, & \sum\limits_{k=1}^{m-1}\sum\limits_{l=1}^{n} e_{kl}^2 \neq 0 \\ 0, & \sum\limits_{k=1}^{m-1}\sum\limits_{l=1}^{n} e_{kl}^2 = 0 \end{cases}$$

and

$$h_{ij}^1 := \begin{cases} \dfrac{f_{ij}^1}{\sum_{k=1}^{m-1}\sum_{l=1}^{n} f_{kl}^1}, & \displaystyle\sum_{k=1}^{m-1}\sum_{l=1}^{n} f_{kl}^1 \neq 0 \\ 0, & \displaystyle\sum_{k=1}^{m-1}\sum_{l=1}^{n} f_{kl}^1 = 0 \end{cases} \quad \text{and} \quad h_{ij}^2 := \begin{cases} \dfrac{f_{ij}^2}{\sum_{k=1}^{m-1}\sum_{l=1}^{n} f_{kl}^2}, & \displaystyle\sum_{k=1}^{m-1}\sum_{l=1}^{n} f_{kl}^2 \neq 0 \\ 0, & \displaystyle\sum_{k=1}^{m-1}\sum_{l=1}^{n} f_{kl}^2 = 0 \end{cases}$$

such that $i \in I_{m-1}$ and $j \in I_n$

Step 5. Obtain $[u_{ij}^1]_{(m-1)\times n}$ and $[u_{ij}^2]_{(m-1)\times n}$ defined by

$$u_{ij}^1 := \max\{g_{ij}^1, h_{ij}^1\} \quad \text{and} \quad u_{ij}^2 := \min\{g_{ij}^2, h_{ij}^2\}$$

such that $i \in I_{m-1}$ and $j \in I_n$

Step 6. Obtain $[v_{i1}^1]_{(m-1)\times 1}$ and $[v_{i1}^2]_{(m-1)\times 1}$ defined by

$$v_{i1}^1 := \sum_{j=1}^{n} u_{ij}^1 \quad \text{and} \quad v_{i1}^2 := \sum_{j=1}^{n} u_{ij}^2$$

such that $i \in I_{m-1}$

Step 7. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ defined by $s_{i1} := \frac{\mu_{i1}^s}{\nu_{i1}^s}$ such that $i \in I_{m-1}$,

$$\mu_{i1}^s := \begin{cases} \dfrac{v_{i1}^1 + \left|\min\limits_{k\in I_{m-1}}\{v_{k1}^1\}\right|}{\max\limits_{k\in I_{m-1}}\{v_{k1}^1 + |v_{k1}^2|\} + \left|\min\limits_{k\in I_{m-1}}\{v_{k1}^1\}\right|}, & \max\limits_{k\in I_{m-1}}\{v_{k1}^1 + |v_{k1}^2|\} + \left|\min\limits_{k\in I_{m-1}}\{v_{k1}^1\}\right| \neq 0 \\ 1, & \max\limits_{k\in I_{m-1}}\{v_{k1}^1 + |v_{k1}^2|\} + \left|\min\limits_{k\in I_{m-1}}\{v_{k1}^1\}\right| = 0 \end{cases}$$

and

$$\nu_{i1}^s := \begin{cases} 1 - \dfrac{v_{i1}^1 + |v_{i1}^2| + \left|\min\limits_{k\in I_{m-1}}\{v_{k1}^1\}\right|}{\max\limits_{k\in I_{m-1}}\{v_{k1}^1 + |v_{k1}^2|\} + \left|\min\limits_{k\in I_{m-1}}\{v_{k1}^1\}\right|}, & \max\limits_{k\in I_{m-1}}\{v_{k1}^1 + |v_{k1}^2|\} + \left|\min\limits_{k\in I_{m-1}}\{v_{k1}^1\}\right| \neq 0 \\ 0, & \max\limits_{k\in I_{m-1}}\{v_{k1}^1 + |v_{k1}^2|\} + \left|\min\limits_{k\in I_{m-1}}\{v_{k1}^1\}\right| = 0 \end{cases}$$

Step 8. Obtain the decision set $\left\{{}^{\mu_{k1}^s}_{\nu_{k1}^s}u_k : u_k \in U\right\}$

The SDM method iBMM12/2 is a generalisation of BMM12/2 [22].

Algorithm 7. iBMM12/2

Step 1. Construct three *ifpifs*-matrices $[a_{ij}]_{m\times n}$, $[b_{ij}]_{m\times n}$, and $[c_{ij}]_{m\times n}$

Step 2. Obtain $[d_{kj}]_{n\times n}$, $[e_{kj}]_{n\times n}$, and $[f_{kj}]_{n\times n}$ defined by $d_{kj} := \frac{\mu_{kj}^d}{\nu_{kj}^d}$, $e_{kj} := \frac{\mu_{kj}^e}{\nu_{kj}^e}$, and $f_{kj} := \frac{\mu_{kj}^f}{\nu_{kj}^f}$ such that $k, j \in I_n$,

$$\mu_{kj}^d := \min\{\mu_{0k}^a, \mu_{0j}^b, \mu_{0j}^c\} \quad \text{and} \quad \nu_{kj}^d := \max\{\nu_{0k}^a, \nu_{0j}^b, \nu_{0j}^c\}$$

$$\mu_{kj}^{e} := \min\{\mu_{0j}^{a}, \mu_{0k}^{b}, \mu_{0j}^{c}\} \quad \text{and} \quad \nu_{kj}^{e} := \max\{\nu_{0j}^{a}, \nu_{0k}^{b}, \nu_{0j}^{c}\}$$

and

$$\mu_{kj}^{f} := \min\{\mu_{0j}^{a}, \mu_{0j}^{b}, \mu_{0k}^{c}\} \quad \text{and} \quad \nu_{kj}^{f} := \max\{\nu_{0j}^{a}, \nu_{0j}^{b}, \nu_{0k}^{c}\}$$

Step 3. Obtain $[u_{ij}^{1}]_{(m-1)\times n}$, $[u_{ij}^{2}]_{(m-1)\times n}$, $[v_{ij}^{1}]_{(m-1)\times n}$, $[v_{ij}^{2}]_{(m-1)\times n}$, $[w_{ij}^{1}]_{(m-1)\times n}$, and $[w_{ij}^{2}]_{(m-1)\times n}$ defined by

$$u_{ij}^{1} := \sum_{k=1}^{n} \min\{\mu_{ik}^{a}, \mu_{kj}^{d}\} \quad \text{and} \quad u_{ij}^{2} := \sum_{k=1}^{n} \max\{\nu_{ik}^{a}, \nu_{kj}^{d}\}$$

$$v_{ij}^{1} := \sum_{k=1}^{n} \min\{\mu_{ik}^{b}, \mu_{kj}^{e}\} \quad \text{and} \quad v_{ij}^{2} := \sum_{k=1}^{n} \max\{\nu_{ik}^{b}, \nu_{kj}^{e}\}$$

and

$$w_{ij}^{1} := \sum_{k=1}^{n} \min\{\mu_{ik}^{c}, \mu_{kj}^{f}\} \quad \text{and} \quad w_{ij}^{2} := \sum_{k=1}^{n} \max\{\nu_{ik}^{c}, \nu_{kj}^{f}\}$$

such that $i \in I_{m-1}$ and $j \in I_n$

Step 4. Obtain $[g_{ij}^{1}]_{(m-1)\times n}$ and $[g_{ij}^{2}]_{(m-1)\times n}$ defined by

$$g_{ij}^{1} := \max\{u_{ij}^{1}, v_{ij}^{1}, w_{ij}^{1}\} \quad \text{and} \quad g_{ij}^{2} := \min\{u_{ij}^{2}, v_{ij}^{2}, w_{ij}^{2}\}$$

such that $i \in I_{m-1}$ and $j \in I_n$

Step 5. Obtain $[h_{i1}^{1}]_{(m-1)\times 1}$ and $[h_{i1}^{2}]_{(m-1)\times 1}$ defined by

$$h_{i1}^{1} := \sum_{j=1}^{n} g_{ij}^{1} \quad \text{and} \quad h_{i1}^{2} := \sum_{j=1}^{n} g_{ij}^{2}$$

such that $i \in I_{m-1}$

Step 6. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ defined by $s_{i1} := \frac{\mu_{i1}^{s}}{\nu_{i1}^{s}}$ such that $i \in I_{m-1}$,

$$\mu_{i1}^{s} = \begin{cases} \dfrac{h_{i1}^{1} + \left|\min\limits_{k\in I_{m-1}}\{h_{k1}^{1}\}\right|}{\max\limits_{k\in I_{m-1}}\{h_{k1}^{1} + |h_{k1}^{2}|\} + \left|\min\limits_{k\in I_{m-1}}\{h_{k1}^{1}\}\right|}, & \max\limits_{k\in I_{m-1}}\{h_{k1}^{1} + |h_{k1}^{2}|\} + \left|\min\limits_{k\in I_{m-1}}\{h_{k1}^{1}\}\right| \neq 0 \\ 1, & \max\limits_{k\in I_{m-1}}\{h_{k1}^{1} + |h_{k1}^{2}|\} + \left|\min\limits_{k\in I_{m-1}}\{h_{k1}^{1}\}\right| = 0 \end{cases}$$

and

$$\nu_{i1}^{s} = \begin{cases} 1 - \dfrac{h_{i1}^{1} + |h_{i1}^{2}| + \left|\min\limits_{k\in I_{m-1}}\{h_{k1}^{1}\}\right|}{\max\limits_{k\in I_{m-1}}\{h_{k1}^{1} + |h_{k1}^{2}|\} + \left|\min\limits_{k\in I_{m-1}}\{h_{k1}^{1}\}\right|}, & \max\limits_{k\in I_{m-1}}\{h_{k1}^{1} + |h_{k1}^{2}|\} + \left|\min\limits_{k\in I_{m-1}}\{h_{k1}^{1}\}\right| \neq 0 \\ 0, & \max\limits_{k\in I_{m-1}}\{h_{k1}^{1} + |h_{k1}^{2}|\} + \left|\min\limits_{k\in I_{m-1}}\{h_{k1}^{1}\}\right| = 0 \end{cases}$$

Step 7. Obtain the decision set $\left\{ {}^{\mu_{k1}^{s}}_{\nu_{k1}^{s}} u_k : u_k \in U \right\}$

The SDM method iBMM12/3 is a generalisation of BMM12/3 [22].

Algorithm 8. iBMM12/3

Step 1. Apply iBMM12 except its Step 4

The SDM methods iCD12/3 and iCD12/4 are generalisations of CD12/3 [22] and CD12/4 [22], respectively.

Algorithm 9. iCD12/3

Step 1. Construct two *ifpifs*-matrices $[a_{ij}]_{m\times n}$ and $[b_{ij}]_{m\times n}$

Step 2. Obtain $[c_{ij}]_{m\times n}$ defined by $c_{ij} := \frac{\mu_{ij}^c}{\nu_{ij}^c}$ such that $i \in I_{m-1}, j \in I_n$,

$$\mu_{0j}^c := \mu_{0j}^a + \mu_{0j}^b - \mu_{0j}^a \mu_{0j}^b \quad \text{and} \quad \mu_{ij}^c := \max\{\mu_{ij}^a, \mu_{ij}^b\}$$

and

$$\nu_{0j}^c := \nu_{0j}^a \nu_{0j}^b \quad \text{and} \quad \nu_{ij}^c := \min\{\nu_{ij}^a, \nu_{ij}^b\}$$

Step 3. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ defined by $s_{i1} := \frac{\mu_{i1}^s}{\nu_{i1}^s}$ such that $i \in I_{m-1}$,

$$\mu_{i1}^s := \frac{1}{m-1}\sum_{j=1}^{n} \mu_{0j}^c \mu_{ij}^c \quad \text{and} \quad \nu_{i1}^s := \frac{1}{m-1}\sum_{j=1}^{n} \nu_{0j}^c \nu_{ij}^c$$

Step 4. Obtain the decision set $\left\{ {}^{\mu_{k1}^s}_{\nu_{k1}^s} u_k : u_k \in U \right\}$

Algorithm 10. iCD12/4

Step 1. Construct two *ifpifs*-matrices $[a_{ij}]_{m\times n}$ and $[b_{ij}]_{m\times n}$

Step 2. Obtain $[c_{ij}]_{m\times n}$ defined by $c_{ij} := \frac{\mu_{ij}^c}{\nu_{ij}^c}$ such that $i \in I_{m-1}, j \in I_n$,

$$\mu_{0j}^c := \mu_{0j}^a \mu_{0j}^b \quad \text{and} \quad \mu_{ij}^c := \min\{\mu_{ij}^a, \mu_{ij}^b\}$$

and

$$\nu_{0j}^c := \nu_{0j}^a + \nu_{0j}^b - \nu_{0j}^a \nu_{0j}^b \quad \text{and} \quad \nu_{ij}^c := \max\{\nu_{ij}^a, \nu_{ij}^b\}$$

Step 3. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ defined by $s_{i1} := \frac{\mu_{i1}^s}{\nu_{i1}^s}$ such that $i \in I_{m-1}$,

$$\mu_{i1}^s := \frac{1}{m-1}\sum_{j=1}^{n} \mu_{0j}^c \mu_{ij}^c \quad \text{and} \quad \nu_{i1}^s := \frac{1}{m-1}\sum_{j=1}^{n} \nu_{0j}^c \nu_{ij}^c$$

Step 4. Obtain the decision set $\left\{ {}^{\mu_{k1}^s}_{\nu_{k1}^s} u_k : u_k \in U \right\}$

The SDM method iFLC12 is a generalisation of FLC12 [22].

Algorithm 11. iFLC12

Step 1. Construct two *ifpifs*-matrices $[a_{ij}]_{m\times n}$ and $[b_{ij}]_{m\times n}$

Step 2. Obtain $[c_{ij}]_{(m-1)\times n}$ and $[d_{ij}]_{(m-1)\times n}$ defined by $c_{ij} := \frac{\mu_{ij}^c}{\nu_{ij}^c}$ and $d_{ij} := \frac{\mu_{ij}^d}{\nu_{ij}^d}$ such that $i \in I_{m-1}, j \in I_n$,

$$\begin{matrix}\mu_{ij}^c \\ \nu_{ij}^c\end{matrix} := \begin{cases} \begin{matrix}\mu_{0j}^a \\ \nu_{0j}^a\end{matrix}, & \sum_{k=1}^{n} \mu_{0k}^a \mu_{ik}^a \geq \sum_{k=1}^{j} \mu_{0k}^a \text{ and } \sum_{k=1}^{n} \nu_{0k}^a \nu_{ik}^a \leq \sum_{k=1}^{j} \nu_{0k}^a \\ \begin{matrix}0 \\ 1\end{matrix}, & otherwise \end{cases}$$

and

$$\begin{matrix}\mu_{ij}^d \\ \nu_{ij}^d\end{matrix} := \begin{cases} \begin{matrix}\mu_{0j}^b \\ \nu_{0j}^b\end{matrix}, & \sum_{k=1}^{n} \mu_{0k}^b \mu_{ik}^b \geq \sum_{k=1}^{j} \mu_{0k}^b \text{ and } \sum_{k=1}^{n} \nu_{0k}^b \nu_{ik}^b \leq \sum_{k=1}^{j} \nu_{0k}^b \\ \begin{matrix}0 \\ 1\end{matrix}, & otherwise \end{cases}$$

Step 3. Obtain $[e_{ip}]_{(m-1)\times n^2}$ defined by $e_{ip} := \begin{matrix}\mu_{ip}^e \\ \nu_{ip}^e\end{matrix}$ such that $i \in I_{m-1}$, $j,k \in I_n$, $p = n(j-1)+k$,

$$\mu_{ip}^e := \min\{\mu_{ij}^c, \mu_{ik}^d\} \quad \text{and} \quad \nu_{ip}^e := \max\{\nu_{ij}^c, \nu_{ik}^d\}$$

Step 4. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ defined by

$$s_{i1} := \begin{cases} e_{il}, & L \neq \emptyset \\ \begin{matrix}0 \\ 1\end{matrix}, & L = \emptyset \end{cases}$$

such that $i \in I_{m-1}$, $L := \{p : \exists i \in I_{m-1}, (\mu_{ip}^e \neq 0 \vee \nu_{ip}^e \neq 1)\}$, and $l := \max L$

Step 5. Obtain the decision set $\left\{ \begin{matrix}\mu_{k1}^s \\ \nu_{k1}^s\end{matrix} u_k : u_k \in U \right\}$

The SDM method iQYZ12(C) is a generalisation of QYZ12 [22]. In the generalised method, C indicates a certain set of indices related to sets of parameters.

Algorithm 12. iQYZ12(C)

Step 1. Apply iRM07a(C) except its Step 5

Step 2. Obtain $[e_{it}^p]_{(m-1)\times|C|}$ and $[f_{it}^p]_{(m-1)\times|C|}$ defined by

$$e_{it}^p := \begin{cases} \mu_{0t}^d, & \mu_{pt}^d > \mu_{it}^d \\ 0, & \mu_{pt}^d = \mu_{it}^d \\ -\mu_{0t}^d, & \mu_{pt}^d < \mu_{it}^d \end{cases} \quad \text{and} \quad f_{it}^p := \begin{cases} \nu_{0t}^d, & \nu_{pt}^d < \nu_{it}^d \\ 0, & \nu_{pt}^d = \nu_{it}^d \\ -\nu_{0t}^d, & \nu_{pt}^d > \nu_{it}^d \end{cases}$$

such that $i,p \in I_{m-1}$ and $t \in I_{|C|}$

Here, $|C|$ denotes the cardinality of C.

Step 3. Obtain $[g_{p1}]_{(m-1)\times 1}$ and $[h_{p1}]_{(m-1)\times 1}$ defined by

$$g_{p1} := \sum_{i=1}^{m-1} \sum_{t=1}^{|C|} e_{it}^p \quad \text{and} \quad h_{p1} := \sum_{i=1}^{m-1} \sum_{t=1}^{|C|} f_{it}^p$$

such that $p \in I_{m-1}$

Step 4. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ defined by $s_{i1} := \begin{matrix}\mu_{i1}^s \\ \nu_{i1}^s\end{matrix}$ such that $i \in I_{m-1}$,

$$\mu_{i1}^{s} = \begin{cases} \dfrac{g_{i1} + \left|\min\limits_{k \in I_{m-1}} \{g_{k1}\}\right|}{\max\limits_{k \in I_{m-1}} \{g_{k1} + |h_{k1}|\} + \left|\min\limits_{k \in I_{m-1}} \{g_{k1}\}\right|}, & \max\limits_{k \in I_{m-1}} \{g_{k1} + |h_{k1}|\} + \left|\min\limits_{k \in I_{m-1}} \{g_{k1}\}\right| \neq 0 \\ 1, & \max\limits_{k \in I_{m-1}} \{g_{k1} + |h_{k1}|\} + \left|\min\limits_{k \in I_{m-1}} \{g_{k1}\}\right| = 0 \end{cases}$$

and

$$\nu_{i1}^{s} = \begin{cases} 1 - \dfrac{g_{i1} + |h_{i1}| + \left|\min\limits_{k \in I_{m-1}} \{g_{k1}\}\right|}{\max\limits_{k \in I_{m-1}} \{g_{k1} + |h_{k1}|\} + \left|\min\limits_{k \in I_{m-1}} \{g_{k1}\}\right|}, & \max\limits_{k \in I_{m-1}} \{g_{k1} + |h_{k1}|\} + \left|\min\limits_{k \in I_{m-1}} \{g_{k1}\}\right| \neq 0 \\ 0, & \max\limits_{k \in I_{m-1}} \{g_{k1} + |h_{k1}|\} + \left|\min\limits_{k \in I_{m-1}} \{g_{k1}\}\right| = 0 \end{cases}$$

Step 5. Obtain the decision set $\left\{ {}^{\mu_{k1}^{s}}_{\nu_{k1}^{s}} u_k : u_k \in U \right\}$

4. DETERMINING GENERALISED SDM METHODS BEING SUCCESSFUL IN FIVE TEST CASES

This section analyses the generalised SDM methods herein by utilising five test cases [8,9] to determine the SDM methods for the real-life problem mentioned in the next section. These tests contain three *ifpifs*-matrices constructed for a decision-making problem fictionalising with four alternatives and four parameters. For more details, see [8,9].

The *ifpifs*-matrices employed in the test case 1 are as follows:

$$[a_{ij}^{1}] := \begin{bmatrix} \substack{0.4\\0.3} & \substack{0.4\\0.3} & \substack{0.4\\0.3} & \substack{0.4\\0.3} \\ \substack{0.7\\0.15} & \substack{0.6\\0.2} & \substack{0.5\\0.25} & \substack{0.4\\0.3} \\ \substack{0.8\\0.1} & \substack{0.7\\0.15} & \substack{0.6\\0.2} & \substack{0.5\\0.25} \\ \substack{0.9\\0.05} & \substack{0.8\\0.1} & \substack{0.7\\0.15} & \substack{0.6\\0.2} \\ \substack{1\\0} & \substack{0.9\\0.05} & \substack{0.8\\0.1} & \substack{0.7\\0.15} \end{bmatrix}, \quad [a_{ij}^{2}] := \begin{bmatrix} \substack{0.4\\0.3} & \substack{0.4\\0.3} & \substack{0.4\\0.3} & \substack{0.4\\0.3} \\ \substack{0.3\\0.35} & \substack{0.2\\0.4} & \substack{0.1\\0.45} & \substack{0\\0.5} \\ \substack{0.4\\0.3} & \substack{0.3\\0.35} & \substack{0.2\\0.4} & \substack{0.1\\0.45} \\ \substack{0.5\\0.25} & \substack{0.4\\0.3} & \substack{0.3\\0.35} & \substack{0.2\\0.4} \\ \substack{0.6\\0.2} & \substack{0.5\\0.25} & \substack{0.4\\0.3} & \substack{0.3\\0.35} \end{bmatrix}, \text{ and } [a_{ij}^{3}] := \begin{bmatrix} \substack{0.4\\0.3} & \substack{0.4\\0.3} & \substack{0.4\\0.3} & \substack{0.4\\0.3} \\ \substack{0.5\\0.25} & \substack{0.4\\0.3} & \substack{0.3\\0.35} & \substack{0.2\\0.4} \\ \substack{0.6\\0.2} & \substack{0.5\\0.25} & \substack{0.4\\0.3} & \substack{0.3\\0.35} \\ \substack{0.7\\0.15} & \substack{0.6\\0.2} & \substack{0.5\\0.25} & \substack{0.4\\0.3} \\ \substack{0.8\\0.1} & \substack{0.7\\0.15} & \substack{0.6\\0.2} & \substack{0.5\\0.25} \end{bmatrix}$$

The *ifpifs*-matrices employed in the test case 2 are as follows:

$$[b_{ij}^{1}] := \begin{bmatrix} \substack{0.8\\0.1} & \substack{0.8\\0.1} & \substack{0.8\\0.1} & \substack{0.8\\0.1} \\ \substack{1\\0} & \substack{0.9\\0.05} & \substack{0.8\\0.1} & \substack{0.7\\0.15} \\ \substack{0.9\\0.05} & \substack{0.8\\0.1} & \substack{0.7\\0.15} & \substack{0.6\\0.2} \\ \substack{0.8\\0.1} & \substack{0.7\\0.15} & \substack{0.6\\0.2} & \substack{0.5\\0.25} \\ \substack{0.7\\0.15} & \substack{0.6\\0.2} & \substack{0.5\\0.25} & \substack{0.4\\0.3} \end{bmatrix}, \quad [b_{ij}^{2}] := \begin{bmatrix} \substack{0.8\\0.1} & \substack{0.8\\0.1} & \substack{0.8\\0.1} & \substack{0.8\\0.1} \\ \substack{0.6\\0.2} & \substack{0.5\\0.25} & \substack{0.4\\0.3} & \substack{0.3\\0.35} \\ \substack{0.5\\0.25} & \substack{0.4\\0.3} & \substack{0.3\\0.35} & \substack{0.2\\0.4} \\ \substack{0.4\\0.3} & \substack{0.3\\0.35} & \substack{0.2\\0.4} & \substack{0.1\\0.45} \\ \substack{0.3\\0.35} & \substack{0.2\\0.4} & \substack{0.1\\0.45} & \substack{0\\0.5} \end{bmatrix}, \text{ and } [b_{ij}^{3}] := \begin{bmatrix} \substack{0.8\\0.1} & \substack{0.8\\0.1} & \substack{0.8\\0.1} & \substack{0.8\\0.1} \\ \substack{0.8\\0.1} & \substack{0.7\\0.15} & \substack{0.6\\0.2} & \substack{0.5\\0.25} \\ \substack{0.7\\0.15} & \substack{0.6\\0.2} & \substack{0.5\\0.25} & \substack{0.4\\0.3} \\ \substack{0.6\\0.2} & \substack{0.5\\0.25} & \substack{0.4\\0.3} & \substack{0.3\\0.35} \\ \substack{0.5\\0.25} & \substack{0.4\\0.3} & \substack{0.3\\0.35} & \substack{0.2\\0.4} \end{bmatrix}$$

The *ifpifs*-matrices employed in the test case 3 are as follows:

$$[c_{ij}^1] := \begin{bmatrix} 0.6 & 0.7 & 0.8 & 0.9 \\ 0.2 & 0.15 & 0.1 & 0.05 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \quad [c_{ij}^2] := \begin{bmatrix} 0.4 & 0.5 & 0.6 & 0.7 \\ 0.3 & 0.25 & 0.2 & 0.15 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \text{ and } \quad [c_{ij}^3] := \begin{bmatrix} 0.2 & 0.3 & 0.4 & 0.5 \\ 0.4 & 0.35 & 0.3 & 0.25 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

The *ifpifs*-matrices employed in the test case 4 are as follows:

$$[d_{ij}^1] := \begin{bmatrix} 0.9 & 0.8 & 0.7 & 0.6 \\ 0.05 & 0.1 & 0.15 & 0.2 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \quad [d_{ij}^2] := \begin{bmatrix} 0.7 & 0.6 & 0.5 & 0.4 \\ 0.15 & 0.2 & 0.25 & 0.3 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \text{ and } \quad [d_{ij}^3] := \begin{bmatrix} 0.5 & 0.4 & 0.3 & 0.2 \\ 0.25 & 0.3 & 0.35 & 0.4 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

The *ifpifs*-matrices employed in the test case 5 are as follows:

$$[e_{ij}^1] := \begin{bmatrix} 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \end{bmatrix}, \quad [e_{ij}^2] := \begin{bmatrix} 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \end{bmatrix}, \quad \text{and} \quad [e_{ij}^3] := \begin{bmatrix} 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \end{bmatrix}$$

Table 1 presents the success performances of the generalised SDM methods in the test cases. These results are obtained using MATLAB R2021b. Bold values in its last column signify the successful SDM methods in all the test cases, i.e., ✓: Successful and –: Unsuccessful. These SDM methods use the following variables in the test cases:

$$C = \{(1,1,1), (2,2,2), (3,3,3), (4,4,4)\}$$

$$\lambda_1 = 0.5$$

$$\lambda_2 = 0.5$$

$$R = I_4$$

Table 1 shows that the SDM methods iRM07a(C), iMBR01/2-based iRM07a(C), iRM07o(C), iMBR01/2-based iRM07o(C), iKWW11/2$(\lambda_1, \lambda_2, R)$, iNS11, iCD12/3, iCD12/4, and iQYZ12(C) are successful for the aforesaid variables in all the test cases. Besides, the SDM methods iBMM12, iBMM12/2, and iBMM12/3 are only unsuccessful in test case 1. On the other hand, iKGW09(C) and iFLC12 are successful only in the test case 5. Consequently, Table 1 manifests that the numbers of the generalised SDM methods passing the test case 1, 2, 3, 4, and 5 are 9, 12, 12, 12, and 14, respectively. Moreover, since the SDM method isMBR01 is a mathematically simplified form of iMBR01, these SDM methods generate the same score matrices and the decision sets. Therefore, only iRM07a(C) and iRM07o(C) are presented herein.

Table 1. Success performances of the SDM methods in the test cases

	Generalised SDM Methods / Test Cases	**Test Case 1**	**Test Case 2**	**Test Case 3**	**Test Case 4**	**Test Case 5**	**Numbers of Tests Passed**
1.	iRM07a(C)	✓	✓	✓	✓	✓	**5**
2.	iMBR01/2-based iRM07a(C)	✓	✓	✓	✓	✓	**5**
3.	iRM07o(C)	✓	✓	✓	✓	✓	**5**
4.	iMBR01/2-based iRM07o(C)	✓	✓	✓	✓	✓	**5**
5.	iKGW09(C)	–	–	–	–	✓	1
6.	iKWW11/2$(\lambda_1, \lambda_2, R)$	✓	✓	✓	✓	✓	**5**
7.	iNS11	✓	✓	✓	✓	✓	**5**
8.	iBMM12	–	✓	✓	✓	✓	4
9.	iBMM12/2	–	✓	✓	✓	✓	4
10	iBMM12/3	–	✓	✓	✓	✓	4
11.	iCD12/3	✓	✓	✓	✓	✓	**5**
12.	iCD12/4	✓	✓	✓	✓	✓	**5**
13.	iFLC12	–	–	–	–	✓	1
14.	iQYZ12(C)	✓	✓	✓	✓	✓	**5**

5. AN APPLICATION OF SUCCESSFUL SDM METHODS TO A PVA PROBLEM

This section applies nine SDM methods being successful in all the test cases to a PVA problem in which the alternatives are seven noise removal filters, namely “Based on Pixel Density Filter (BPDF)” [24], “Decision-Based Algorithm (DBAIN)” [25], “Modified Decision-Based Unsymmetrical Trimmed Median Filter (MDBUTMF)” [26], “Noise Adaptive Fuzzy Switching Median Filter (NAFSMF)” [27], “Different Applied Median Filter (DAMF)” [28], “Adaptive Weighted Mean Filter (AWMF)” [29], and “Adaptive Riesz Mean Filter (ARmF)” [30], and the parameters are nine noise densities. In other words, the set of alternatives is $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$ such that u_1 = “BPDF”, u_2 = “DBAIN”, u_3 = “MDBUTMF”, u_4 = “NAFSMF”, u_5 = “DAMF”, u_6 = “AWMF”, and u_7 = “ARmF”. Moreover, the set of parameters is $E = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$ such that $x_1 =$ "noise density 10%", $x_2 =$ "noise density 20%", $x_3 =$ "noise density 30%", $x_4 =$ "noise density 40%", $x_5 =$ "noise density 50%", $x_6 =$ "noise density 60%", $x_7 =$ "noise density 70%", $x_8 =$ "noise density 80%", and $x_9 =$ "noise density 90%". This section, firstly, presents the *ifpifs*-matrices $[a_{ij}]_{8\times 9}$ provided in [8], $[b_{ij}]_{8\times 9}$ and $[c_{ij}]_{8\times 9}$ in [9], obtained by the Structural Similarity (SSIM) [31], Peak Signal-to-Noise Ratio (PSNR), and Visual Information Fidelity (VIF) [32] results of these filters, respectively.

$$[a_{ij}] = \begin{bmatrix}
0.05 & 0.15 & 0.25 & 0.35 & 0.5 & 0.65 & 0.75 & 0.85 & 0.9 \\
0.9 & 0.8 & 0.7 & 0.6 & 0.5 & 0.3 & 0.2 & 0.1 & 0.05 \\[4pt]
0.9657 & 0.9335 & 0.8856 & 0.8269 & 0.7503 & 0.6452 & 0.5159 & 0.3648 & 0.1259 \\
0.0062 & 0.0142 & 0.0270 & 0.0450 & 0.0759 & 0.1165 & 0.1887 & 0.2998 & 0.4895 \\[4pt]
0.9666 & 0.9424 & 0.9047 & 0.8552 & 0.7917 & 0.7104 & 0.6060 & 0.4880 & 0.3518 \\
0.0031 & 0.0080 & 0.0168 & 0.0297 & 0.0478 & 0.0762 & 0.1223 & 0.1858 & 0.2766 \\[4pt]
0.9642 & 0.9228 & 0.7833 & 0.7539 & 0.7855 & 0.7572 & 0.6950 & 0.6000 & 0.3492 \\
0.0050 & 0.0509 & 0.1319 & 0.1593 & 0.1167 & 0.0551 & 0.0575 & 0.1359 & 0.5228 \\[4pt]
0.9606 & 0.9216 & 0.8767 & 0.8305 & 0.7800 & 0.7211 & 0.6540 & 0.5766 & 0.4578 \\
0.0086 & 0.0169 & 0.0267 & 0.0357 & 0.0465 & 0.0595 & 0.0790 & 0.1082 & 0.2173 \\[4pt]
0.9700 & 0.9518 & 0.9270 & 0.8953 & 0.8563 & 0.8072 & 0.7465 & 0.6667 & 0.5415 \\
0.0018 & 0.0045 & 0.0088 & 0.0139 & 0.0204 & 0.0291 & 0.0423 & 0.0624 & 0.1148 \\[4pt]
0.9551 & 0.9440 & 0.9209 & 0.8948 & 0.8611 & 0.8148 & 0.7551 & 0.6736 & 0.5469 \\
0.0067 & 0.0076 & 0.0095 & 0.0122 & 0.0166 & 0.0240 & 0.0370 & 0.0574 & 0.1052 \\[4pt]
0.9718 & 0.9532 & 0.9272 & 0.8971 & 0.8630 & 0.8239 & 0.7663 & 0.6819 & 0.5515 \\
0.0013 & 0.0030 & 0.0054 & 0.0087 & 0.0137 & 0.0214 & 0.0348 & 0.0554 & 0.1038
\end{bmatrix}$$

$$[b_{ij}] = \begin{bmatrix}
0.05 & 0.15 & 0.25 & 0.35 & 0.5 & 0.65 & 0.75 & 0.85 & 0.9 \\
0.9 & 0.8 & 0.7 & 0.6 & 0.5 & 0.3 & 0.2 & 0.1 & 0.05 \\[4pt]
0.8315 & 0.7582 & 0.6932 & 0.6373 & 0.5787 & 0.5286 & 0.4599 & 0.3667 & 0.1864 \\
0.0035 & 0.0549 & 0.0995 & 0.1429 & 0.1828 & 0.2345 & 0.3008 & 0.4048 & 0.6592 \\[4pt]
0.8726 & 0.7964 & 0.7314 & 0.6733 & 0.6226 & 0.5654 & 0.5042 & 0.4462 & 0.3712 \\
0.0007 & 0.0264 & 0.0597 & 0.0969 & 0.1359 & 0.1834 & 0.2303 & 0.2874 & 0.3706 \\[4pt]
0.7891 & 0.7263 & 0.6166 & 0.6000 & 0.6445 & 0.6368 & 0.5995 & 0.5129 & 0.3021 \\
0.0073 & 0.0752 & 0.1258 & 0.1485 & 0.1527 & 0.1645 & 0.1863 & 0.2639 & 0.4933 \\[4pt]
0.8215 & 0.7434 & 0.6916 & 0.6538 & 0.6204 & 0.5922 & 0.5606 & 0.5280 & 0.4751 \\
0.0159 & 0.0723 & 0.1025 & 0.1254 & 0.1577 & 0.1813 & 0.2039 & 0.2376 & 0.3204 \\[4pt]
0.9678 & 0.8775 & 0.8103 & 0.7628 & 0.7186 & 0.6735 & 0.6298 & 0.5749 & 0.5052 \\
0 & 0.0300 & 0.0602 & 0.0780 & 0.1125 & 0.1392 & 0.1657 & 0.2014 & 0.2650 \\[4pt]
0.8497 & 0.8241 & 0.7958 & 0.7625 & 0.7273 & 0.6867 & 0.6405 & 0.5825 & 0.5101 \\
0.0282 & 0.0369 & 0.0549 & 0.0658 & 0.0862 & 0.1099 & 0.1390 & 0.1764 & 0.2387 \\[4pt]
0.9983 & 0.9074 & 0.8478 & 0.7946 & 0.7471 & 0.6983 & 0.6471 & 0.5857 & 0.5112 \\
0 & 0.0154 & 0.0380 & 0.0522 & 0.0767 & 0.1028 & 0.1348 & 0.1745 & 0.2384
\end{bmatrix}$$

and

$$
[c_{ij}] = \begin{bmatrix}
\substack{0.05 \\ 0.9} & \substack{0.15 \\ 0.8} & \substack{0.25 \\ 0.7} & \substack{0.35 \\ 0.6} & \substack{0.5 \\ 0.5} & \substack{0.65 \\ 0.3} & \substack{0.75 \\ 0.2} & \substack{0.85 \\ 0.1} & \substack{0.9 \\ 0.05} \\
\substack{0.6854 \\ 0.0805} & \substack{0.5350 \\ 0.1708} & \substack{0.4293 \\ 0.2711} & \substack{0.3425 \\ 0.3820} & \substack{0.2652 \\ 0.5028} & \substack{0.1826 \\ 0.6338} & \substack{0.1119 \\ 0.7727} & \substack{0.0579 \\ 0.8869} & \substack{0.0217 \\ 0.9544} \\
\substack{0.7456 \\ 0.0506} & \substack{0.6003 \\ 0.1168} & \substack{0.4793 \\ 0.2152} & \substack{0.3713 \\ 0.3135} & \substack{0.2875 \\ 0.4248} & \substack{0.2115 \\ 0.5392} & \substack{0.1452 \\ 0.6667} & \substack{0.0884 \\ 0.7857} & \substack{0.0395 \\ 0.8974} \\
\substack{0.7174 \\ 0.0539} & \substack{0.5640 \\ 0.1861} & \substack{0.4333 \\ 0.3799} & \substack{0.3703 \\ 0.4318} & \substack{0.3427 \\ 0.4046} & \substack{0.2927 \\ 0.4196} & \substack{0.2353 \\ 0.4956} & \substack{0.1574 \\ 0.6293} & \substack{0.0466 \\ 0.8868} \\
\substack{0.6356 \\ 0.0887} & \substack{0.5081 \\ 0.1682} & \substack{0.4290 \\ 0.2555} & \substack{0.3624 \\ 0.3313} & \substack{0.3048 \\ 0.4094} & \substack{0.2507 \\ 0.4948} & \substack{0.1969 \\ 0.5805} & \substack{0.1306 \\ 0.6703} & \substack{0.0683 \\ 0.8058} \\
\substack{0.7742 \\ 0.0250} & \substack{0.6564 \\ 0.0650} & \substack{0.5698 \\ 0.1215} & \substack{0.4896 \\ 0.1772} & \substack{0.4180 \\ 0.2330} & \substack{0.3501 \\ 0.2972} & \substack{0.2847 \\ 0.3813} & \substack{0.2100 \\ 0.4843} & \substack{0.1255 \\ 0.6670} \\
\substack{0.6396 \\ 0.0801} & \substack{0.5992 \\ 0.0988} & \substack{0.5489 \\ 0.1299} & \substack{0.4861 \\ 0.1671} & \substack{0.4245 \\ 0.2160} & \substack{0.3599 \\ 0.2792} & \substack{0.2895 \\ 0.3682} & \substack{0.2119 \\ 0.4767} & \substack{0.1262 \\ 0.6613} \\
\substack{0.7908 \\ 0.0214} & \substack{0.6833 \\ 0.0489} & \substack{0.6006 \\ 0.0884} & \substack{0.5186 \\ 0.1328} & \substack{0.4442 \\ 0.1894} & \substack{0.3718 \\ 0.2583} & \substack{0.2966 \\ 0.3533} & \substack{0.2152 \\ 0.4677} & \substack{0.1278 \\ 0.6569}
\end{bmatrix}
$$

Secondly, this section applies the SDM methods passing all the test cases in the previous section to aforesaid *ifpifs*-matrices. Table 2 presents the intuitionistic fuzzy decision sets of these SDM methods. These results are produced on MATLAB R2021b. Moreover, the variables in the SDM methods are as follows:

$$C = \{(1,1,1), (2,2,2), (3,3,3), (4,4,4), (5,5,5), (6,6,6), (7,7,7), (8,8,8), (9,9,9)\}$$

$$\lambda_1 = 0.5$$

$$\lambda_2 = 0.5$$

$$R = I_9$$

Here, the SDM methods iKWW11/2$(\lambda_1, \lambda_2, R)$, iNS11, iCD12/3, and iCD12/4 utilise the *ifpifs*-matrices $[a_{ij}]$ and $[b_{ij}]$, and the others employ the *ifpifs*-matrices $[a_{ij}]$, $[b_{ij}]$, and $[c_{ij}]$.

Table 2. Intuitionistic fuzzy decision sets of the successful SDM methods

Successful SDM Methods	**Decision Sets**
iRM07a$(\boldsymbol{C})$	$\{{}^{0}_{0.7555}\text{BPDF}, {}^{0.1488}_{0.8150}\text{DBAIN}, {}^{0.2898}_{0.5459}\text{MDBUTMF}, {}^{0.2083}_{0.6391}\text{NAFSMF}, {}^{0.4683}_{0.3622}\text{DAMF}, {}^{0.5343}_{0.3596}\text{AWMF}, {}^{0.6779}_{0}\text{ARmF}\}$
iMBR01/2-based iRM07a$(\boldsymbol{C})$	$\{{}^{0}_{0.7584}\text{BPDF}, {}^{0.1435}_{0.8305}\text{DBAIN}, {}^{0.2675}_{0.6221}\text{MDBUTMF}, {}^{0.2201}_{0.5799}\text{NAFSMF}, {}^{0.4617}_{0.3604}\text{DAMF}, {}^{0.5396}_{0.3838}\text{AWMF}, {}^{0.6766}_{0}\text{ARmF}\}$
iRM07o$(\boldsymbol{C})$	$\{{}^{0}_{0.8316}\text{BPDF}, {}^{0.1791}_{0.7981}\text{DBAIN}, {}^{0.2032}_{0.5575}\text{MDBUTMF}, {}^{0.1979}_{0.6390}\text{NAFSMF}, {}^{0.4626}_{0.3503}\text{DAMF}, {}^{0.5334}_{0.3810}\text{AWMF}, {}^{0.6791}_{0}\text{ARmF}\}$
iMBR01/2-based iRM07o$(\boldsymbol{C})$	$\{{}^{0}_{0.8330}\text{BPDF}, {}^{0.1576}_{0.8263}\text{DBAIN}, {}^{0.1496}_{0.6365}\text{MDBUTMF}, {}^{0.2126}_{0.6009}\text{NAFSMF}, {}^{0.4440}_{0.3722}\text{DAMF}, {}^{0.5252}_{0.4091}\text{AWMF}, {}^{0.6660}_{0}\text{ARmF}\}$
iKWW11/2$(\boldsymbol{\lambda_1}, \boldsymbol{\lambda_2}, \boldsymbol{R})$	$\{{}^{0.2868}_{0.5602}\text{BPDF}, {}^{0.3388}_{0.4621}\text{DBAIN}, {}^{0.3557}_{0.3634}\text{MDBUTMF}, {}^{0.3802}_{0.3440}\text{NAFSMF}, {}^{0.5201}_{0.0697}\text{DAMF}, {}^{0.5323}_{0.0462}\text{AWMF}, {}^{0.5718}_{0}\text{ARmF}\}$
iNS11	$\{{}^{0}_{0.1697}\text{BPDF}, {}^{0.3305}_{0.1016}\text{DBAIN}, {}^{0.4485}_{0.1279}\text{MDBUTMF}, {}^{0.5298}_{0.1353}\text{NAFSMF}, {}^{0.6222}_{0.0290}\text{DAMF}, {}^{0.6871}_{0.0606}\text{AWMF}, {}^{0.6519}_{0}\text{ARmF}\}$
iCD12/3	$\{{}^{0.4820}_{0.0118}\text{BPDF}, {}^{0.5536}_{0.0073}\text{DBAIN}, {}^{0.5665}_{0.0273}\text{MDBUTMF}, {}^{0.5825}_{0.0094}\text{NAFSMF}, {}^{0.6456}_{0.0032}\text{DAMF}, {}^{0.6491}_{0.0040}\text{AWMF}, {}^{0.6552}_{0.0022}\text{ARmF}\}$
iCD12/4	$\{{}^{0.1618}_{0.1108}\text{BPDF}, {}^{0.2048}_{0.0762}\text{DBAIN}, {}^{0.2161}_{0.1000}\text{MDBUTMF}, {}^{0.2307}_{0.0920}\text{NAFSMF}, {}^{0.2585}_{0.0614}\text{DAMF}, {}^{0.2614}_{0.0566}\text{AWMF}, {}^{0.2652}_{0.0442}\text{ARmF}\}$
iQYZ12$(\boldsymbol{C})$	$\{{}^{0}_{0.7555}\text{BPDF}, {}^{0.1488}_{0.8150}\text{DBAIN}, {}^{0.2898}_{0.5459}\text{MDBUTMF}, {}^{0.2083}_{0.6391}\text{NAFSMF}, {}^{0.4683}_{0.3622}\text{DAMF}, {}^{0.5343}_{0.3596}\text{AWMF}, {}^{0.6779}_{0}\text{ARmF}\}$

Table 3 provides the ranking orders of the successful SDM methods. These ranking orders are obtained using Proposition 2.1. According to the table, iRM07a(C), iMBR01/2-based iRM07a(C), iRM07o(C), and iQYZ12(C) generate the same ranking orders just as iMBR01/2-based iRM07o(C), iKWW11/2(λ_1, λ_2, R), and iNS11 do. Besides, iCD12/3 and iCD12/4 have the same ranking orders. Though the skills of sorting the noise removal filters of all the SDM methods in this table relatively differ, all indicate that ARmF, AWMF, and DAMF are the top three filters with the highest noise removal performance, respectively. Further, all the SDM methods show that BPDF has the lowest noise removal performance.

Table 3. Ranking orders of the successful SDM methods

Successful SDM Methods	**Ranking Orders**
iRM07a($\boldsymbol{C}$)	BPDF ≺ DBAIN ≺ NAFSMF ≺ MDBUTMF ≺ DAMF ≺ AWMF ≺ ARmF
iMBR01/2-based iRM07a($\boldsymbol{C}$)	BPDF ≺ DBAIN ≺ NAFSMF ≺ MDBUTMF ≺ DAMF ≺ AWMF ≺ ARmF
iRM07o($\boldsymbol{C}$)	BPDF ≺ DBAIN ≺ NAFSMF ≺ MDBUTMF ≺ DAMF ≺ AWMF ≺ ARmF
iMBR01/2-based iRM07o($\boldsymbol{C}$)	BPDF ≺ DBAIN ≺ MDBUTMF ≺ NAFSMF ≺ DAMF ≺ AWMF ≺ ARmF
iKWW11/2($\boldsymbol{\lambda_1}, \boldsymbol{\lambda_2}, \boldsymbol{R}$)	BPDF ≺ DBAIN ≺ MDBUTMF ≺ NAFSMF ≺ DAMF ≺ AWMF ≺ ARmF
iNS11	BPDF ≺ DBAIN ≺ MDBUTMF ≺ NAFSMF ≺ DAMF ≺ AWMF ≺ ARmF
iCD12/3	BPDF ≺ MDBUTMF ≺ DBAIN ≺ NAFSMF ≺ DAMF ≺ AWMF ≺ ARmF
iCD12/4	BPDF ≺ MDBUTMF ≺ DBAIN ≺ NAFSMF ≺ DAMF ≺ AWMF ≺ ARmF
iQYZ12($\boldsymbol{C}$)	BPDF ≺ DBAIN ≺ NAFSMF ≺ MDBUTMF ≺ DAMF ≺ AWMF ≺ ARmF

6. CONCLUSIONS

The current study generalised 12 SDM methods provided in [22] to *ifpifs*-matrices space. It then analysed the generalised SDM methods by utilising the five test cases proposed in [8,9]. Thus, the study determined the SDM methods generating a valid ranking order in all the test cases. Afterwards, it applied the determined SDM methods to a PVA problem related to sorting the performances of seven well-known noise removal filters.

Further, this paper is among the pioneering studies generalising the SDM methods constructed with *fpfs*-matrices to operate in *ifpifs*-matrices space. Hence, future studies can focus on generalising the SDM methods employed in this space to its superstructure, such as interval-valued intuitionistic fuzzy parameterized interval-valued intuitionistic fuzzy soft matrices (*d*-matrices) [33]. Additionally, the *ifpifs*-matrices-based classification algorithms can be proposed with the help of the SDM methods generalised herein and others [8,9], inspired by the classification algorithms developed by using the SDM methods constructed with the *fpfs*-matrices and provided in [34,35].

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A STUDY ON ABSOLUTE SERIES SPACE $|\bar{N}_p^\theta|(\mu)$ AND CERTAIN MATRIX TRANSFORMATIONS

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Abstract:

In this study, some characterizations of matrix operators from the paranormed space $|\bar{N}_p^\theta|(\mu)$ which has been introduced and studied by Gökçe and Sarıgöl, to the classical sequence spaces c, c_0, l_∞ are obtained. Then, we show that the matrix operators between the spaces $|\bar{N}_p^\theta|(\mu)$ and c, c_0, l_∞ are bounded operators.

Keywords: Absolute summability, weighted mean matrix, matrix transformation, bounded linear operators.
General area of research: Mathematics
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1. INTRODUCTION

The sequence spaces and matrix transformations play important roles in summability theory, a wide field of mathematics, which have several applications in approximation theory, calculus, and essentially in functional analysis. The classical theory deals with the generalization of the concept of convergence for sequences and series. The aim is to assign a limit for non-convergent sequences and series by making use of an operator defined by infinite matrices. The reason why matrices are used for a general linear operator is that a linear operator from a sequence space to another one can be given by an infinite matrix. A large literature has grown up concerned with characterizing completely all matrices which transform one given sequence space into another, and also, many sequence spaces and related matrix operators have been studied by several authors [1,2,3,4].

Let denote the set of all complex (or real) valued sequences by ω. Any vector subspace of ω is called as a sequence space. Assume that X, Y are arbitrary sequence spaces and $A = (a_{nv})$ is any infinite matrix of complex components. By $A(x) = \big(A_n(x)\big)$, we denote the A-transform of the sequence $x = (x_v)$ if the series

$$A_n(x) = \sum_{v=0}^{\infty} a_{nv} x_v$$

is convergent for any integer n. If $A(x) \in Y$, whenever $x \in X$, then it is said that A defines a matrix transformation from X into Y, and the class of all infinite matrices A such that $A : X \to Y$ is represented by (X, Y). Besides, the concept of matrix domain of an infinite matrix A in a sequence space X is defined by the set

$$X_A = \{x \in \omega : A(x) \in X\}$$

which is also a sequence space.

The Maddox's space defined by

$$l(\mu) = \left\{x = (x_n) : \sum_{n=0}^{\infty} |x_n|^{\mu_n} < \infty\right\}$$

has an important role in summability theory. Note that $l(\mu)$ is an FK space which is a complete metrizable locally convex space with continuous coordinates $r_n: X \to \mathbb{C}$ defined by $r_n(x) = x_n$ for all $n \in \mathbb{N}$, according to its paranorm given by

$$g(x) = \left(\sum_{k=0}^{\infty} |x_k|^{\mu_k}\right)^{1\backslash M}$$

where $M = \max\{1; \sup_k \mu_k\}$. Besides, this space has AK property i.e., every sequence $x \epsilon l(\mu)$ has a unique representation $x = \sum_{k=0}^{\infty} x_k e^{(k)}$ where $e^{(k)}$ is the sequence whose only non-zero term is 1 in the kth place for each $k \epsilon \mathbb{N}$ ([6], [7], [8]).

Let $\sum a_v$ be an infinite series with the sequence of partial sum (s_n), $\theta = (\theta_n)$ be any sequence of positive real numbers and $\mu = (\mu_n)$ be any bounded sequence of positive real numbers. The series $\sum a_v$ is said to be summable $|A, \theta_n|(\mu)$ if

$$\sum_{n=1}^{\infty} \theta_n^{\mu_{n-1}} |A_n(s) - A_{n-1}(s)|^{\mu_n} < \infty$$

[3]. It should be noted that the concept of the summability $|A, \theta_n|(\mu)$ includes some well known summability methods for special cases of the sequences μ, θ and the matrix A (see [1], [2], [3],[4]). One of them is the summability method $|\bar{N}, p_n, \theta_n|(\mu)$, which determines the main subject of this study : combining the concept of absolute summability and weighted mean matrix given by

$$a_{nv} = \begin{cases} p_v/P_n, & 0 \le v \le n \\ 0, & v > n \end{cases}$$

where (p_n) be a positive sequence with $P_n = p_0 + p_1 + \cdots + p_n \to \infty$ as $n \to \infty$, $(P_{-1} = p_{-1} = 0)$, we get the absolute summability $|\bar{N}, p_n, \theta_n|(\mu)$ and its related space $\left|\bar{N}_p^\theta\right|(\mu)$ defined as the set of all series summable by the absolute summability $|\bar{N}, p_n, \theta_n|(\mu)$, [3].
The space $\left|\bar{N}_p^\theta\right|(\mu)$ can be express the space more clearly as :

$$\left|\bar{N}_p^\theta\right|(\mu) = \left\{a : \sum_{n=1}^{\infty} \theta_n^{\mu_{n-1}} \left|\frac{p_n}{P_n P_{n-1}} \sum_{v=1}^{n} P_{v-1} a_v\right|^{\mu_n} < \infty\right\}.$$

On the other hand, with the notation of domain, the space can be redefined by $\left|\bar{N}_p^\theta\right|(\mu) = \big(l(\mu)\big)_{T(\theta,\mu,p)}$, where the matrix $T(\theta, \mu, p)$ is given by

$$t_{nv}(\theta, \mu, p) = \begin{cases} 1, & n = 0, v = 0 \\ \theta_n^{1/\mu_n^*} \dfrac{p_n P_{v-1}}{P_n P_{n-1}}, & 1 \le v \le n \\ 0, & v > n, \end{cases}$$

whose inverse transformation $S(\theta, \mu, p)$ is

$$s_{00}(\theta, \mu, p) = 1,$$

$$s_{nv}(\theta,\mu,p)=\begin{cases}-\theta_{n-1}^{-1/\mu_{n-1}^{*}}\dfrac{P_{n-2}}{p_{n-1}}, v=n-1\\ \theta_{n}^{-1/\mu_{n}^{*}}\dfrac{P_{n}}{p_{n}}, \quad v=n\\ 0, \quad v\neq n-1,n\end{cases}$$

where μ_n^* is the conjugate of μ_n, i.e. $1/\mu_n+1/\mu_n^*=1$, $\mu_n>1$, and $1/\mu_n^*=0$ for $\mu_n=1$.

Now, we remind certain lemmas which have important role in the proofs of main theorems:
Lemma 1 ([5]) Let $\mu=(\mu_v)$ be any bounded sequence of strictly positive numbers.
(i) If $\mu_v\leq 1$, for all v, then,

$$A\in(l(\mu),c)\Leftrightarrow(a)\ \lim_{n\to\infty}a_{nv}\ \text{exists for each}\ v,\qquad (b)\sup_{n,v}|a_{nv}|^{\mu_v}<\infty$$

$$A\in(l(\mu),c_0)\Leftrightarrow(c)\ \lim_{n\to\infty}a_{nv}=0\ \text{for each}\ v,\qquad (b)\ \text{holds}$$

and

$$A\in(l(\mu),l_\infty)\Leftrightarrow(b)\ \text{holds}.$$

(ii) If $\mu_v>1$ for all v, then,
$A\in(l(\mu),c)\Leftrightarrow(a')\ \lim_{n\to\infty}a_{nv}$ exists for each v, (b') there is a number $M>1$ such that

$$\sup_n\sum_{v=0}^{\infty}|a_{nv}M^{-1}|^{\mu_v^*}<\infty,$$

$$A\in(l(\mu),c_0)\Leftrightarrow(c')\ \lim_{n\to\infty}a_{nv}=0\ \text{for each}\ v,\qquad (b')\ \text{holds}$$

and

$$A\in(l(\mu),l_\infty)\Leftrightarrow(b')\ \text{holds}.$$

Lemma 2 ([9]) Let X be an FK space with AK, U be triangle, S be its inverse and Y be an arbitrary subset of ω. Then, we have $A\in(X_U,Y)$ if and only if $\widetilde{A}\in(X,Y)$ and $V^{(n)}\in(X,c)$ for all n, where

$$\tilde{a}_{nv}=\sum_{j=v}^{\infty}a_{nj}s_{jv}, n,v=0,1,\ldots$$

$$v_{mv}^{(n)}=\begin{cases}\sum_{j=v}^{m}a_{nj}s_{jv}, 0\leq v<\infty\\ 0, \quad v>m.\end{cases}$$

Lemma 3 ([10]) Let U be a triangle. Then, for $X,Y\subset\omega, A\in(X,Y_U)$ if and only if $B=UA\in(X,Y)$.
Lemma 4 ([11, Theorem 4.2.8]) Matrix transformations between FK-spaces are continuous.

Theorem 1. ([3]) Let (θ_n) be a sequence of positive numbers and (μ_n) be bounded sequence of positive numbers. Then, the set $\left|\bar{N}_p^{\theta}\right|(\mu)$ becomes a linear space with the coordinate-wise addition and scalar multiplication, and also it is an FK-space with AK with respect to the paranorm $h(x)=g(T(x))$ with

$$g(T(x))=\left(\sum_{n=0}^{\infty}\theta_n^{\mu_{n-1}}|T_n(\theta,\mu,p)(x)|^{\mu_n}\right)^{1/M}$$

where $\theta_0=1$ and $M=\max\{1,\sup_n\mu_n\}$.

Theorem 2. ([3]) Assume that $\mu=(\mu_n)$ be a bounded sequence of positive numbers. Then, the space $\left|\bar{N}_p^\theta\right|(\mu)$ is linear isomorphic to $l(\mu)$, that is, $\left|\bar{N}_p^\theta\right|(\mu)\cong l(\mu)$.

2. MAIN RESULTS

Theorem 3 Let $A=(a_{nv})$ be an infinite matrix of complex numbers, $\theta=(\theta_n)$ be any sequence of positive real numbers and $\mu=(\mu_v)$ be any bounded sequence of positive real numbers with $\mu_v\leq 1$ for all v. Then

(a) A $\epsilon(\left|\bar{N}_p^\theta\right|(\mu),c)$ if and only if

$$\sup_v\left\{\left|\frac{P_v}{\theta_v^{1/\mu_v^*}p_v}\left(a_{nv}-\frac{P_{v-1}}{P_v}a_{n,v+1}\right)\right|^{\mu_v}+\left|\frac{P_v a_{nv}}{\theta_v^{1/\mu_v^*}p_v}\right|^{\mu_v}\right\}<\infty \tag{1}$$

$$\lim_{n\to\infty}\left(\frac{P_v}{\theta_v^{1/\mu_v^*}p_v}\left(a_{nv}-\frac{P_{v-1}}{P_v}a_{n,v+1}\right)\right)\text{ exists for all } v \tag{2}$$

$$\sup_{n,v}\left|\frac{P_v}{\theta_v^{1/\mu_v^*}p_v}\left(a_{nv}-\frac{P_{v-1}}{P_v}a_{n,v+1}\right)\right|^{\mu_v}<\infty. \tag{3}$$

(b) A $\epsilon(\left|\bar{N}_p^\theta\right|(\mu),c_0)$ if and only if the conditions (1), (3) and

$$\lim_{n\to\infty}\left(\frac{P_v}{\theta_v^{1/\mu_v^*}p_v}\left(a_{nv}-\frac{P_{v-1}}{P_v}a_{n,v+1}\right)\right)=0\quad\text{for all } v \tag{4}$$

hold.

(c) A $\epsilon(\left|\bar{N}_p^\theta\right|(\mu),l_\infty)$ if and only if the conditions (1), (3) hold.

Proof Let $\mu_v\leq 1$ for all integer v. Since $\left|\bar{N}_p^\theta\right|(\mu)=(l(\mu))_{T(\theta,\mu,p)}$, it follows from Lemma 2 A $\epsilon(\left|\bar{N}_p^\theta\right|(\mu),c)$ if and only if $\tilde{A}\,\epsilon\,(l(\mu),c)$ and $V^{(n)}\,\epsilon\,(l(\mu),c)$ where

$$\hat{a}_{nv}=\frac{P_v}{\theta_v^{1/\mu_v^*}p_v}\left(a_{nv}-\frac{P_{v-1}}{P_v}a_{n,v+1}\right), n,v=0,1,\ldots$$

and

$$v_{mv}^{(n)}=\begin{cases}\hat{a}_{nv}, 0\leq v\leq m-1\\ \dfrac{P_m a_{nm}}{\theta_m^{1/\mu_m^*}p_m}, \quad v=m, m\geq 1\\ 0, \qquad v>m.\end{cases}$$

So, using Lemma 1, we get that $\tilde{A}\,\epsilon\,(l(\mu),c)$ if and only if the conditions (2), (3) hold. Again, applying Lemma 1 to the matrix $V^{(n)}$, we obtain that $V^{(n)}\,\epsilon\,(l(\mu),c)$ if and only if the condition (1) holds which concludes the first part of the proof.
The remaining part of the proof can be proved similar way. So, it left to reader.

Theorem 4 Let $A=(a_{nv})$ be an infinite matrix of complex numbers, $\theta=(\theta_n)$ be any sequence of positive real numbers and $\mu=(\mu_v)$ be any bounded sequence of positive real numbers with $\mu_v>1$ for all v. Then

(a) A $\epsilon(\left|\bar{N}_p^\theta\right|(\mu),c)$ if and only if there exists an integer $M>1$ such that

$$\sup_n \sum_{v=0}^{\infty} \left| M^{-1} \frac{P_v}{\theta_v^{1/\mu_v^*} p_v} \left(a_{nv} - \frac{P_{v-1}}{P_v} a_{n,v+1} \right) \right|^{\mu_v^*} < \infty \tag{5}$$

$$\sup_m \left\{ \sum_{v=0}^{m-1} \left| M^{-1} \frac{P_v}{\theta_v^{1/\mu_v^*} p_v} \left(a_{nv} - \frac{P_{v-1}}{P_v} a_{n,v+1} \right) \right|^{\mu_v^*} + \left| M^{-1} \frac{P_m a_{nm}}{\theta_m^{1/\mu_m^*} p_m} \right|^{\mu_m^*} \right\} \tag{6}$$

and the condition (2) hold.

(b) A $\epsilon(|\bar{N}_p^\theta|(\mu), c_0)$ if and only if the conditions (4), (5) and (6) hold.

(c) A $\epsilon(|\bar{N}_p^\theta|(\mu), l_\infty)$ if and only if the conditions (5), (6) hold.

Proof Let $\mu_v > 1$ for all v. Since $|\bar{N}_p^\theta|(\mu) = (l(\mu))_{T(\theta,\mu,p)}$, it follows from Lemma 2 A $\epsilon(|\bar{N}_p^\theta|(\mu), c_0)$ if and only if $\tilde{A} \in (l(\mu), c_0)$ and $V^{(n)} \in (l(\mu), c)$ where the matrices $\tilde{A}$ and $V^{(n)}$ are defined as in the above theorem. It follows from Lemma 1, $\tilde{A} \in (l(\mu), c_0)$ if and only if the condition (5) holds. Also, $V^{(n)} \in (l(\mu), c)$ equals to the conditions (4) and (6) hold. So, it completes the proof of (b).
The other part left to reader to avoid repetition.
Take the matrix $L = (l_{nv})$ defined by

$$l_{nv} = \begin{cases} 1, & 0 \le v \le n \\ 0, & v > n. \end{cases}$$

Since $c_s = \{c\}_L$ and $b_s = \{l_\infty\}_L$, with Lemma 3, the matrix classes $(|\bar{N}_p^\theta|(\mu), c_s)$ and $(|\bar{N}_p^\theta|(\mu), b_s)$ can be characterized as follows:

Theorem 5 Let $A = (a_{nv})$ be an infinite matrix of complex numbers, $\theta = (\theta_n)$ be any sequence of positive real numbers, $\mu = (\mu_v)$ be any bounded sequence of positive numbers. Put $a(n,v) = \sum_j^n a_{jv}$ instead of a_{nv} in Theorem 3 and Theorem 4.
If $\mu_v \le 1$ for all v,

$$(|\bar{N}_p^\theta|(\mu), c_s) \Leftrightarrow (1), (2), \text{and } (3) \text{ hold},$$
$$(|\bar{N}_p^\theta|(\mu), b_s) \Leftrightarrow (1) \text{ and } (3) \text{ hold}.$$

If $\mu_v > 1$ for all v,

$$(|\bar{N}_p^\theta|(\mu), c_s) \Leftrightarrow (2), (5), \text{and } (6) \text{ hold},$$
$$(|\bar{N}_p^\theta|(\mu), b_s) \Leftrightarrow (5) \text{ and } (6) \text{ hold}.$$

Theorem 6 Let $A = (a_{nv})$ be an infinite matrix of complex numbers, $\theta = (\theta_n)$ be any sequence of positive real numbers, $\mu = (\mu_v)$ be any bounded sequence of positive numbers and $\Lambda \epsilon \{c, c_0, l_\infty\}$. If A $\epsilon(|\bar{N}_p^\theta|(\mu), \Lambda)$, then A defines a bounded linear operator.
Proof Since the spaces c, c_0, l_∞ are BK- spaces, normed FK-spaces, using Lemma 4 and Theorem 1, the proof of Theorem can be immediately obtained.

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A NOTE ON ABSOLUTE NORLUND SUMMABILITY FACTORS

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Abstract:

The set of all sequences $\varepsilon = (\varepsilon_n)$ such that the series $\sum x_n \varepsilon_n$ is summable by the summability method Γ whenever the series $\sum x_n$ is summable by the summability method Λ is denoted by (Λ, Γ) and the sequence ε is called as a summability factor type (Λ, Γ). In the present study, we establish the general summability factor theorems related to the absolute Nörlund summabilities $|N, \theta, p|(\mu)$ where θ is any sequence of positive numbers and μ is any bounded sequence of positive numbers.

Keywords: Absolute summability, Nörlund matrix, summability factor, matrix transformations
General area of research: Mathematics
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1. INTRODUCTION

Let $A = (a_{nv})$ be an infinite matrix of complex numbers and ω be the set of all sequences of complex numbers. By $A(x) = (A_n(x))$, we stand for the A- transform of a sequence $x = (x_v)$, if the series

$$A_n(x) = \sum_{v=0}^{\infty} a_{nv} x_v$$

is convergent for $n \geq 0$. If $Ax \in Y$ whenever $x \in X$ then A is called a matrix transformation from X into Y, where X and Y are arbitrary sequence space, subsets of ω, and it is written that $A : X \to Y$. Also, we mean the class of all infinite matrices A such that $A : X \to Y$ by (X, Y).

Let $\sum x_v$ be a given infinite series with its nth partial sum s_n, $\theta = (\theta_n)$ be a sequence of positive real numbers and $\mu = (\mu_n)$ be any bounded sequence of positive numbers. The series $\sum x_v$ is said to be summable $|A, \theta|(\mu)$ if ([2])

$$\sum_{n=1}^{\infty} \theta_n^{\mu_n - 1} |A_n(s) - A_{n-1}(s)|^{\mu_n} < \infty.$$

It is noted that the summability method includes several well known summability methods. For instance, combining the concept of absolute summability and Nörlund matrix given by

$$a_{nv} = \begin{cases} \frac{p_{n-v}}{P_n}, & 0 \leq v < n \\ 0, & v > n \end{cases}$$

the absolute Nörlund summability method $|N,\theta,p|(\mu)$ and the space of all series summable by the summability method

$$|N_p^\theta|(\mu) = \left\{ x = (x_v) : \sum_{v=0}^{\infty} \theta_n^{\mu_n - 1} \left| \sum_{v=0}^{n} \left(\frac{P_{n-v}}{P_n} - \frac{P_{n-1-v}}{P_{n-1}} \right) x_v \right|^{\mu_n} < \infty \right\}$$

have been defined and studied by Gökçe and Sarıgöl [1]. It is seen immediately that there is a close relation between the Absolute Nörlund spaces $|N_p^\theta|(\mu)$ and the space $l(\mu)$ such that $|N_p^\theta|(\mu) = (l(\mu))_{L^{(\mu)}(\theta)\circ T(p)}$ where the terms of the matrices $L^\mu(\theta)$ and $T(p)$ as follows:

$$l_{nv}^{(\mu)}(\theta) = \begin{cases} \theta_n^{1/\mu_n^*}, & v = n \\ -\theta_n^{1/\mu_n^*}, & v = n-1 \\ 0, & v \neq n, n-1, \end{cases}$$

$$t_{nv} = \begin{cases} \frac{P_{n-v}}{P_n}, & 0 \leq v < n \\ 0, & v > n. \end{cases}$$

Throughout the whole paper, we assume that μ_n^* is conjugate of μ, i.e., $\frac{1}{\mu_n} + \frac{1}{\mu_n^*} = 1$ for $\mu_n > 1$ and $\frac{1}{\mu_n^*} = 0$ for $\mu_n = 1$, for all n.

The series-to-sequence transformation corresponding to the summability $|N,\theta,p|(\mu)$ define the sequence $(T_n(\theta))$ such that

$$T_n(\theta)(x) = \theta_n^{1/\mu_n^*} \sum_{v=0}^{n} \left(\frac{P_{n-v}}{P_n} - \frac{P_{n-1-v}}{P_{n-1}} \right) x_v.$$

Let Λ, Γ be arbitrary summability methods. If the series $\sum x_n \varepsilon_n$ is summable by the summability method Γ whenever the series $\sum x_n$ is summable by the summability method Λ, then it is said that the sequences $\varepsilon = (\varepsilon_n)$ is a summability factor type (Λ, Γ) and it is written that $\varepsilon \in (\Lambda, \Gamma)$.

A lot of studies about the absolute summability factor and the comparison of summability methods, which have an important role in Approximation Theory and Fourier Analysis, have taken their place in the literature. In this study, we note that these problems correspond to the special matrix transformations such as identity matrix and diagonal matrix. In this context, applying the main theorems given by Gökçe and Sarıgöl in [1] to summability factors, we give the necessary and sufficient conditions on the sequence ε for $\varepsilon \in (|N,\theta,p|(\mu), |N,\varphi,q|)$ and $\varepsilon \in (|N,\theta,p|(\mu), |N,\varphi,q|(\xi))$ without using standard operations where θ, φ are any sequences of positive numbers, μ and ξ are any bounded sequences of positive numbers. Throughout the paper, for simplicity, we write

$$\sigma_{jv} = \sum_{v=j}^{n} C_{n-v} P_v$$

where (C_n) is a sequence such that

$$\sum_{v=0}^{n} P_{n-v} C_v = \begin{cases} 1, n = 0 \\ 0, n > 0. \end{cases}$$

Followings theorems are given by Gökçe and Sarıgöl [1]:

Theorem 1. Let $A=(a_{nv})$ be an infinite matrix of complex numbers, θ,φ be sequences of positive numbers and μ,ξ be any bounded sequences of positive numbers with $\mu_n\leq 1$ and $\xi_n\geq 1$ for all n. Also, define the matrix $\hat{A}=L^{(\xi)}\circ T(q)\circ\tilde{A}$ by

$$\tilde{a}_{nv}=\theta_v^{-1/\mu_v^*}\sum_{j=v}^{\infty}a_{nj}\sigma_{jv}\,,n,v=0,1,....$$

If p_0 is non-zero number, then $A\epsilon\left(\left|N_p^\theta\right|(\mu),\left|N_q^\varphi\right|(\xi)\right)$ if and only if exists an integer $M>1$ such that for $n=0,1,...$

$$\theta_v^{-1/\mu_v^*}\sum_{j=v}^{\infty}a_{nj}\sigma_{jv}\ \text{ converges for each } v \tag{1}$$

$$\sup_{m,v}\left|\theta_v^{-1/\mu_v^*}\sum_{j=v}^{m}a_{nj}\sigma_{jv}\right|^{\mu_v}<\infty \tag{2}$$

$$\sup_v\sum_{n=0}^{\infty}\left|M^{-1/\mu_v}\hat{a}_{nv}\right|^{\xi_n}<\infty\,. \tag{3}$$

Theorem 2. Assume that $A=(a_{nv})$ is an infinite matrix of complex numbers, θ,φ are sequences of positive numbers and μ is any bounded sequence of non-negative numbers with $\mu_n>1$ for all n. If p_0 is non-zero number, then $A\epsilon\left(\left|N_p^\theta\right|(\mu),\left|N_q^\varphi\right|\right)$ if and only if exists an integer $M>1$ such that for $n=0,1,...$

$$\theta_v^{-1/\mu_v^*}\sum_{j=v}^{\infty}a_{nj}\sigma_{jv}\ \text{ converges for each } v \tag{4}$$

$$\sup_m\sum_{v=0}^{\infty}\left|\theta_v^{-1/\mu_v^*}\sum_{j=v}^{m}a_{nj}\sigma_{jv}\,M^{-1}\right|^{\mu_v^*}<\infty \tag{5}$$

$$\sum_{v=0}^{\infty}\left(\sum_{n=0}^{\infty}\left|M^{-1}\hat{a}_{nv}\right|\right)^{\mu_v^*}<\infty\,. \tag{6}$$

2. MAIN RESULTS

In this section, we will obtain the necessary and sufficient conditions $\varepsilon\,(|N,\theta,p|(\mu),|N,\varphi,q|)$ and $\varepsilon\in(|N,\theta,p|(\mu),|N,\varphi,q|(\xi))$ applying the main theorems given by Gökçe and Sarıgöl [1] to summability factor without using standard operations.

Theorem 3. Let $A=(a_{nv})$ be an infinite matrix of complex numbers, p_0 be non-zero number, θ,φ be sequences of positive numbers and μ,ξ be any bounded sequences of positive numbers with $\mu_n\leq 1$ and $\xi_n\geq 1$ for all n. Then, the necessary and sufficient condition for $\varepsilon\,(|N,\theta,p|(\mu),|N,\varphi,q|(\xi))$ is

$$\sup_v \sum_{n=v}^{\infty} \left| M^{-1/\mu_v} \frac{\varphi_n^{1/\xi_n^*}}{\theta_v^{1/\mu_v^*}} \left(\sum_{j=v}^{n} \left(\frac{P_{n-j}}{P_n} - \frac{P_{n-1-j}}{P_{n-1}} \right) \varepsilon_j \, \sigma_{jv} \right) \right|^{\xi_n} < \infty . \qquad (7)$$

Proof. If we take the diagonal matrix $W = (w_{nv})$ defined by

$$w_{nv} = \begin{cases} \varepsilon_n, & n = v \\ 0, & n \neq v \end{cases}$$

instead of A in Theorem 1, then the condition (1) is reduced to the condition

$$\theta_v^{-1/\mu_v^*} \varepsilon_n \sigma_{nv}, n \geq v$$

which is trivial. Also, the condition (2) is directly satisfied. On the other hand, with $A = W$, the term of $\hat{A}$ is obtained as follows:

$$\hat{a}_{nv} = \frac{\varphi_n^{1/\xi_n^*}}{\theta_v^{1/\mu_v^*}} \left(\sum_{j=v}^{n} \left(\frac{P_{n-j}}{P_n} - \frac{P_{n-1-j}}{P_{n-1}} \right) \varepsilon_j \, \sigma_{jv} \right)$$

which is completed the proof of theorem.

Theorem 4. Assume that $A = (a_{nv})$ is an infinite matrix of complex numbers, p_0 is non-zero number, θ, φ are sequences of positive numbers and μ is any bounded sequence of positive numbers with $\mu_n > 1$ for all n. Then, the necessary and sufficient condition for $\varepsilon \in (|N, \theta, p|(\mu), |N, \varphi, q|)$ is

$$\sum_{v=0}^{\infty} \left(\sum_{n=v}^{\infty} \left| \frac{M^{-1}}{\theta_v^{1/\mu_v^*}} \left(\sum_{j=v}^{n} \left(\frac{P_{n-j}}{P_n} - \frac{P_{n-1-j}}{P_{n-1}} \right) \varepsilon_j \, \sigma_{jv} \right) \right| \right)^{\mu_v^*} < \infty . \qquad (8)$$

Proof. Let p_0 be non-zero number, θ, φ be sequences of positive numbers and μ be any bounded sequence of positive numbers with $\mu_n > 1$ for all n. Taking W instead of A in Theorem 2 the condition (4) and (5) are directly satisfied. On the other hand, the condition (6) is reduced to (8).

Corollary 5. Let $A = (a_{nv})$ be an infinite matrix of complex numbers, p_0 be non-zero number, θ, φ be sequences of positive numbers and $1 \leq k < \infty$. Then, the necessary and sufficient condition for $\varepsilon \in (|N, \theta, p|, |N, \varphi, q|_k)$ is

$$\sup_v \sum_{n=v}^{m} \left| M^{-1/\mu_v} \varphi_n^{1/k^*} \left(\sum_{j=v}^{n} \left(\frac{P_{n-j}}{P_n} - \frac{P_{n-1-j}}{P_{n-1}} \right) \varepsilon_j \, \sigma_{jv} \right) \right|^{k} < \infty .$$

Corollary 5. Let $A = (a_{nv})$ be an infinite matrix of complex numbers, p_0 be non-zero number, θ, φ be sequences of positive numbers and $1 \leq k < \infty$. Then, the necessary and sufficient condition for $\varepsilon \in (|N, \theta, p|_k, |N, \varphi, q|)$ is

$$\sum_{v=0}^{\infty}\left(\sum_{n=v}^{\infty}\frac{1}{\theta_v}\left|\left(\sum_{j=v}^{n}\left(\frac{P_{n-j}}{P_n}-\frac{P_{n-1-j}}{P_{n-1}}\right)\varepsilon_j\,\sigma_{jv}\right)\right|\right)^{k^*} < \infty .$$

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ASSOCIATION BETWEEN COL9A2 AND COL9A GENES POLYMORPHISMS AND DISC DEGENERATIVE DISORDER

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Abstract:

Background: The extracellular matrix of intervertebral discs is mainly composed of different types of collagen fibers. Polymorphisms in collagen genes may play a vital role in the development of degenerative disc disease (DDD). In this study, we investigate the association between single nucleotide polymorphisms on COL9A2 and COL9A3 genes and Disc Degenerative Disorder (DDD) epidemiology.
Material and Methods: One hundred and two Jordanian DDD patients and sixty-three health controls were matched for age, gender, and body mass index. 4 ml of whole blood was collected in EDTA coated vacutainer tubes for DNA extraction using an Omega kit (E.Z.N.A. Blood DNA Mini Kit). The primers for SNPs in COL9A2 and COL9A3 were manually designed using Ensemble database sequence and genotyped by DNA sequencing for the target regions, then analyzed manually.
Results: Our results reported a significant association between rs2228564 polymorphism of the COL9A2 gene and rs35908728 of the COL9A3 and the development of DDD. On the other hand, rs12077871 and rs2228567 polymorphisms of the COL9A2 gene and rs61734651 and rs142639450 polymorphism of COL9A3 gene showed no significant association with the development of DDD. Nevertheless, our findings in rs2228567 and rs35908728 demonstrated a significant association between those variations and cases greater than 40 years.
Conclusion: Our findings underscore the possible association between the development of DDD and the polymorphic variants of rs2228564, which suggests further genetic investigations on collagen genes and their role in DDD development

Keywords: COL9A3, COL9A2, rs2228564, rs35908728, rs12077871, rs222856 and DDD
General area of research: Biology and Genetics
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FRACTIONAL-ORDER PROBLEM COUPLED WITH A SECOND-ORDER PERTURBED MOREAU SWEEPING PROCESS

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Abstract:

The advancement of science in the field of biology and environment, has given rise to other mathematical problems such as the study of coupled systems of fractional-order differential inclusion. We refer the reader to references [1, 4] for the recent results on the topic. Motivated by [3, 5] and inspired by [4], we investigate the existence of an absolutely continuous solution, in a Hilbert space, of a non local multi-point boundary value problem coupled with a second-order perturbed time and state dependent Moreau's sweeping process [2] by applying Schauder's fixed point theorem. The sets in the process are considered to be subsmooth. To the best of our knowledge, a very few study is available in the fractional-order differential inclusions coupled with a time and state dependent sweeping process, which are a special kind of coupled system of differential inclusions.

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Keywords: Clarke normal cone, Hadamard fractional derivative, subsmoothness, fixed point theorem, differential
inclusions.
General area of research: Mathematics
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COMPUTER STUDY OF OPTICAL FIELD LOCALIZATION AND PHASE TRANSITION IN A THREE-DIMENSIONAL PERCOLATION NANOSYSTEM

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Abstract:

Disordered photonic materials can diffuse and localize light through random multiple scattering that leads to formation of the electromagnetic modes depending on the structural correlations, scattering strength, and dimensionality of the system, see [1] and references therin. However, in real systems the non-negligible interactions between light and medium can take place. Therefore important aspect of the optical localization is the interplay between nonlinear interactions and linear localization effect. We systematically study the optical field localization in active three-dimensional (3D) disordered percolating system where the excited light nanoemitters are incorporated into the percolating clusters. The feature of a situation that in such a medium the field clusters joint a fractal radiating system, where the light is both emitted and scattered by the disordered structures. The analysis reveals the non-trivial dynamics of such localization-related phenomenon allowing the propagation of the localized field bunches in a percolating system. We have studied the average localization strength of such states and carry out an analysis of the dynamic properties as a function of the occupation probability of disordered clusters. We have found the critical transition separating the propagating states and narrow point-like fields pinned to the emitters. It is shown that stable optical bunches can propagate in the system if the percolating probability does not exceed the threshold value (Fig.1). The study of field transition in 3D optical systems still has not been conclusive despite considerable efforts. The localization transition may be difficult to reach for the light waves due to various effects in dense disordered media required to achieve strong scattering . The optical field localization in 3D percolating disorder, where the percolating clusters are filled by the light nanoemitters in the excited state may be suggested as an extension to 3D optical case the localization released in one-dimensional waveguide in the presence of a controlled disorder. In 3D disordered percolating systems the optical transport was observed and also it is found that the random lasing assisted by nanoemitters incorporated into such a disordered structure can occur.

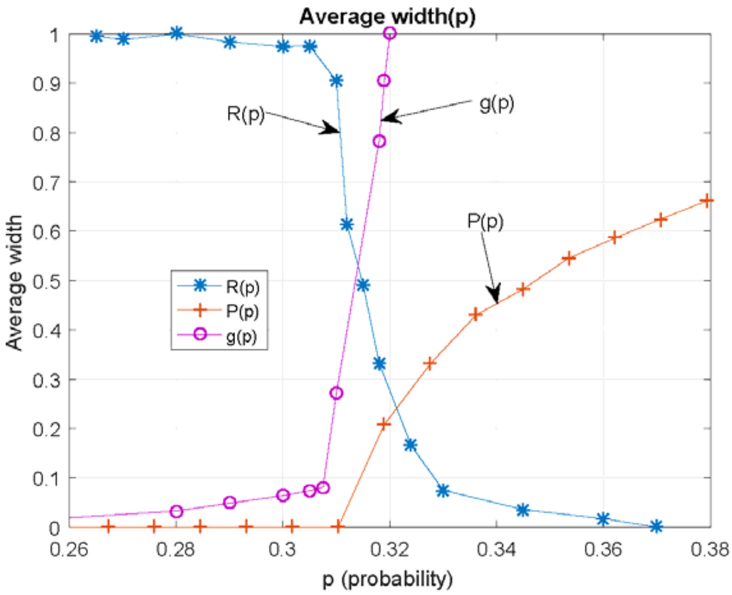

Fig. 1 The average radius of the localized optical bunches R=R(p) and amplitude a=a(p) as function of the percolation probability p.

1. G. Burlak, The Dynamic Three-Dimensional Localization of Fields in Active Percolating Systems, Adv.Math.Phys., 5867012, (2019).

Acknowledgement: This study is supported in part by CONACYT (Mexico) under the Grant No. A1-S-9201.

Keywords: Optical localization, percolation 3D, computer simulations
General area of research: Physics and applications.
ICFAS2022-ID: 1005

ON GENERALIZED DERIVATIONS INVOLVING PRIME IDEALS OF $*$-RINGS

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Abstract:

Let R be an associative ring with involution $*$ and P a prime ideal of R. An additive mapping $d: R \to R$ is said to be a derivation of R if it satisfies $d(xy) = d(x)y + xd(y)$ for all $x, y \in R$. An additive mapping $F: R \to R$ is said to be a generalized derivation of R if there exists a derivation d of R such that $F(xy) = F(x)y + xd(y)$ for all $x, y \in R$. In the present paper, we study the action of generalized derivations on prime ideals of $*$-rings and investigate the structure of the quotient ring R/P.

Keywords: Prime ideal, derivation, generalized derivation, involution
General area of research: Mathematics
ICFAS2022-ID: 1008

SOLUTION OF ANGULAR PART OF THE CONFORMABLE SCHRODINGER EQUATION

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Abstract:

In this work, the conformable Schrodinger equation in spherical coordinates is separated into two parts; radial and angular part, the angular part of the Schrodinger equation is solved. The normalized Spherical harmonics function is obtained as a solution of the angular part.

Keywords: Conformable derivative, spherical harmonics, Schrodinger equation, conformable partial derivative
General area of research: Mathematics
ICFAS2022-ID: 1011

A SECURE ROUTING APPROACH FOR EFFECTIVE COMMUNICATION IN WIRELESS SENSOR NETWORK

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Abstract:

In wireless sensor networks (WSN), security is a crucial fact for energy conservation. Moreover, trust management is a problematic issue in WSN because trust is used when participation is necessary to ensure reliable communication. Secret information, such as military operations, hospital monitoring and environment, must often be communicated urgently in a military application employing WSN. On the other hand, existing routing algorithms don't take care of security during the routing method. Furthermore, because security is a critical component of WSNs, security considerations must be factored into routing algorithms. In the literature, different ways to provide security are discussed, including trust management, firewalls, as well as key management. When compared to other security solutions, trust management is one of them that can provide enhanced security. In this paper, we propose a new trust-based secure routing algorithm where the trust evaluation is use to find malicious nodes beneficially in WSN. Based on the experiment and works, it is clear that the proposed trust-based routing algorithm outperforms existing systems in terms of security, energy consumption, and packet delivery ratio.

Keywords: Wireless sensor network, routing algorithm, energy consumption, packet delivery ratio
General area of research: Information Science
ICFAS2022-ID: 1012

ON KIRCHHOFF TYPE PROBLEMS WITH CONVECTION

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Abstract:

We study elliptic equations whose main operator is a degenerate Kirchhoff-type p(x)-operator, imposing different boundary conditions. The appropriate framework space to develop the study is based on the notion of generalized Sobolev space with variable exponent. We establish existence results of solutions, in a weak sense, using the Nemitsky map associated to the reaction term, and a consequence of Brouwer fixed point theorem assested for a Galerkin basis of the framework space, which is a Banach space with regularities. The technical assumptions are useful to control the growth of the reaction term, obtaining a priori bounds to integral terms and sign constraints to the involved operators.

Keywords: Variable exponent space, Kirchhoff term, p(x)-Laplace operator
General area of research: Mathematics
ICFAS2022-ID: 1014

ANISOTROPIC PROBLEMS IN VARIABLE EXPONENT SOBOLEV SPACES

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Abstract:

We investigate the solutions to certain anisotropic Dirichlet boundary value problems, with a gradient dependent nonlinearity. Precisely, we deal with elliptic equations driven by the sum of two differential operators of p(x)-Laplacian type and q(x)-Laplacian type, on a open bounded domain Ω, with smooth boundary. The exponents are continuous functions, which are related each other, and separately are bounded and bounded away from 1. The techniques of proofs are based on the properties of pseudomonotone operators, jointly with suitable a priori estimates in Lebesgue and Sobolev spaces. Thus, we establish both the existence and uniqueness of weak solutions, using a topological approach.

Keywords: Variable exponent space, anisotropic regularity, gradient dependent reaction
General area of research: Mathematics
ICFAS2022-ID: 1017

MULTIPLICATIVE b−GENERALIZED DERIVATIONS ON DENSE IDEALS OF PRIME RINGS

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Abstract:

Consider R to be an associative prime ring and K to be a nonzero dense ideal of R. A non additive mapping F : R → Qmr associated with derivation d : R → R is called a multiplicative b−generalized derivation if $F(\alpha\delta) = F(\alpha)\delta + b\alpha d(\delta)$ holds for all $\alpha, \delta \in R$ and for any fixed $0 \neq b \in Qs \subseteq Qmr$. In this manuscript, we study the commutativity of prime rings when the map b−generalized derivation satisfies the strong commutativity preserving condition and moreover, we investigate the commutativity of prime rings that admitting multiplicative b−generalized derivation which improves many results in the literature

Keywords: Prime ring, derivation, generalized derivation, multiplicative b−generalized derivation
General area of research: Mathematics
ICFAS2022-ID: 1018

ON JAIN OPERATORS AND THEIR GENERALIZATIONS

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Abstract:

On the last six decades the interest of the study of positive approximation processes have emerged with growing evidence. A special place is occupied by in-depth study of classical operators. The most eloquent example is Bernstein operator which represents a permanent challenge for the researches in mentioned field.
However, in this talk we focused on presenting a class of operators introduced by G. C. Jain that have long been in a shadowy cone. In recent years many papers have appeared about their properties and many generalizations have been analyzed. In our approach, there is no question of an exhaustive treatment, but only of collecting some published results by various authors including those of the undersigned that prove the importance of this class through the generous possibilities offered by the approximation of signals from different spaces functions. In distinct sections we will present approximation properties related to discrete operators, respectively integral generalizations thereof. In this direction, among others, Voronovskaya type theorems, the rate of convergence expressed by using different smoothness modules as well as statistical convergence are inspected. Another goal followed was that the presentation to be essentially self containd.

Keywords: linear positive operator; Poisson distribution; Korovkin theorem; weighted space; statistical convergence.
General area of research: Mathematics
ICFAS2022-ID: 1019

CONTRIBUTIONS REGARDING THE RUBBER SPRING SUSPENSIONS STUDY FOR THE RAILWAY VEHICLES

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Abstract:

In the article, the authors enlist the main elastic and amortization properties of the material (rubber) and implementation possibilities for the railway vehicles which are rolling in extreme temperatures conditions.
In the first part of the study, there are presented the constructive types of suspensions that are utilizing rubber, and their main characteristics. The main properties of rubber are analyzed, including the situations when suspensions, exclusively made of rubber, are being used. The analysis also includes suspensions with metal springs that are working simultaneously with the rubber components.
The second part of the article consists of experimental determinations for temperatures <20°C and >40 °C. Considering that rubber durability is strongly influenced by temperature, the paper offers recommendations regarding the suspension's auxiliary systems, which maintain the temperatures within acceptable ranges, so the elastic characteristics will comply with the railway safety and quality of rolling requirements.

Keywords: Spring suspension, rubber, temperature
General area of research: Mechanical Engineering
ICFAS2022-ID: 1020

CONTRIBUTIONS REGARDING THE QUALITY OF ROLLING AND PASSENGER COMFROT FOR RAILWAY VEHICLES

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Abstract:

In the article, the authors enlist the main properties regarding the quality of rolling, for railway vehicles, followed by an analysis of the quality determination methods. The article continues with a technical presentation of the main embedded hardware, responsible with the rolling quality, installed on the subway vehicles. The last part of the article contains the conclusions drawn from the study.

Keywords: Quality, comfort, rolling
General area of research: Mechanical Engineering
ICFAS2022-ID: 1021

RESEARCH REGARDING THE AIR SUSPENSIONS OF THE RAILWAY VEHICLES

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Abstract:

In the article, there are presented the damping and elastic properties of the air suspensions, used in the railway vehicles industry. Within the doctoral school at Politehnica University of Bucharest, Faculty of Transportation Engineering, this specific domain has been addressed as a matter of priority, given that this type of suspension is widely used for subway vehicles, and for railway vehicles.

There are presented experimental determinations and a mathematical model for studying the vibrations of the railway vehicles is also proposed.

A further analysis regarding two important conditions in railway vehicles industry: safety of rolling on the track and rolling quality assurance.

Keywords: Air suspension, vibrations, railway safety
General area of research: Mechanical Engineering
ICFAS2022-ID: 1022

THE ANALYSIS OF THE MAIN FACTORS THAT ARE LIMITING THE MAXIMUM SPEED OF A RAILWAY VEHICLE

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Abstract:

In this article, it is being presented the influence of resistances in relation to the rolling speed of a railway vehicle and the hunting oscillation phenomenon, which have a direct impact on the maximum speed limit.
Furthermore, it is being analyzed the vehicle performance in connection with the adherence between the wheels and rails.
It is being presented also a mathematical model that permits the determination of the critical speed when the hunting becomes unstable.

Keywords: hunting, adherence, critical speed
General area of research: Mechanical Engineering
ICFAS2022-ID: 1023

NONLINEAR GENERALIZED JORDAN n-DERIVATIONS OF UNITAL ALGEBRAS WITH IDEMPOTENTS

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Abstract:

Let $\mathcal{A}$ be a unital algebra with a nontrivial idempotent. We show that under certain conditions every nonlinear generalized Jordan n-derivation $\Delta: A \to A$ is of the form $\Delta(x) = zx + \delta(x)$, where $z \in Z(A)$ and $\delta: A \to A$ is a Jordan n-derivation. The main result is then applied to some classical examples of unital algebras with nontrivial idempotents such as triangular algebras, matrix algebras, nest algebras, and algebras of bounded linear operators.

Keywords: Unital algebra, matrix algebra, nest algebra, nonlinear Jordan \$n\$-derivation, nonlinear generalized Jordan \$n\$-derivation
General area of research: Mathematics
ICFAS2022-ID: 1024

THEORETICAL AND SPECTROSCOPIC STUDY OF THE INCLUSION PROCESS OF BENZOXAZOLINONE (BOA) IN β-CYCLODEXTRIN

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Abstract:

The inclusion complexation behavior, characterization and binding ability of β-cyclodextrin (β-CD) with Benzoxazolinone (BOA) have been investigated both in solution and solid state by means of UV-Vis absorption, FT-IR, and molecular modeling methods. The stability constant of inclusion complexes was determined by spectroscopic data and 1:1 stoichiometry was confirmed by Benesi-Hildebrand plots. Solid state characterization by FT-IR spectroscopy provided remarkable evidences of the formation of an inclusion system (BOA/β-CD). In addition to that, the complexation energy, thermodynamic parameters (ΔG, ΔH and ΔS) and HOMO, LUMO orbital investigations using PM3 level of theory, PM6, and hybrid method ONIOM2 confirm the stability of the inclusion system and indicate that the complex formation of (BOA/β-CD) was spontaneous and exothermic process. Moreover, the interactions between BOA and β-CD were discussed from NBO analysis. The major driving forces leading to complexation have been proposed to include electrostatic and van -der Waals interaction, hydrophobic interaction, and hydrogen bonding.

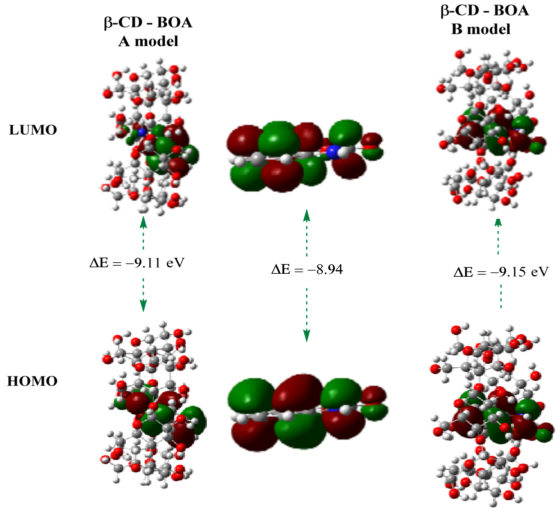

Fig. 1 Typical contour plots of HOMO and LUMO of the BOA and the b-CD/BOA Complexes (A model and B model) with their energy gaps Δ (HOMO–LUMO) in acetonitrile

Keywords: Benzoxazolinone , β-Cyclodextrin, inclusion complex, Benesi-Hildebrand plots, molecular modeling, PM3, ONIOM2, HOMO, LUMO, hydrophobic interaction
General area of research: Chemistry
ICFAS2022-ID: 1025

SOME PROPERTIES OF WEAKLY ZERO-DIVISOR GRAPH OF COMMUTATIVE RINGS

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Abstract:

Let R be a commutative ring and $Z(R)$ be its zero-divisors set. The weakly zero-divisor graph of R, denoted by $W\Gamma(R)$, is an undirected graph with vertex set $Z(R)^*$ and two distinct vertices x and y are adjacent if and only if there exists $a \in Ann(x)$ and $b \in Ann(y)$ such that $ab = 0$. In this paper, first we characterized finite rings R for which $W\Gamma(R)$, is isomorphic to some well-known graphs such as a tree, a unicycle graph, a split graph or an outerplanar graph and then we classify all the finite rings R for which $W\Gamma(R)$ is planar, toroidal or double toroidal. Finally, we classify the finite rings R for which the graph $W\Gamma(R)$ has crosscap at most two.

Keywords: Zero-divisor graph: weakly zero-divisor graph: planar graph: genus of a graph: crosscap of a graph
General area of research: Mathematics
ICFAS2022-ID: 1026

QUANTUM CODES FROM CONSTACYCLIC CODES OVER A CLASS OF NON-CHAIN RINGS

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Abstract:

Let $R_r = F_q + v_1 F_q + \cdots + v_r F_q$, where q is the power of prime, $v_i^2 = v_i,\ v_i v_j = v_j v_i = 0$ for $1 \leq i, j \leq r$ and $r \geq 1$. In this paper, the structure of λ-constacyclic codes over the ring R_r is studied and a Gray map ϕ from R^n to $F_q^{(r+1)n}$ is given. The necessary and sufficient conditions for these codes to contain their Euclidean duals are determined. As an application, we obtain many new better quantum codes from dual-containing λ -constacyclic codes over R_r, for $r = 1,2,3,$ that improve on the known existing quantum codes.

Keywords: Quantum codes, constacyclic codes, self-orthogonal codes, Gray map
General area of research: Mathematics
ICFAS2022-ID: 1027

MODELING THE TREATMENT DESERTION OF TUBERCULOSIS IN LOW- AND MIDDLE-INCOME COUNTRIES

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Abstract:

Pulmonary tuberculosis (TB) is an urgent public health problem that constitutes one of the leading causes of morbidity and mortality worldwide. TB is caused by a species of pathogenic bacteria called *Mycobacterium tuberculosis* or Koch's bacilli, which is transmitted from person to person through microscopic airborne droplets from the cough or sneeze of a person carrying active bacilli. Latent people whose immune systems are weakened by malnutrition or comorbidities (influenza, pneumonia, COVID19, kidney disease, diabetes, etc.) or those receiving immuno-suppressive treatments for HIV, cancer, or transplants may develop an active TB form anytime and thus become infectious.
Nowadays, there are different pharm products to treat both latent and active forms of tuberculosis. Still, their high costs, together with a long duration of the treatment, negatively affect the rate of treatment completion and provoke treatment abandonment. To reduce the costs of TB treatment, in many low- and middle-income countries, tuberculosis control is mainly centered on preventing latent TB infections from developing into active disease forms. In practice, only actively infected patients (bearing symptoms) are diagnosed and treated. During the first phase of the treatment, the patients start feeling much better as the severe symptoms of the disease disappear. Further, a considerable share of them stops taking medicine, thinking they have already been cured. However, by not completing the treatment, such patients remain infectious and spread the disease.
In this presentation, we propose and justify a simplified version of the TB transmission model where the treatment abandonment is explicitly modeled. In contrast to other models featuring treatment abandonment, our model has only four standard classes (susceptible, latent, infective, and treated), while people abandoning treatment are not separated into an additional class. Furthermore, we also model the non-sterilizing treatment, which is typical for low- and middle-income countries.

Keywords: Tuberculosis transmission model, non-sterilizing treatment, treatment abandonment, global stability, Lyapunov function
General area of research: Applied Mathematics, Mathematical applications of Biology
ICFAS2022-ID: 1028

BIOCONTROL OF DENGUE BY RELEASING STERILE MALE *Aedes Aegypti* MOSQUITOES

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Abstract:

The presence and abundance of *Aedes aegypti* mosquitoes are strongly correlated with the persistence and proliferation of different arboviral infections, such as yellow fever, dengue, chikungunya, and zika. *Aedes aegypti* females transmit the virus during their blood meals taken on human individuals, while male mosquitoes do not ingest human blood and cannot transmit viral infections. Therefore, the disease spread among people can be controlled via suppression of the mosquito population by mechanical, chemical, or biological control.
This presentation proposes an epidemiological model (SIR-SI) based on a sex-structured entomological model encompassing only adult mosquitoes. First, the dynamic of the dengue transmission without external control interventions is studied, and the basic reproductive number is calculated. Second, we introduce the biological control based on releasing the sterile male mosquitoes to suppress the total mosquito population. This method is widely known as the "Sterile Insect Technique" (SIT).
Our main result consists in identifying the constant number of sterile males to be released daily to lessen the effective reproductive number below one. Also, we explore the relation between the total cost of intervention and the constant number of sterile insects to be released daily.

Keywords: *Aedes aegypti* mosquitoes, sex-structured model, dengue transmission model, Sterile Insect Techniques, biological control
General area of research: Applied mathematics/Mathematical epidemiology
ICFAS2022-ID: 1029

A CLASS OF SPACES CONTAINING BOTH THE CLASSES OF RELATIVE NORMALITY AND RELATIVE COMPACTNESS

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Abstract:

In 2005, Kohli and Das introduced and studied some variants of normality namely (weakly) θ-normal spaces and obtained few factorizations of normality. In the present paper, these notions are studied in relative sense. It is observed that these classes contains both the classes of relative normality and relative compactness. Relative versions of these notions are investigated and their relationship with existing relative topological properties such as relative paracompactness, relative Lindelofness, and some other variants of relative normality like relative κ-normality, relative β-normality are studied. Behavior of these spaces under mappings are also studied.

Keywords: Relative topological properties, normality, θ-normality
General area of research: Mathematics
ICFAS2022-ID: 1030

A STUDY OF QUANTUM CODES FROM CYCLIC CODE OVER THE FINITE RING $F_p[u,v]/\langle u^2-\alpha^2, v^3-\beta^2 v, uv-vu \rangle$

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Abstract:

Let F_p be the field of order p, where p be an odd prime and α, β be the non zero elements of the field F_p. The primary goal of this paper is to study the structural properties of cyclic codes over the non chain ring $R = F_p[u,v]/\langle u^2-\alpha^2, v^3-\beta^2 v, uv-vu \rangle$, where $u^2 = \alpha^2$, $v^3 = \beta^2 v$ and $uv = vu$, p. We define a Gray map and construct the better quantum codes than the previously known quantum codes. Also, we also obtain the necessary and sufficient condition for cyclic codes that contain their duals.

Keywords: Cyclic code, dual code, Gray map, Quantum codes
General area of research: Mathematics
ICFAS2022-ID: 1032

A STUDY OF GENERALISED DERIVATIONS ON PRIME IDEALS IN RINGS WITH INVOLUTION

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Abstract:

In this paper, we make use of generalized derivations to scrutinise the deportment of prime ideal satisfying contain algebraic $*$-identities in rings with involution. In specific cases, the structure of the quotient ring R/P will be resolved, where R is an arbitrary ring and P is a prime ideal of R and we also find he behaviour of derivations associated with generalized derivations satisfying algebraic $*$-identities involving prime ideals.

Keywords: Prime ideals, derivarion
General area of research: Mathematics
ICFAS2022-ID: 1033

ON DERIVATIONS IN PRIME RINGS WITH INVOLUTION

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Abstract:

Let R be an associative ring (algebra) and N be the set of all non-negative integers. For all $x, y \in R$, the symbol $[x, y]$ stands for the commutator xy-yx. A mapping $f : R \rightarrow R$ is said to be commuting on R if $[f(x), x] = 0$ for all $x, y \in R$. A family of additive mappings $(d_i)_{i \in N}$ on a ring R is said to be higher derivation of order n, if $d_0(x) = x$ and $d_n(xy) = \sum_{i+j=n} d_i(x)d_j(y)$ for all $x, y \in R$. In a prime ring, if the commutator of each element and its image under a non-zero derivations is central, then the ring is commutative. In this paper, we shall discuss the *-version of Posner's second theorem for higher derivations in prime rings with involution. Moreover, some related results are also discussed.

Keywords: Prime ring, *-ideal, involution, derivation, higher derivation
General area of research: Mathematics
ICFAS2022-ID: 1034

ON A CLASS OF λ-MODULES

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Abstract:

Smith (2014) introduced maps between the lattice of ideals of a commutative ring and the lattice of submodules of an R-module M, i.e., μ and λ mappings. The definitions of the maps were motivated by the definition of multiplication modules. Moreover, some sufficient conditions for the maps to be lattice homomorphisms were studied. We define a class of λ-modules and indicate the properties of this class. We also present sufficient conditions for the module and the ring under which the class λ is a hereditary pretorsion class

Keywords: Lattice of ideals, lattice of submodules, multiplication module, hereditary pretorsion class
General area of research: Mathematics
ICFAS2022-ID: 1035

PROJECTIVEU MODULE AS A GENERALIZATION OF PROJECTIVE MODULE

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Abstract:

Let $\mathcal{U}$ and $\mathcal{N}$ be two families of *R*-modules, *V* a submodule of a direct sum of some elements in $\mathcal{U}$, and *X* a submodule of a direct sum of some elements in $\mathcal{N}$. An *R*-module *N* is $\mathcal{U}_V$-generated if there is an epimorphism from *V* to *N*. The family $\mathcal{N}$ is an *X*-sub-linearly independent to an *R*-module *M* if there is a monomorphism from *X* to *M*. Furthermore, the concept of $\mathcal{U}_V$-generated module and *X*-sublinearly independent are used to define $\mathcal{U}$-basis and $\mathcal{U}$-free module. A $\mathcal{U}$-free module is a generalization of a free module. In this paper, we define projective$_{\mathcal{U}}$ module as a direct summand of a $\mathcal{U}$-free module. This concept is a generalization of the projective module. We prove that the direct sum of some projective$_{\mathcal{U}}$ modules is projective$_{\mathcal{U}}$. Moreover, we show that the properties of the family $\mathcal{U}$ of *R*-modules affect the properties of projective$_{\mathcal{U}}$ module.

Keywords: Free module; projective module; projective$_{\mathcal{U}}$ module.
General area of research: Mathematics
ICFAS2022-ID: 1036

DETECTING AFFINE EQUIVALENCES AND SYMMETRIES OF IMPLICIT PLANE ALGEBRAIC CURVES

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Abstract:

We present a computational method to detect affine equivalences of implicit plane algebraic curves. The method proceeds by first reducing the problem to the case of affine equivalences where the orgin is fixed, using the Hessian curves of the input curves. Then, the Hessian is again used to derive certain differential invariants that allow us to compute the components of the equivalences by just using resultants and polynomial factoring. In special cases, for instance curves with infinitely many self-affine equivalences, equivalences are computed using a more direct method involving polynomial system solving, but with polynomials of smaller degree. A comparison of the algorithm with other methods, including brute force, is also given.

Keywords: Affine equivalence, affine symmetry, implicit curves, computational geometry
General area of research: Mathematics
ICFAS2022-ID: 1037

STMMETRIES AND SIMILARITIES OF RATIONAL PLANE ALGEBRAIC CURVES IN COMPLEX REPRESENTATION

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Abstract:

This study is devoted to investigate the problem of detecting symmetries and similarities of rational plane algebraic curves given in complex representation. Unlike other studies addressing the same problem, we provide a different approach which uses complex differential invariants of the input curves. Our method takes advantage of the complex representation $z(t) = x(t) + iy(t)$ of a rational plane curve $\boldsymbol{C}$ and the fact that the curves $\boldsymbol{C_1}$ and $\boldsymbol{C_2}$ properly parametrized by $z_1(t)$ and $z_2(t)$ are similar if and only if there exist complex numbers a,b and a Möbius transformation φ such that $az_1(t) + b = z_2(\varphi(t))$. In order to determine a and b, we first determine the Möbius transformation φ. It can be easily done by using an rational complex differential invariant function $I(z)$ defined on a curve $\boldsymbol{C}$ properly parametrized by z. Finally the symmetries of a curve $\boldsymbol{C}$ can be determined by the same setup by taking $|a| = 1$ and $\boldsymbol{C_1} = \boldsymbol{C_2}$.

Keywords: Symmetries, similarities, rational curves, computational geometry
General area of research: Mathematics
ICFAS2022-ID: 1038

EMBEDDING OF THE SUBORBITAL GRAPHS CORRESPONDING TO THE NORMALIZER

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Abstract:

It is known that an embedding of a graph $\mathcal{F}$ on a surface S corresponds to a representation of $\mathcal{F}$ on S, where the edges of $\mathcal{F}$ splits S into polygonal cells while the vertices of $\mathcal{F}$ exactly correspond to the punctures of S. Let $\mathcal{F}$ be embedded on a surface S. The complement of vertices and edges on S forms a family of regions called faces. If each of these faces is homeomorphic to an open disk then the embedding is called a map, and if the map has identical regular polygonal faces, then it is called regular. Indeed the regular maps are the natural generalizations of the well-known platonic solids. This study is devoted to investigating the embedding of the graphs arising from the action of the normalizer of $\Gamma_0(N)$ in $PSL(2,R)$, which acquire significance since the investigation of the monstrous moonshine. We define a suitable chain of map subgroups of the normalizer and study the regular maps corresponding to these subgroups.

Keywords: Suborbital graphs, graph embedding, regular maps
General area of research: Mathematics
ICFAS2022-ID: 1039

HYBRINOMIALS RELATED TO HYPER-FIBONACCI AND HYPER-LUCAS NUMBERS

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Abstract:

The hybrid numbers which are accepted as a generalization of complex, hyperbolic and dual numbers, have attracted the attention of many researchers recently. In this paper, hybrinomials related to hyper-Fibonacci and hyper-Lucas numbers are defined. Then some algebraic and combinatoric properties of these hybrinomials are examined such as the recurrence relations, summation formulas and generation functions. Additionally, hybrid hyper-Fibonacci and hybrid hyper-Lucas numbers are defined by using the hybrinomials related to hyper-Fibonacci and hyper-Lucas numbers.

Keywords: Hyper-Fibonacci numbers, hyper-Lucas numbers, polynomials, hybrinomials
General area of research: Mathematics
ICFAS2022-ID: 1041

COMPATIBLE STRUCTURE IN IDEAL CECH CLOSURE SPACES

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Abstract:

let (X, f, I) be a Cech closure space with an ideal I. For a subset A of X$, the set ∼f(A) called a Cech touch points is defined as follows: ∼ f(A) = { x in X: A ⋂N not in I for every N in N(x)}. Several characterizations of these sets will also be discussed through this paper. Moreover, we obtain characterizations of ∼ f-operator in an ideal Cech closure space (X, f, I) and we investigate the notion of f-compatibility with an ideal I, and obtain several characterizations of the compatibility.

Keywords: Cech closure operator, ideal Cech closure space, f-operator, f-compatible
General area of research: Mathematics
ICFAS2022-ID: 1042

SUBCLASS OF ANALYTIC FUNCTIONS WITH NEGATIVE COEFFICIENTS RELATED WITH MILLER-ROSS-TYPE POISSON DISTRIBUTION SERIES

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Abstract:

The purpose of the present paper is to find a necessary and sufficient condition for Miller-Ross-type Poisson distribution series to be in the class $P^*(\alpha,\beta,\gamma)$ of analytic functions with negative coefficients .Also, we investigate several inclusion properties of the classes $QCV(A,B)$, $CS^*(A,B)$ and $CK(A,B)$ associated of the operator defined by this distribution. Further, we consider an integral operator related to Miller-Ross-type Poisson distribution series. Several corollaries and consequences of the main results are also considered.

Keywords: Analytic functions, starlike functions, convex functions, Hadamard product, Miller-Ross-type Poisson distribution series
General area of research: Mathematics
ICFAS2022-ID: 1043

POSITIVE SOLUTIONS FOR A FRACTIONAL BOUNDARY VALUE PROBLEM WITH SEQUENTIAL DERIVATIVES

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Abstract:

We investigate the existence of positive solutions for a Riemann-Liouville fractional differential equation with sequential derivatives, a positive parameter and a nonnegative singular nonlinearity, subject to nonlocal boundary conditions which contain Riemann-Stieltjes integrals and various fractional derivatives. In the proof of the main result, we use the fixed point index theory.

Keywords: Riemann-Liouville fractional differential equation; nonlocal boundary conditions; singular functions; positive solutions.
General area of research: Mathematics
ICFAS2022-ID: 1044

REMOVAL OF CRYSTAL VIOLET USING ZIRCONIUM SILICATE AS AN ADSORBENT

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Abstract:

Various kinds of synthetic dyestuffs and pigments have been used in many industries like food, textile, paper, rubber, plastics and cosmetics [1]. Releases of coluored effluents containing dyes into environmental and mainly into natural water sources have a negative impact on the environment [2].
Crystal violet is a triphenylmethane dye which is mainly used in textile dying and paper printing. The dye is also used as a dermatological agent, veterinary medicine and an indicator.
Crystal violet is harmful when ingested or inhaled and can cause digestive system irritability, skin irritation, eye irritation, kidney failure and permanent blindness [3]. Thus, removal of the analyte from the environmental samples is crucial.
This study aims to investigate the adsorption behavior of crsytal violet on zirconium silicate. Various parameters for the batch adsorption studies such as initial pH, adsorption time, adsorbent dose, desorption solvent and desorption time for the adsorption of crsytal violet have been optimized.

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2. Saeed, A., Sharif, M., Iqbal M., Application potential of grapefruit peel as dye sorbent: Kinetics, equilibrium and mechanism of crystal violet adsorption. Journal of Hazardous Materials 179 (2010) 564–572.
3. Oladipo, A. A. and Gazi, M., Enhanced removal of crystal violet by low cost alginate/acid activated bentonite composite beads: Optimization and modelling using non-linear regression technique Journal of Water Process Engineering 2 (2014) 43–52.

Keywords: Crystal violet, adsorption, zirconium silicate
General area of research: Chemistry
ICFAS2022-ID: 1045

NEW QUANTUM AND LCD CODES FROM CYCLIC CODES OVER A FINITE NON-CHAIN RING

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Abstract:

In this work, we study cyclic codes of length n over a finite commutative non-chain ring $S = \frac{K_q[u,v]}{<u^2-\alpha u, v^2-\beta v, uv-vu>}$, where $\alpha, \beta \in K_q^*$. Then certain constraints are imposed on the generator polynomials of cyclic codes so that these codes become LCD codes. We then verify that the Gray image of an LCD code of length n over S is an LCD code of length 4n over K_q by establishing a Gray map. Finally, we determine numerous enhanced quantum codes compared to the previously known codes in recent references by applying the CSS construction to cyclic codes over S that contain their Euclidean duals**.**

Keywords: LCD codes; Hamming distance; Quantum codes
General area of research: Mathematics, Coding Thoery
ICFAS2022-ID: 1047

COMPUTATIONALLY EFFICIENT MODEL SELECTION PROCEDURE FOR RANDOM FOREST CAUSAL MODEL

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Abstract:

Multiple testing procedures have had an increasing attention in the recent years by researchers from different scientific domains such as econometrics, machine learning, epidemiology. More recently, the conditional randomization test (CRT), which is developed as a recent multiple testing, has been proposed to decide relationship between variables accurately. One of the recent applications of this approach has been done in random forest (RF) method. The RF method is widely used for the purpose of dimension reduction, but can be also applied for nonparametric causal model. From previous study, it has been shown that CRT inserted RF causal model is successful in the application of limited real biological systems whose dimensions are large and sample sizes are small. In this study, we extend this analysis by suggesting a comprehensive simulation study in such a way that we can evaluate the performance of this model in various simulated biological networks under different sparsity and dimensions. We assess the results via distinct accuracy measures which are calculated by well-known model selection criteria such as Akaike Information Criterion and Information and Complexity Selection Criterion

Keywords: Random forest method, conditional randomization etst, model selection criteria
General area of research: Mathematics
ICFAS2022-ID: 1048

TOTALLY GOLDIE*-SUPPLEMENTED MODULES

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Abstract:

Let R be a ring and M be a right R-module. A module M is said to be Goldie*-supplemented module, if, for each submodule X of M, there exists a supplement submodule S of M such that $X\beta^*S$ [1]. Clearly, Goldie*-supplemented modules are supplemented. A module M is called totally Goldie*-supplemented module, if every submodule of M is Goldie*-supplemented. We show that factor module of totally Goldie*-supplemented module is totally Goldie*-supplemented. Moreover, finite direct sum of totally Goldie*-supplemented is totally Goldie*-supplemented under certain condition. We also discuss the the some properties of this module and we give the relation between totally Goldie*-supplemented and Goldie*-supplemented.

1. Birkenmeier, G.F., Mutlu, F.T., Nebiyev, C., Sokmez, N., Tercan, A., Goldie*-supplemented modules, Glasgow Math. Journal, 52(A), (2010), 41 – 52.

Keywords: Supplemented modules, Goldie*-supplemented modules, totally Goldie*-supplemented modules
General area of research: Mathematics
ICFAS2022-ID: 1049

COMPLETELY INTEGRALLY CLOSED MODULES OVER INTEGRAL DOMAINS

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Abstract:

Let M be a torsion-free module over an integral domain D with quotient field K. We define a concept of completely integrally closed modules as a D-module M whose integral submodules (submodules N where $KN = KM$ holds) satisfy $O_K(N) = D$. We prove that M is completely integrally closed modules if and only if $O_K(M) = D$ and all v-submodules of M are v-invertible. Moreover, the complete integral closedness of M over D is equivalent to the complete integral closedness of $M[x]$ over $D[x]$ and $M[[x]]$ over $D[[x]]$.

Keywords: Completely integrally closed modules, polynomial modules
General area of research: Mathematics
ICFAS2022-ID: 1053

CONTINUOUS COMODULES

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Abstract:

Let R be a commutative ring with unity and C be an R-coalgebra. The ring R is clean if every r in R is a sum of a unit and an idempotent element of R. An R-module M is clean if the endomorphism ring of M over R is clean. Moreover, every continuous module is clean. We modify this idea to the comodule and coalgebra cases. A C-comodule M is called a clean comodule if the C-comodule endomorphism of M is a clean ring. We introduced continuous comodules and proved that every continuous comodules is a clean comodule.

Keywords: Clean modules, clean comodules, continuous modules, continuous comodules
General area of research: Mathematics
ICFAS2022-ID: 1054

COMPARISON BETWEEN THE ECCENTRIC CONNECTIVITY INDEX AND FIRST ZAGREB INDEX OF GRAPH

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Abstract:

For a connected grapg G, the eccentric index and the first zagreb index of G are defined as $\xi^c(G) = \sum_{v_i \in V(G)} d_i \varepsilon_i$ and $M_1(G) = \sum_{v_i \in V(G)} d_i^2$, respectively, where d_i is the degree of v_i in G and ε_i denotes the eccentricity of vertex v_i in G. Recently, Das and Trinajstic (2011), compared the eccentric connectivity index and zagreb indices for chemical tree and molecular graphs. However, the comparison between the eccentric connectivity index and Zagreb indices, in case of general trees and general graphs, is very hards and remains unsolved till now. In this paper, we compare the eccentric connectivity index and the first Zagreb index of graph, where $\ominus(T) = \xi^c(T) - M_1(T)$ for any tree T. As a results, we proved that $\ominus(T)$ is minimum for T is caterpillar.

1. K. C. Das, N. Trinajstic, Relation between the eccentric connectivity index and Zagreb indices, Comput. Math. Appl. 62:1758-1764, 2011.
2. H. Hua, K. C. Das, The relationship between the eccentric connectivity index and Zagreb indices, Discrete Appl. Math., 161:2480-2491, 2013.
3. K. C. Das, Comparison between Zagreb eccentricity indices and the eccectric connectivity index, the second geometric-arithmetic index and the Graovac-Ghorbani index, Croat. Chem. Acta, 89(4), 505-510, 2016.

Keywords: Tree graph, caterpillar graph, eccentric connectivity index, first Zagreb index
General area of research: Chemical Graph Theory, Mathematics
ICFAS2022-ID: 1056

THE EXISTENCE OF GRADED $*-$RINGS

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Abstract:

A semiprime ring A is called a $*-$ring if $\beta(A/I) = A/I$ for any nonzero ideal I of A, where β is the prime radical class. In this paper, we show that every polynomial ring over a $*-$ring is not a $*-$ring. Furthermore, a ring A is called a graded ring if $A = \bigoplus_{g\in G} A_g$ where the set $\{A_g | g \in G\}$ is the additive subgroups of A such that $A_g A_h \subseteq A_{gh}$ for all $g, h \in G$. We also construct some graded $*-$rings in this paper.

Keywords: *-ring: graded ring: prime radical
General area of research: Mathematics
ICFAS2022-ID: 1057

FIRST ORDER NORMAL DIFFERENTIAL OPERATORS WITH AN INVOLUTION

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Abstract:

In this work, some spectral properties related with the first order linear differential-operator expression with an involution in the Hilbert space of vector-functions on finite symmetric interval are investigated. Firstly, the minimal and maximal operators which are generated by mentioned above linear differential operator expression in this Hilbert space are described. Then, the deficiency indices of the minimal operator are calculated and the space of boundary values of the minimal operator is consructed. Later on, by using the method of Calkin-Gorbachuk, the general form of all normal extensions of the formally normal minimal operator in terms of boundary values is found. Finally, the spectrum set of these extensions is researched.

Keywords: Differential operator with an involution, normal differential operatör, deficiency indices, space of boundary values, spectrum
General area of research: Mathematics
ICFAS2022-ID: 1058

AN EFFICIENT WAVELET COLLOCATION METHOD BASED ON HERMITE POLYNOMIAL FOR A CLASS OF 2D QUASI-LINEAR ELLIPTIC EQUATIONS

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Abstract:

An efficient wavelet collocation method based on Hermite polynomial is proposed to investigate a class of 2D quasi-linear elliptic partial differential equations (PDEs) frequently arising in the field of applied mathematics, electromagnetic theory, nonlinear optics, weather forecasting, etc. The application of Hermite wavelets and their integrations to 2D quasi-linear elliptic PDEs returned a system of equations which on solving provides the approximate solution. For the theoretical aspect, the upper bound of error norm is established to assure the convergence of method. The present method has an exponential rate of convergence and hence converges very fast. The proposed method can be adapted uniformly to investigate the solution of 2D near singular elliptic PDEs without any modification in the present scheme. Some numerical simulations have been done to validate the theoretical findings. The maximum absolute errors are calculated for different numbers of collocation grids. The comparison of numerical findings with the existing methods concludes the superiority of the proposed method.

Keywords: Hermite wavelet, Collocation grids, Quasi-linear elliptic partial differential equation, Convergence
General area of research: Mathematics
ICFAS2022-ID: 1059

THE ADSORPTION PERFORMANCE AND CHARACTERIZATION OF THE ACTIVATED CARBON PRODUCED FROM PEPPER STALKS

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Abstract:

This study presented the evaluation of the adsorption of Eriochrome Black T (EBT) on the activated carbon (trH-BS) produced from pepper stalk by activating with 50% phosphoric acid solution (H_3PO_4) and then by carbonizing at 650 ºC for 30 min. in atmosphere of nitrogen (N_2) was presented. The isotherm of trH-BS was Type IV, representing micro, meso and macroporous structures. The micropore volume and micropore surface area values of trH-BS were 0.39 cc/g and 1107.294 m^2/g, respectively. Also, its meso and macropores had a volume of about 15-17 cc/g and an average surface area of 400 m^2/g. The adsorption energy was 18.156 kJ/mol and the BET surface area was 756.257 m^2/g. The structure of trH-BS was clarified structurally and morphologically and the formations like nano rod were seen at Scanning Electron Microscopy (SEM) images. By using trH-BS, the adsorption of EBT from the aqueous medium was investigated using 5 different parameters. As a result, it was found that the adsorbents obtained from pepper stalks are suitable for use in EBT adsorption with a ratio of about 50% under suitable conditions.

Keywords: Pepper Stalk, phosphoric Acid, activated carbon, Eriochrome Black T, adsorption
General area of research: Chemistry
ICFAS2022-ID: 1061

AMIT'S CONJECTURE FOR WORDS OVER TWO VARIABLES

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Abstract:

To measure the commutativity of two elements in a group, we can define a commutator by $[x,y]=x^{-1}y^{-1}xy$ for all x,y in G. Two elements, x, and y, in G, are commute if and only if $[x,y]=e$, where e is the identity element of G. Then, we want to determine the probability of two elements x and y in a finite group such that $[x,y]=e$. By using the free group, we can define a probability associated with a verbal subgroup of G denoted as $P_{G,w}(g)$. Up until now, there are many open problems about the structure of $P_{G,w}(g)$ and its implication for the underlying group structure. One of them is Amit's Conjecture which says the value of $P_{G,w}(e)$ never be less than $1/|G|$ for every finite nilpotent group G.
This paper proves Amit's Conjecture for any words over two variables. As an application, we give a bound for the number of edges of the commuting and non-commuting graphs of finite nonabelian groups and also bounds for some topological indices.

Keywords: Commutator of a group, probability of group elements, number of edges
General area of research: Mathematics
ICFAS2022-ID: 1062

ON VIETORIS' HYBRID NUMBER SEQUENCE

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Abstract:

A hybrid number $z = a + b\mathbf{i} + c\epsilon + d\mathbf{h}, a,b,c,d \in R$ which come to exist by means of complex, dual and hyperbolic numbers is introduced by M. Özdemir. Here $\mathbf{i}^2 = -1, \epsilon^2 = 0, \mathbf{h}^2 = 1, \mathbf{ih} = -\mathbf{hi} = \epsilon + \mathbf{i},$ where 1, $\mathbf{i}$, ϵ, and $\mathbf{h}$ represent real, complex, dual and hyperbolic units, respectively, and called as hybrid units.

Besides, the *nth* element of the Vietoris number sequence $\{v_n\}_{n\geq 0}$ is given by the formula $v_n = \frac{1}{2^n}\binom{n}{\left\lfloor \frac{n}{2} \right\rfloor}$ where $\binom{n}{\left\lfloor \frac{n}{2} \right\rfloor}$ is the central binomial coefficient and the notion $\lfloor , \rfloor$ represents the floor function. This a rational sequence and the first several values of this sequence are (related with the sequence A283208 in OEIS): $1, \frac{1}{2}, \frac{1}{2}, \frac{3}{8}, \frac{3}{8}, \frac{5}{15}, \frac{5}{16}, \frac{35}{128}, \frac{35}{128}, \frac{63}{256}, \frac{63}{256}, \ldots$.

Motivated by properties of Vietoris' sequence and hybrid numbers, this work aims to bring together Vietoris' sequence and hybrid numbers to construct Vietoris' hybrid number sequence. Then some properties of the hybrid numbers with Vietoris' number coefficients are examined. Some relations between this hybrid number and its norm, the recurrence relations, the generating function and Binet's formula are also calculated. Furthermore, a determinantal approach is considered to obtain elements of Vietoris' hybrid number sequence.

Keywords: Vietoris' number sequence, recurrence relation, hybrid number
General area of research: Mathematics
ICFAS2022-ID: 1064

MODELING OF PRECIPITATION AMOUNT AND FLOW RATE WITH ADVANCED DATA PROCESSING METHODS, THE ROLE OF CLIMATE CHANGE

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Abstract:

Water is the primary source of life for living things. Although three quarters of our world is covered with water, there is a limited structure in terms of fresh water resources. Only 2.5% of the total amount of water on earth is found as fresh water in rivers and lakes. Rapidly growing population and consumption increase, pressures on water resources as a result of human activities and threats caused by climate change etc. For these reasons, it is predicted that water resources will be insufficient in the future. For this reason, the economic use of fresh water potential and the determination of the effects of climate change on stream flow rate are of great importance for future generations. In our country, daily flow velocity measurements are made at flow observation stations in rivers. Forward estimations can be made using the data of the local and temporal variation of the flow rate. In this study, the flow rate until 2050 was estimated by considering the monthly average flow rate of the entrance of Isparta Eğirdir Lake, which was selected as the study area, the monthly total precipitation amount of the region and the monthly average air temperature values. With the data between December 1978-September 1987 and October 1997-December 2020, the current using machine learning methods Linear Regression, Support Vector, Decision Tree, Random Forest, Extra Trees, Artificial Neural Networks and Wavelet-Neural Networks Mixed models The temporal variation of the velocity was estimated. The performances of the applied models were compared. It has been determined that the Extra Trees, Random Forest and Wavelet-Artificial Neural Network mixed models are more successful than other methods in creating monthly change scenarios to 2050 in estimating the flow rate and precipitation amount.

Keywords: Climate Change, water potential, precipitation, flow rate, water management, deep learning, wavelet
General area of research: Machine Learning
ICFAS2022-ID: 1066

STABILITY ANALYSIS OF A CAPUTO FRACTIONAL WATERBORNE INFECTIOUS DISEASE MODEL WITH INFECTIOUS SATURATION EFFECT ON BACTERIAL DISEASE TRANSMISSION

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Abstract:

In this presentation we propose a Caputo type fractional waterborne bacterial infection model with saturation effect of infectious population on the transmission of disease caused by waterborne bacteria. The total population size $N(t)$ is divided into two compartments: susceptible individual $S(t)$ and infectious with symptoms $I(t)$ at time $t \geq 0$. Furthermore, we consider a compartment $B(t)$ that reflects the bacterial concentration at time t. We assume positive natural death rate μ. Susceptible individuals can become infected by contact with infected individuals at rate $\rho I(t)/(1+m_1 I(t))$. Susceptible individuals can become infected with waterborne disease, like cholera by contact with contaminated sources at rate $\beta B(t)/(1+m_2 I(t))$, where $\beta > 0$ is ingestion rate of the bacteria through contaminated sources. We assume nonnegative death rate δ caused by infection. Each infected individual contributes to the increase of the bacterial concentration at rate ξ. On the other hand, the bacterial concentration can decrease at mortality γ. With these assumptions we have the following Caputo type fractional waterborne disease model

$$\begin{cases} D^{\alpha}S(t) = \Lambda - \left(\dfrac{\rho I(t)}{1+m_1 I(t)} + \dfrac{\beta B(t)}{1+m_2 I(t)}\right) S(t) - \mu S(t), \\ D^{\alpha}I(t) = \left(\dfrac{\rho I(t)}{1+m_1 I(t)} + \dfrac{\beta B(t)}{1+m_2 I(t)}\right) S(t) - \delta I(t) - \mu I(t), \\ D^{\alpha}B(t) = \xi I(t) - \gamma B(t). \end{cases} \tag{1}$$

where D^{α} denotes the Caputo fractional derivative of order $\alpha, 0 < \alpha \leq 1$. For this model with initial conditions

$$S(0) \geq 0, I(0) \geq 0, B(0) \geq 0, \tag{2}$$

we first find existence and uniqueness of the solution of system (1) with initial conditions (2) which remains in a positively invariant region

$$\Omega = \left\{(S,I,B) \in \mathbb{R}_+^3 : \quad S + I \leq \frac{\Lambda}{\mu}, \quad 0 \leq B \leq \frac{\xi\Lambda}{\mu\gamma}\right\}.$$

Secondly, we find the existence of disease-free equilibrium point $E_0 = (\Lambda/\mu, 0, 0)$ and endemic epidemic equilibrium point $E_* = (S_*, I_*, B_*)$ and define basic reproduction number R_0 of this system. Next, we investigate the local stabilities of equilibriums E_0 and E_* by using Jacobian matrix of system (1) at these equilibriums and global stability of the disease-free equilibrium using Lyapunov's method.

Keywords: Fractional calculus, Caputo derivatives, epidemiology, equilibrium, stability
General area of research: Mathematics
ICFAS2022-ID: 1067

COMMUTING PRODUCT OF AUTOMORPHISMS AND *b*-GENERALIZED SKEW DERIVATIONS ON MULTILINEAR POLYNOMIALS

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Abstract:

Let R be a prime ring of characteristic different from 2 and 3, Q_r its right Martindale quotient ring and C its extended centroid. Suppose that β is an automorphism of R, F a non-zero b-generalized skew derivation of R and $f(x_1,\ldots\ldots, x_n)$ a non-central multilinear polynomial over C with n non-commuting variables, such that

$$[\beta(f(r_1,\ldots\ldots, r_n))F(f(r_1,\ldots\ldots, r_n)),f(r_1,\ldots\ldots, r_n)]=0$$

for all $r_1,\ldots\ldots, r_n \in R$, then

1. either β is the identity map on R and there exists $\lambda\in C$ such that $F(x)= \lambda x$, for any $x\in R$.
2. or $f(x_1,\ldots\ldots, x_n)^2$ is central valued on R and there exists $\lambda\in C$ such that $F(x)= \lambda\beta(x)$, for any $x\in R$.
3. or $f(x_1,\ldots,x_n)^2$ is central valued on R, $\beta=$ identity and $F(x)=ax+xa+\lambda x$, with $a\in Q$ and $\lambda\in C$.

Keywords: *b*-Generalized skew derivation, multilinear polynomial, prime ring
General area of research: Mathematics
ICFAS2022-ID: 1068

PRIME RINGS WITH PERIODIC VALUES ON LIE IDEALS

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Abstract:

Here we discuss the relationship between the structure of a ring R and the periodic values of some appropriate subsets of R. More precisely, let F and G be two generalized derivations satisfying the following condition:

$$(F(x)x - xG(x))^n = F(x)x - xG(x)$$

for all $x, y \in L$, where L is a non-central Lie ideal of R and $n > 1$ a fixed integer.
We prove that if R is prime and char(R) is either zero or $p > 0$ such that $p \nmid (2^n - 2)$, then one of the following holds:

1. there exists $\lambda \in C$ (the extended centroid of R) such that $F(x) = G(x) = \lambda x$, for all $x \in R$;
2. there exist $a, c \in Q_r$ (the right Martindale quotient ring of R) such that $F(x) = xa$ and $G(x) = cx$, for all $x \in R$, where $(a - c)^n = a - c \in C$. Moreover both C is a periodic field and L is a periodic Lie ideal;
3. $R \subseteq M_2(K)$, the ring of all 2×2 matrices over a field K and there exist $a, c \in R$ such that $F(x) = ax + xc$ and $G(x) = cx + xa$, for all $x \in R$;
4. $R \subseteq M_2(K)$, the ring of all 2×2 matrices over a field K and there exist $a, b, u \in R$ such that $F(x) = ax + xb$ and $G(x) = bx + xu$, for all $x \in R$, where $(a - u)^n = a - u$. Moreover K is a periodic field.

Keywords: Prime ring, generalized derivation, differential identities
General area of research: Mathematics
ICFAS2022-ID: 1069

ALGEBRA GAGE MATRIX PHYSICS

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Abstract:

With absolute vacuum and zero stringmetrics, physics maybe extended to zero-point fluctuations analogous to a "zitterbewegung" oscillations that are fundamental universal musings!!
It is shown here algebra gage developed earlier provides algorithms to advance mathematical quantification of gage discontinuum dissipative physics that will unify grand theory of the four superforces electromagnetism, gravity, nuclear strong-weak. Signal/noise matrix formed out of the probability gage unitary matrix essentially gives critical density matrix characterizing gravitational component in the form of fluctuating oscillations, with lower regions decomposing to decohered multiple prime factorized magic square symmetry matrix output, while upper regions combining entangled matrix. These appear with peer-reviewed publications ongoing accepted referential papers articles, demonstrating theoretical observables capable of viable experimentally measurable observations applying to Science, Engineering, Technology, Mathematical, and Quantum Computing Applied Fields Global.

Keywords: Matrix Physics
General area of research: Mathematics
ICFAS2022-ID: 1074

ELABORATION OF STOCHASTIC MATHEMATICAL MODELS FOR THE RELATIVE HUMIDITY LEVELS PREDICTION USING ARTIFICIAL NEURAL NETWORKS

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Abstract:

The artificial neural networks (ANN) are generally, used to deal with mathematical problems, especially in statistical problems where the variables are nonlinear. This work provides the development of a powerful artificial neural network model, for the prediction of relative humidity levels, using other meteorological parameters of the Rabat-Kenitra region. The treatment was applied to a database containing a daily history of five meteorological parameters of 9 stations covering this region for a period from 1979 to mid-2014.
We have shown that for the prediction of relative humidity in this region, the best performing three-layer ANN (input, hidden and output) mathematical model is the multi-layer perceptron (MLP) model. This neural model using the Levenberg-Marquard algorithm, having an architecture [5-11-1] and the transfer functions Tansig in the hidden layer and Purelin in the output layer was able to estimate values for relative humidity very close to those observed. Indeed, this was affirmed by a low mean squared error (MSE) and a high correlation coefficient (R), compared to the statistical indicators relating to the other models developed as part of this study.

Keywords: Modeling, ANN, MLP, learning algorithm, relative humidity
General area of research: Mathematics
ICFAS2022-ID: 1075

NITROGEN-DOPED CARBON QUANTUM DOTS-GELLAN GUM AS AN INNOVATIVE SELF-HEALABLE HYDROGEL COMPOSITE

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Abstract:

Strain sensors can be widely applied to detect body movements and 1onitör physiological signals. Hydrogels with conductive properties draw attention among the studies in this field. However, their application is limited because hydrogels can be easily damaged during use. In this study, a self-healing conductive hydrogel was produced by adding nitrogen-doped carbon quantum dots (NCQDs) to gellan gum (GG) polymer. The self-healing property of the hydrogen bonds in the prepared polymeric matrix network to a certain extent and the conductivity were supported by the addition of NCQDs. The electrical recovery process of the hydrogel in the 1, 2, and 3 cutting/healing cycles was illustrated by a visually designed LED bulb serial circuit. As a result of connecting the obtained 3D hydrogel to a real-time resistance change measurement system, the resistance changes in the cutting/healing cycles were monitored. The duration of the total cut-healing process, including cut and contact time, was 2.12 s. In addition, a free-standing gel bridge was formed after joining the two cut pieces of cylindrical hydrogels. Due to the resulting hydrogel composite properties, it has promising potential in various applications such as personal health diagnosis, human activity monitoring, and human-motion sensors.

Acknowledgments
This work was supported by the Scientific Research Projects Commission of Sakarya University (Project number: 2019-6-23-223).

Keywords: Self-healable, hydrogel composite, conductivity
General area of research: Chemistry
ICFAS2022-ID: 1076

SOME PROPERTIES OF COMPACT OPERATORS ON SOME DIFFERENCE SEQUENCE SPACES

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Abstract:

In the present work, we are going to present some characteristics of the matrix classes $\left(l_1, l_p\left(\hat{B}\right)\right)$ in which $l_p\left(\hat{B}\right)$ is a sequence space which is the matrix domain of a generalized difference matrix $\hat{B}$ in sequence space l_p. We are at the same time going to find out the estimates about the norms for the bounded linear operators L_A which are described by those matrix trasformations. While we are also try to find the conditions required for obtaining the corresponding to the subclasses of compact matrix operators by means of the Housdorff measure of noncompactness.

1. Alotaibi A, Mursaleen M, AS Alamsi B and Mohiuddine S.A; Compact operators on some Fibonacci difference sequence spaces; JIA; 2015:203, 8 pages.
2. Candan M; A new sequence space isomorphic to the space $l(p)$ and compact operators; J. Math. Comput. Sci. 4(2014), No.2, 306-334.

Keywords: Difference sequence spaces, compact operators, Hausdorff measure of noncompactness
General area of research: Mathematics
ICFAS2022-ID: 1078

A NOVEL SEQUENCE SPACE ISOMORPHIC TO THE SPACE $l(p)$

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Abstract:

As a major issue in this work, we present the paranormed sequence space $l(u,v,p,F,\widetilde{B})$ consisting of all sequences of which the trasformation which is going to be detailed in this presentation in the linear space $l(p)$ introduced by Maddox [Quart. J. Math. Oxford 18 (1967, 345-355)], where $u=(u_k)$ and $v=(v_k)$ are sequences such that $u_k \neq 0$, $v_k \neq 0$ for all $k \in IN$, $1 \leq p < \infty$ and also F denotes Fibonacci band matrix and $\widetilde{B}$ denotes double sequential band matrix. Here; it is proved linear isomorphic of the sequence spaces $l(u,v,p,F,\widetilde{B})$ and $l(p)$. In addition to this, we are going to present the basis of this space and find its $\alpha-, \beta-$ and $\gamma-$duals.

1. Candan M; Domain of the double sequential band matrix in the classical sequence spaces; JIA 2012(1), 1-15.
2. Candan M; Almost convergence and double sequential band matrix; Acta Math. Sci. 34(2), 354-366.
3. Candan M; A new sequence space isomorphic to the space $l(p)$ and compact operators; J. Math. Comput. Sci. 4(2014), No.2, 306-334.
4. Candan M; Domain of the sequential band matrix in the spaces of convergent and null sequences; Adv in Dif. Equ. 2014(1), 1-18.

Keywords: Paranormed sequence spaces, double sequential band matrix, $\alpha-, \beta-$ and $\gamma-$duals
General area of research: Mathematics
ICFAS2022-ID: 1079

THEORETICAL INVESTIGATION OF ORIENTATION EFFECT ON ELECTRONIC PROPERTIES OF Pt/Co/Ir/Co/Pt THIN FILMS

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Abstract:

Based on the first principles, the energy band and density of states of Pt/Co/Ir/Co/Pt thin films were modeled by the Materials Studio (MS) program. The energy band, total density of states (TDOS) of Pt/Co/Ir/Co/Pt with different crystal orientation (100), (110) and (111) were studied, respectively. The results suggested that the Fermi energy (EF) passes through the energy band with dense distribution of energy levels. It has been shown by the obtained data that using platinum buffer layer more induced (111) texture.

Acknowledgment: This work was supported by Research Projects with Foundation Number BAP-20-1003-007, Bandırma Onyedi Eylül University, Scientific Research Commission, Turkey.

Keywords: Density of States, CASTEP, Crystal Orientation, Multilayered Thin Films
General area of research: Physics
ICFAS2022-ID: 1081

EFFECTS OF HETEROGENEOUS AND NONLINEAR LAYER LYING BETWEEN TWO NONLINEAR HALF SPACES ON LOVE-TYPE WAVE PROPAGATION

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Abstract:

The propagation of Love-type waves in a homogeneous, nonlinear, elastic two half spaces separated by a heterogeneous, nonlinear, elastic layer with regular interfaces is considered. It is assumed that shear modulus, nonlinear material parameter and density of the layer vary exponentially in the vertical direction of wave propagation whereas phase velocity remains unchanged. By using the derivative expansion method, the nonlinear modulation of the waves is described by a nonlinear Schrödinger (NLS) equation, the coefficients of which depend on the material properties of the media, wave number as well as the heterogeneity parameters of the layer. The effects of both heterogeneity and nonlinearity of the layer on the envelope and dark solitary wave solutions of the NLS equation are demonstrated graphically.

Keywords: Heterogeneous layer, solitary SH waves, nonlinear elasticity
General area of research: Mathematics
ICFAS2022-ID: 1082

IDEAL-BASED GRAPHS

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Abstract:

Let R be a commutative ring with identity and I a proper ideal of R. In this talk we introduce the ideal-based quasi zero divisor graph $Q\Gamma_I(R)$ with respect to I which is an undirected graph with vertex set $V = \{a \in R \backslash \sqrt{I}: ab \in I \; for\ some\ a \in R \backslash \sqrt{I}\}$ and two distinct vertices a and b are adjacent if and only if $a, b \in I$. We first give the basic properties of this graph such as diameter, girth, dominaton number, etc. Then, we will talk on the interplay between the ring theoretic properties of a Noetherian multiplication ring R and graph-theoretic properties of $Q\Gamma_I(R)$.

Keywords: Ideal-based zero divisor graph, quasi primary ideal, zero divisor graph
General area of research: Mathematics
ICFAS2022-ID: 1083

NOISE REDUCTION PARTIAL DIFFERENTIAL EQUATIONS

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Abstract:

A primary source of image degradation is thermal noise entering the time domain. Noise is also often described as the corrupted data or artificial imprints added to digital images by sensors. To overcome such problems, the use of nonlinear diffusion filters have been proposed. These filters involve the introduction of a small nonlinear diffusion term with large gradient. One may minimize a norm of the image under some given conditions. This idea is connected to analyzing nonlinear diffusion filters in order to obtain algorithms to aid noise removal of digital images. In this talk, we consider the class of nonlinear diffusion filters - comprising of a fourth-order partial differential equation noise suppression model.

Keywords: Variational principle, Noether symmetries and exact solutions
General area of research: Mathematics
ICFAS2022-ID: 1085

SOLUTION OF NONLINEAR VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS OF SECOND KIND BY SHEHU DECOMPOSITION METHOD

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Abstract:

Many scientists are interested in both Volterra integro-differential equations and new methods which pertain to their solutions. In this study, we propose a combination form of the Shehu transform and Adomian decomposition method to solve nonlinear Volterra integro-differential equation of the second kind. The result reveals that the proposed method is very efficient, simple and can be applied to other applications.

Keywords: Shehu transform, Adomian decomposition method, Volterra integro-differential equations
General area of research: Mathematics
ICFAS2022-ID: 1086

APPLICATION OF SHEHU DECOMPOSITION METHOD TO SOLVE NONLINEAR SYSTEM VOLTERRA INTEGRO-DIFFERENTIAL EQUATIONS

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Abstract:

Both linear and nonlinear Volterra integral equations arise in many different branches of applied sciences. There are numerous studies for the solutions of Volterra integro-differential equations and new solution methods continue to attract the attention of scientists. In this study, a combined form of Shehu transform method with Adomian decomposition method is effectively used to obtain approximate solutions for nonlinear systems of Volterra integro-differential equations.

Keywords: Shehu transform, Adomian decomposition method, nonlinear system Volterra integro-differential equations
General area of research: Mathematics
ICFAS2022-ID: 1087

ON ONE BOUNDARY VALUE PROBLEM WITH SYMMETRIC DOUBLE WELL POTENTIAL AND QUADRATIC SPECTRAL PARAMETER IN THE BOUNDARY CONDITION

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Abstract:

In this paper, we obtain asymptotic estimates of eigenvalues for regular Sturm-Liouville problems having the quadratic eigenvalue parameter in the boundary condition. The potential of the problem is symmetric double well potential that continuous, symmetric about to the midpoint and also quarter point of the interval and non-increasing on the quarter related interval.

Keywords: Sturm-Liouville problems, quadratic eigenvalue, asymptotics, symmetric double Well potential
General area of research: Mathematics
ICFAS2022-ID: 1088

ON ONE BOUNDARY VALUE PROBLEM WITH SYMMETRIC POTENTIAL AND A SPECTRAL PARAMETER IN THE BOUNDARY CONDITIONS

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Abstract:

We present the asymptotic estimate of the eigenvalue for regular Sturm-Liouville problem having the functions of the eigenvalue parameter in the boundary conditions by using the solution of a Riccati equation. Also the potential of the problem is integrable and symmetric in the related interval.

Keywords: Boundary value problems, spectral parameter, asymptotic estimates, integrable potential
General area of research: Mathematics
ICFAS2022-ID: 1089

STABILITY BY FIXED POINT THEORY FOR NONLINEAR FRACTIONAL DIFFERENTIAL EQUATIONS

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Abstract:

We study the stability properties of nonlinear fractional differential equations and give conditions to ensure that the zero solution is asymptotically stable by applying the large contraction principle and Schauder's fixed point theorem. We first convert the fractional differential equation as an integral equation by applying the variation of parameters formula in terms of Mittag-Leffler functions that are completely monotone. This allows us to define a mapping function by the right-hand side of the integral equation. We show that this function has a fixed point that is a solution of the original differential equation tending to zero as time approaches infinity. Stability and asymptotic stability theorems are proved. This talk consists of a contraction mapping and compact fixed point foundation for fractional differential equations. The reader will also see how very complete, simple, and rigorous analysis on a highly challenging stability problem can be achieved using fixed point theory in the space of continuous functions with the supremum norm.

Keywords: Asymptotic stability, fractional differential equations, fixed point theory
General area of research: Mathematics
ICFAS2022-ID: 1090

GOLD NANOFLOWERS GROWN IN-SITU ON CU GRID AS A SERS-ACTIVE SUBSTRATE

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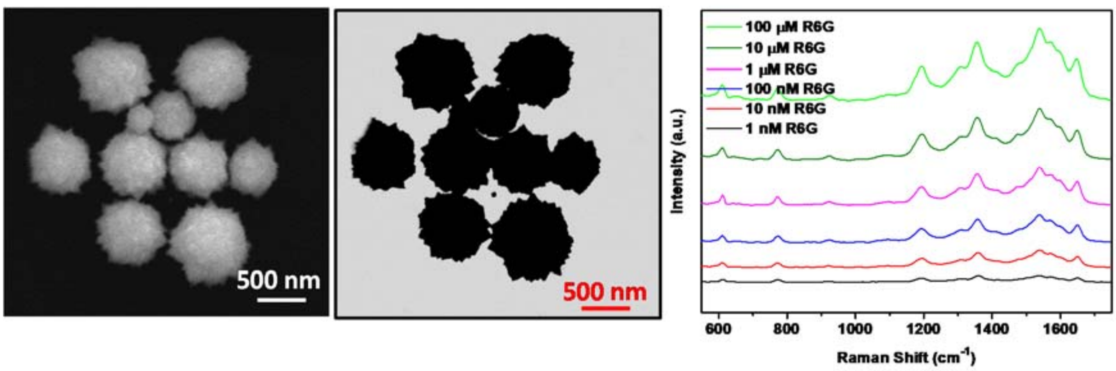
Abstract:

Due to its reliability, sensitivity, and non-destructive nature, surface-enhanced Raman spectroscopy is used in many different sciences such as materials science, chemistry, medicine, and pharmacy. Plasmonically active nanoparticles built on solid substrates are usually obtained by a top-down approach. Lithography methods are often used for this. While these processes require high cost and experienced manpower, they involve long process steps. The chemical synthesis method, which is a more economical and simple way to construct plasmonic nanostructures on solid substrates, could also be used.
In this study, Au nanoparticles were obtained in situ by wet chemical synthesis method using Cu grid as solid substrate and used as SERS-active substrate. Thanks to the in situ growth of nanoparticles, the agglomeration problem encountered in colloidal nanoparticles has been avoided. The agglomeration of nanoparticles negatively affects the formation of hot spots. Growing Au nanoparticles in situ increases the number of hot spots and enables us to receive a stronger signal.
Thanks to hydroquinone, a weak reducing agent, Au nanoflowers were obtained on the Cu grid without using any seed. The protrusions around the nanoflower acted as hot spots, causing the electromagnetic field to increase on the surface and further strengthen the signal. In this way, the determination of the rhodamine 6G dye molecule, which is an organic pollution, was achieved even at a very low concentration of 1 nM. In addition, the determination of Thiram, one of the pesticides harmful to living organisms, was carried out at very low concentrations.
Accordingly, it is seen that the plasmonically active nanostructures are successfully obtained in situ thanks to the Cu grid. Cu grids could be used as an ideal platform for in situ growth of different nanostructures for different applications.



Keywords: Cu grid, Gold nanoparticles, SERS
General area of research: Material Science
ICFAS2022-ID: 1091

LINEAR POSITIVE OPERATORS ON A MOBILE INTERVAL

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Abstract:

Important features of the approach to continuous functions are introduced with a sequences of linear positive operators defined on a mobile interval. In addition, the rate of approximation is calculated numerically by obtaining significant equations for the defined operator.

Keywords: Linear positive operators, modulus of continuity, rate of convergence
General area of research: Mathematics
ICFAS2022-ID: 1092

MALEVIS DATASET MALWARE SOFTWARE DETECTION WITH CNN AND DEEP LEARNING

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Abstract:

The number and types of malicious software used today are increasing day by day. It is important to determine the type of software as well as whether it is malicious or not. Recent studies have shown that malware detection can be considered as an image processing problem. MaleVis (https://web.cs.hacettepe.edu.tr/~selman/malevis/) data set was used in the testing of this study. With Keras library models and image size changes are tested in the testing environment.

Keywords: CNN, DenseNet201, Injector. RMSprop(lr=1e-4)
General area of research: Computer Science
ICFAS2022-ID: 1094

ON SOME PROPERTIES OF A PASCAL-TYPE MATRIX

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Abstract:

Pascal's triangle appears in many fields of mathematics such as algebra, combinatorics and number theory. Pascal matrices are constructed from this triangle of binomial coefficients, which forms elementary matrices with several interesting properties. We define a matrix which satisfies similar properties owned by Pascal matrices. It is shown that the defined matrix can be factorized by some special matrices. We find an explicit formula for the inverse and k-th power and produce some combinatorial identities.

Keywords: Hermite polynomial, factorization of a matrix, matrix inversion, Toeplitz matrix
General area of research: Mathematics
ICFAS2022-ID: 1095

DYNAMICAL BEHAVIOR OF RATIONAL DIFFERENCE EQUATION $x_{n+1}=\frac{x_n x_{n-7}}{x_{n-6}(\pm 1 \pm x_n x_{n-7})}$

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Abstract:

We explore the dynamics of adhering to rational difference formula

$$x_{n+1}=\frac{x_n x_{n-7}}{x_{n-6}(\pm 1 \pm x_n x_{n-7})}, n \in \mathbb{N}_0$$

where the initials are arbitrary nonzero real numbers. Specifically, we examine global asymptotically stability. Additionally, we provide examples and solutions graphs of some special cases.

Keywords: Difference equations, local stability, recursive sequences
General area of research: Mathematics
ICFAS2022-ID: 1096

EXPRESSIONS AND DYNAMICAL BEHAVIOR OF RATIONAL DIFFERENCE EQUATIONS

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Abstract:

This paper studies the dynamics of the solutions of recursive sequences satisfying

$$x_{n+1} = \frac{x_{n-23}}{\pm 1 \pm x_{n-3}x_{n-7}x_{n-11}x_{n-15}x_{n-19}x_{n-23}}, n \in \mathbb{N}_0$$

where the initial conditions are arbitrary nonzero real numbers. Also, we get explicit forms of the solutions.

Keywords: Difference equations, local stability, recursive sequences
General area of research: Mathematics
ICFAS2022-ID: 1097

FINITE DIFFERENCE METHOD FOR NUMERICAL SOLUTION OF TWO-INTERVAL BOUNDARY VALUE TRANSMISSION PROBLEMS

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Abstract:

Boundary value problems (BVPs) arise as mathematical models of many problems in physics and engineering. It is clear that not all BVPs can be resolved analytically. Even if a BVP can be solved analytically, the implicit form of the analytical solution can take some complex form that is useless to use. Therefore, we have to apply various numerical methods to determine the approximate solution closest to the real solution. One of them, the finite difference method, can be applied to a wide range of BVPs, provided that the problem under consideration has complete continuity and boundary conditions.
In this study, we will consider a new type of BVP with a singular point. Such singular problems arise in physical problems, such as, in heat and mass transfer problems, in vibrating string problems when the string loaded additionally with point masses, in thermal conduction problem for a thin laminated plate.
The main feature of this problem is that there are boundary conditions that include not only the endpoints of the range under consideration, but also an interior point of the singularity. Naturally, such singular problems are much more difficult to solve than normal problems. We will develop a new modification of the classical FDM to solve BVPs with additional transition conditions at the singularity point.

Keywords: Finite difference method, transmission conditions, interior singular point, two interval BVP
General area of research: Mathematics
ICFAS2022-ID: 1098

A NEW SEMI-ANALYTICAL METHOD AND ITS APPLICATION

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Abstract:

The history of boundary value problems for differential equations starts with the well-known studies of D. Bernoulli, J. D'Alambert, C. Sturm, J. Liouville, L. Euler, G. Birkhoff and V. Steklov. The greatest success in spectral theory of ordinary differential operators has been achieved for Sturm–Liouville problems. There are various numerical methods for solving many problems in mathematical physics. The concept of the Differential Transform Method (DTM) was first proposed by Zhou [1] for solving some initial value problems appearing in electric circuit analysis.
This work is aimed at computing the eigenvalues of singular two-interval Sturm–Liouville problems. Based on traditional DTM we developed a new approximation method for solving various type boundary value problems which we call many-parameterized differential transform method (MPDTM). The present MPDTM differs from the well known DTM in calculating the recurrence terms. In the special case when the number of parameters is equal to one, our method reduces to the classical DTM, that is the MPDTM is a generalization of the classical DTM.

Keywords: The differential transform method, boundary value problem, approximation methods, eigenvalues, numerical solution
General area of research: Mathematics
ICFAS2022-ID: 1099

A STUDY ON GAUSSIAN PELL QUATERNION POLYNOMIALS

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Abstract:

In this study, we aim to investigate some properties of Gaussian Pell quaternion polynomials. Using Binet's formula of these polynomials, we give the exponential generating function and the Poisson generating function for Gaussian Pell quaternion polynomials. Furthermore, we derive binomial sum formulas for these polynomials.

Keywords: Pell numbers, Gaussian Pell numbers, Gaussian Pell polynomials, Gaussian Pell quaternions
General area of research: Mathematics
ICFAS2022-ID: 1102

CALCULATION ON SOMBOR AND SOMBOR-TYPE INDICES OVER TENSOR AND CARTESIAN PRODUCT OF A MONOGENIC SEMIGROUP GRAPH

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Abstract:

Sombor index is one of the newest vertex-degree based topological index introduced in 2021 by Gutman. Its connection with geometry makes Sombor index different from other topological indices. Following the invention of the Sombor index, the Nirmala and Banhatti-Sombor indices are devised by Kulli in 2021.
Motivated by zero divisor graphs, monogenic semigroup graphs was invented by Das et al. in 2013. Afterwards, many studies were carried out on the subject of monogenic semigroup graphs.
In this study we found exact formulas for the Sombor and two Sombor-type indices (named Nirmala and Banhatti-Sombor) over the tensor and Cartesian product of a graph of a monogenic semigroup. Algorithms are given concerning calculations and the main theorems are supported by the examples.

Keywords: Monogenic semigroups, graphs, tensor product, cartesian product, indices
General area of research: Mathematics
ICFAS2022-ID: 1103

DETECTION OF NON-CDS REGION USING ERROR-CORRECTING CODES

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Abstract:

We know that on DNA, which is the genetic material of living things, there are regions that participate in protein synthesis and that do not, respectively, called CDS and non-CDS. In this study, an algorithm was created by utilizing the structure of error correcting codes in the detection of non-CDS regions, and results that overlap with the literature at various lengths were obtained.

Keywords: Error correcting codes, non-CDS region,
General area of research: Mathematics
ICFAS2022-ID: 1104

RUNGE-KUTTA METHOD FOR STOCHASTIC OPTIMAL CONTROL PROBLEMS AND WEAK ORDER CONDITIONS

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Abstract:

In this work, we obtain weak order-2 conditions of Runge-Kutta method for the optimal control of stochastic differential equations (SDEs) which occurs in many areas of economics and finance, and recently in cognitive sciences and neuroscience. We get the order conditions that a stochastic
Runge–Kutta technique must meet to have weak order two by comparing the stochastic expansion of the approximation with the associated Taylor scheme. Moreover, we present a numerical example which verifies the theoretical results.

Keywords: Optimal control, Stochastic differential equations, Weak order Taylor expansion
General area of research: Mathematics
ICFAS2022-ID: 1105

DE MOIVRE-TYPE IDENTITIES FOR THE PADOVAN NUMBERS

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Abstract:

At this work, we give a method for constructing the Perrin and Padovan sequences and obtain the De Moivre-type identity for Padovan numbers. Also, we define a Padovan sequence with new initial conditions and find some identities between all of these auxiliary sequences. Furthermore, we give quadratic approximations for these sequences.

Keywords: De Moivre-type identity, Padovan numbers: Perrin numbers, quadratic approximation
General area of research: Mathematics
ICFAS2022-ID: 1109

TENSOR PRODUCT OF PHASE RETRIEVABLE FRAMES

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Abstract:

Frame vectors in the tensor product of Hilbert spaces that accomplish phase retrieval can be characterized. In this article, we determine the conditions under which the tensor product of vectors may do phase retrieval. Given that tensor product of two frames always implies a frame in the tensor product of Hilbert spaces, we particularly concentrate on finding conditions for phase retrieval in the tensor product of Hilbert spaces.

Keywords: Frame vectors, phase retrieval, tensor product
General area of research: Mathematics
ICFAS2022-ID: 1111

ON THE REGULARIZED TRACE FORMULA OF A DISCONTINUOUS ONE-POINT BOUNDARY VALUE PROBLEM WITH RETARDED ARGUMENT

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Abstract:

In this study we found the regularized trace formula fort he eigenvalues of the boundary value problem formed by the differential equation with discontinuity at $x = \frac{\pi}{2}$, boundary conditions including spectral parameter λ and transmission conditions.

Keywords: Sturm-Liouville, trace, regularized trace formula, differential equation with retarded argument
General area of research: Mathematics
ICFAS2022-ID: 1112

A WORK ON RELATIVE HOMOLOGY GROUPS OF MA-SPACES

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Abstract:

The Marcus-Wyse topological spaces play important roles in the fields of pure and applied topology [4,5,6,9]; topological invariants such as homology, homotopy, and cohomology are also crucial for comparing digital spaces in the study of digital topology. The digital singular homology theory for MA-spaces is introduced previously in [8]. The present work focuses on the relative homology groups of topological spaces which are digital spaces equipped with the Marcus-Wyse topology, besides the reduced homology groups of MA-spaces are introduced.

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Keywords: Marcus-Wyse topology, MA-space, digital singular homology, relative homology.
General area of research: Mathematics
ICFAS2022-ID: 1113

APPLICATION OF ADOMIAN DECOMPOSITION METHOD AND DIFFERENTIAL TRANSFORMATION METHOD IN SOLVING BAGLEY-TORVIK EQUATION

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Abstract:

In this paper, we investigate solving of the Bagley-Torvik Equation defined by

$$\frac{d^2y}{dx^2} + 0,5\frac{d^{\frac{3}{2}}y}{dx^{\frac{3}{2}}} + 0,5\,y = f(x)$$

satisfying the initial conditions $y(0) = 0$ and $y'(0) = 0$. Where $f(x) = \begin{cases} 8 & , \quad 0 < x < 1 \\ 0 & , \quad 1 < x \end{cases}$.

We apply Adomian decomposition method and fractional differential transformation method to solve this fractional differential equation. We measure the efficiency of the methods by comparing the results.

Keywords: Fractional Differential Equations, Bagley-Torvik Equation, Adomian decomposition, differential transformation
General area of research: Mathematics
ICFAS2022-ID: 1116

SOLUTION OF FRACTIONAL DİFFERENTİAL EQUATION WITH SHEHU TRANSFORM

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Abstract:

In this paper, we investigate a new integral transform called the Shehu Transform. On the following set of functions

$$A = \left\{ \frac{f(t)}{\exists N}, \eta_1, \eta_2 > 0, |f(t)| < N exp\left(\frac{|t|}{\eta_j}\right), \text{ eğer } t \in (-1)^i \times [0, \infty) \right\}$$

The Shehu Transform of an exponential function f(t) is given by the following integral:

$$\mathbb{S}[f(t)] = F(s,u) = \int_0^\infty exp\left(-\frac{st}{u}\right) f(t)dt, \;\; t > 0$$

Keywords: Shehu Tranform, fractional differantional equation, integral operator
General area of research: Mathematics
ICFAS2022-ID: 1117

A CHARACTERIZATION OF THE SELF-MODULE HOMOMORPHISMS OF A σ-FINITE BOOLEAN RING $\mathscr{R}$

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Abstract:

Let $\mathscr{R}$ be sequentially complete σ-finite subring of a complete Boolean algebra $\mathfrak{C}$. We prove that any pseudomultiplicative function γ on $\mathscr{R}$, i.e., $\gamma(ab) = a\gamma(b)$ for all a, b in $\mathscr{R}$, is an endomorphism and is of the form $\gamma(a) = ac$ for all $a \in \mathscr{R}$, for some c in $\mathfrak{C}$.

Keywords: Boolean algebra, endomorphism, measure space
General area of research: Mathematics
ICFAS2022-ID: 1118

STABILITY FOR A LASOTA WAZEWSKA FRACTIONAL MODEL WITH PIECEWISE CONSTANT ARGUMENT

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Abstract:

In this presentation, we introduce a conformable type Lasota Wazekska fractional model with piecewise constant argument of the form

$$T_\alpha N(t) = -\mu N(t) + \beta e^{-\gamma N(\theta(t))}, \qquad t \geq 0. \tag{1}$$

where T_α denotes the conformable fractional derivative of order α, $0 < \alpha \leq 1$. Moreover, $N(t)$ is the number of red blood cells at time t; $\mu \in (0, 1)$ is the death probability of red blood cell; positive constants β and γ are related to the production rate of red blood cell in unit time; and $\theta(t)$ is the piecewise constant argument. In this model, the piecewise constant argument is also considered in three seperate groups, namely delay $\theta(t) = [t]$, advanced $\theta(t) = [t + 1]$ and advanced-delay $\theta(t) = [t + 1/2]$, where $[.]$ denotes the greatest integer function. For each piecewise constant argument we first obtain the corresponding discrete equation. Then, we investigate the existence and stability of the equilibrium points using this equation.

Keywords: Conformal fractional derivative, Lasota Wazewska model, equilibrium, stability
General area of research: Mathematics
ICFAS2022-ID: 1119

A NOTE ON 2-ABSORBING VAGUE IDEALS OF COMMUTATIVE SEMIRINGS

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Abstract:

The purpose of this study is to investigate the algebraic structure of 2-absorbing ideals, as well as how they may be applied to vague sets and the connections and algebraic properties that exist between them. This study adds to the literature by looking at the 2-absorbing vague ideal. In this study, 2-absorbing vague ideals are established and cases and theorems are shown using 2- absorbing ideals and vague sets.

Keywords: Vague sets, vague ideals, 2-absorbing ideals, 2-absorbing vague ideals
General area of research: Mathematics
ICFAS2022-ID: 1120

A STUDY ON 1-ABSORBING INTUITIONISTIC FUZZY IDEALS OF COMMUTATIVE SEMIRINGS

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Abstract:

The goal of this research is to look at the algebraic structure of 1-absorbing ideals and how they may be applied to intuitionistic fuzzy sets, as well as the linkages and algebraic features that exist between them. The 1-absorbing intuitionistic fuzzy ideal is examined in this study as an addition to the literature. In this research, intuitionistic fuzzy 1-absorbing ideals are defined, and 1-absorbing ideals and intuitionistic fuzzy sets are used to illustrate instances and theorems.

Keywords: Fuzzy sets, fuzzy ideals, 1-absorbing ideals, 1-absorbing fuzzy ideals, 1-absorbing intuitionistic fuzzy ideals
General area of research: Mathematics
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