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ORIGINAL ARTICLE

A hybrid sigma-pi neural network for combined intuitionistic fuzzy time series prediction model

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Abstract

Intuitionistic fuzzy time series models consider observations hesitation degree but they use memberships and non-membership values together as inputs in the prediction system. The usage of membership and non-membership values as inputs in separate prediction models and combining the outputs of these separate models will provide a more flexible computational approach. Thus, different effects of membership and non-membership degrees on the predictions can be revealed. In this paper, an intuitionistic fuzzy time series prediction model (IFTS-PM) has been proposed. The proposed IFTS-PM uses a new hybrid sigma-pi neural network (HSP-NN), introduced for the first time in the literature, to determine nonlinear relationships between inputs and outputs. In addition, this newly proposed HSP-NN has the ability to multiply linear functions of inputs by unequal weights and convert them to nonlinear relationships. The structure of the proposed IFTS-PM consists of three parts. Two different HSP-NNs generate predictions by taking into account the different contribution levels of memberships and non-memberships. The last part is the part where these predictions are combined. Modified particle swarm optimization is performed to obtain optimal weights of HSP-NNs as well as the combination weights. And by taking the advantage of intuitionistic fuzzy C-means, fuzzy clusters, membership and non-membership values of observations are obtained. Performance of the proposed model is verified by applying it on 48 time series data sets. With all used indications, it has been clearly observed that proposed model has produced outstanding predictions compared to some other state-of-the-art prediction tools.

Keywords Intuitionistic fuzzy time series · Hybrid sigma-pi neural network · Prediction combination · Modified particle swarm optimization

1 Introduction

The prediction of time series has a crucial effect on our daily life in both theoretical and practical aspects. Having a wide usage area such as finance, health care, environment, and energy makes time-series analysis methods quite attractive. The way to get satisfactory prediction results is only possible with an appropriate and competent prediction tool. From a general perspective, techniques for time series analysis could be grouped under three categories as traditional statistical approaches, non-traditional computational methods, and fuzzy-based techniques. Although statistical prediction models, specifically autoregressive (AR), Box–Jenkins or autoregressive integrated moving average (ARIMA) and multiple regressions (MR) are well-known and widely used conventional statistical time series modelling approaches until recently, requiring some strict assumptions such as model assumption, normal distribution and number of observations cause these models to fail in the analysis of complex real-world time series. In contrast, artificial neural networks (ANNs), support vector machines and some other prediction tools from non-traditional approaches, have been researched and utilized as viable alternatives to handle complex dynamic processes. On the other hand, traditional statistical approaches and

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computational methods may not be effective for some prediction problems in which the data are imprecise and vague or linguistic. In these cases, the usage necessity of the advanced fuzzy time series (FTS) prediction models is inevitable to get better prediction results. From this point of view, fuzzy time series prediction models have become one of the most attractive prediction tools for complex time series prediction during the last few decades or so. Some of the models in which fuzzy-based methods exhibit superior prediction performance than computational methods are presented in [1–4] studies. In the literature, the first fuzzy time series (FTS) approach [5] is based on Zadeh's fuzzy set theory [6]. Although fuzzy time series models (FTSM) have a wide literature, they still have some problems to be solved.

When FTSMs are evaluated in terms of the fuzzy theory they are based on, it is obvious that fuzzy sets, in so many cases, fail to satisfy or characterize the uncertainty of the data in a comprehensive manner. The reason behind this problem is that these fuzzy sets-based models do not consider the neutrality degree of time series. Inevitably, the analysis process is adversely affected by the means of not taking into account both the information from the degrees of non-membership and the degrees of neutrality of the time series. As it is known, hesitations always exist in the analysis and decision-making process, since information related to the decision is generally under the influence of some factors such as environmental, business, psychological behaviour. Conventional fuzzy clusters are incapable of dealing with this situation. Based on this information, Atanassov introduced the first intuitionistic fuzzy set definition that consists of memberships and hesitation degree together [7]. Thus, both membership and non-membership information and hesitation information were evaluated together.

The main feature of intuitionistic prediction models, which have been limitedly introduced in the literature, is that they determine the relationships between inputs and outputs either according to fuzzy rules, with the help of fuzzy functions or through the sigma-pi neural network. In all these models, memberships and non-membership values are evaluated together as inputs in the prediction system. At this point, the usage of membership and non-membership values as inputs in separate prediction models and combining the outputs of these separate models will provide a more flexible computational approach. Thus, this situation will lead to a higher prediction performance.

The primary objective of this study is to examine the situation. Moreover, the sigma-pi neural network, used as a prediction tool in the literature, multiplies the linear functions of inputs with equal weights and converts them into nonlinear relationships. However, these linear functions may have a different level of contributions in determining nonlinear relationships. Failure to take this into account will negatively affect prediction performance. At this point, each element in this multiplication transaction has different weights, making it easier to adapt the model to the nonlinear surface of the data. The high adaptation of the model to the nonlinear surface also enhances the prediction performance. In this study, a hybrid sigma-pi neural network is proposed to determine intuitionistic fuzzy relationships instead of the traditional sigma-pi neural network. Traditional multilayer perceptron (MLP) generally, used to determine the fuzzy relations, utilizes an additive aggregation function. Moreover, the single multiplicative neuron model (SMNM), introduced by Yadav [8], uses a multiplicative aggregation function in determining fuzzy relations. The proposed hybrid sigma-pi can be referred to as a hybrid model combining MLP and SMNM since it uses both aggregation functions.

In a general framework, the prediction process set forth in this study has the following features:

- IFTS-PM takes into separately account memberships and non-memberships having different effects on predictions in the prediction process.
- To model these effects, HSP-NN has been proposed for the first time in the literature.
- The proposed HSP-NN has the ability to multiply linear functions of inputs by unequal weights and convert them to nonlinear relationships.
- The membership values are used as an input with lagged variables of real observations in one HSP-NN. Moreover, the non-membership values are used as an input with lagged variables of real observations in other HSP-NN. Both NNs generate predictions that include the effects of membership values and non-membership values. These two produced predictions are combined by weights determined by considering the different effects of memberships and non-memberships. Thus, final predictions are produced.
- The membership and non-membership values of time-series observations are obtained by intuitionistic fuzzy C-means, fuzzy clusters technique.
- Modified particle swarm optimization is performed to obtain optimal weights of HSP-NNs as well as the combination weights

To reveal the prediction performance of the proposed IFTS-PM, Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX), Dow Jones Industrial Average Index (DJI), Shanghai Securities Composite Index (SSEC) and Istanbul Stock Exchange (IEX) data sets have been comprehensively analysed. The obtained results have been comparatively evaluated with some state-of-the-art prediction tools, using some error criteria, basic properties of linear regression analysis and plots.

The rest of the paper is fictionalized as follows: In the second section, the literature review is presented. With the third section I-FCM is given. The fourth chapter presents the methodology, including the proposed hybrid sigma-pi neural network and the proposed intuitionistic fuzzy time series prediction model, with their detailed features. The comprehensive results of implementations are investigated in section five with comparatively comments and evaluations. Finally, the obtained findings and conclusions are discussed in the last section.

2 Literature review

Time series prediction has always been an attractive issue in the most scientific area. Prediction plays a crucial role in making correct and logical decisions about the future. For this reason, it is important to have a correct and logical perspective on the future in terms of determining policies and strategies for all institutions, operations and even country governments. There are many prediction methods put forward to determine the correct strategies in the literature. Although some of these are known as classical prediction methods, especially in recent years, fuzzy-based and computational-based methods have become frequently preferred methods.

On the other hand, having some distinguishing features makes fuzzy-based methods quite popular in the literature. FTS models are one of these popular models and have been originally designed for time series estimation purposes. The analysis process of FTS models basically consists of three main steps; fuzzification of observation, identification of fuzzy relationships and defuzzification. In the first FTS models, the fuzzification stage was performed at equal intervals determined by the arbitrary decision [9–13]. While Huarng used average and distribution-based methods [14] to determine intervals, some other researchers utilized optimization-based approaches [15, 16]. Also, with a ratio-based approach [17] and the idea of optimization of the ratio [18], inconstant interval lengths were specified. Better results were obtained with the use of heuristic optimization algorithms in the fuzzification transaction [19–27]. Some researchers have chosen to use some entropy-based and other methodologies to determine ranges in high-order prediction models [28–32]. Moreover, to be able to obtain intervals in a more systematic way, fuzzy C-means (FCM) [33] and some other clustering techniques were utilized in the fuzzification stage [15, 34–43].

The stage which the internal relationship of the fuzzy time series is determined has an effective role in the prediction performance of the models. While in the first studies fuzzy logic relation matrix was preferred to determine the fuzzy relations, (Song and Chissom [19, 44, 45] subsequently, Sullivan and Woodall [46] used transition matrices consisted of Markov chain in the same stage. On the other hand, over time, artificial neural networks (ANNs) have become a popular tool for determining fuzzy relationships. A first-order FTS approach that takes the advantage of a feed-forward artificial neural network (FFANN) was proposed by Huarng et al. [47]. Moreover, in many studies, FFANNs were used in the identification of fuzzy relations [15, 17, 32, 34, 48–54]. Aladag [55] proposed a FTS model which uses a single multiplicative neuron model in this stage. FCM technique was used in various studies to determine the membership values systematically [42, 56, 57]. Although the usage of ANN has many advantages, determination of the hidden layer neuron number and the excessive number of parameters are considered as the disadvantages of these models. Aladag [55] managed to eliminate this problem by using an artificial neural network with single multiplicative neuron model neural network (SMNM-NN) in the determination of fuzzy relations, however, in this study membership values were not considered. Bas et al. [58] introduced a fuzzy time series model with a network structure.

In the defuzzification step which is last stage of FTS model, centroid method was preferred by many researchers [14, 17, 29]. On the other hand, ANN [45, 59] and the adaptive expectation method [15] were used in this stage. In particular in recent years, in some studies, these stages have been evaluated together. Cagcag Yolcu and Alpaslan [2] proposed a study which evaluates the stages in one process synchronously. A hybrid FTS model based on granular computation was proposed to reduce the stock price [60]. A new fuzzy prediction model which uses both similarity measures and PSO was presented for the analysis of TAIEX [61]. A FTS model using ARMA-type fuzzy logic relationships in stock data prediction was proposed [62].

Apart from FTS models, ANFIS has been widely used for the prediction of time series in the literature as a prediction tool. While a temporal neuro-fuzzy system (NFS), inspired by ANFIS, was proposed for multi-factor time series estimation [63], an ANFIS was proposed for river flow prediction [64]. Also to predict wave parameters a hybrid ANFIS which uses genetic algorithm was presented [65]. A prediction model that combines autoregressive integrated moving average (ARIMA) models, fuzzy logic and ANNs was proposed for financial markets prediction [66]. Two different ANFISs were presented to find natural gas demand [67] and to predict stock exchanges' short-term trends [68]. A study that evaluates both the hybrid ANFIS based on the genetic algorithm and the performance of the ANNs was discussed for the prediction of energy consumption [69]. To be able to get better prediction results for the TAIEX data sets a hybrid ANFIS which uses

AR models and volatility was put forward [70]. A combination study based on ANFIS and PSO was discussed to be able to get electricity prices estimations [71]. Also, an ensemble was presented which takes the advantage of ARIMA and ANFIS for the prediction of Iran's annual energy consumption [72]. In addition, many studies have recently been conducted in many areas, revealing ANFIS-based time series prediction models. In some of them: It is aimed at the high accuracy prediction of the ionosphere total electron content (TEC) using ANFIS [73]. ANFIS was used to simulate the nonlinear behaviour of various water quality parameters by estimating the re-aeration coefficient of Yamuna River, Delhi [74]. Long-term prediction of blood pressure time series was carried out by an ANFIS system based on DKFCM clustering [75]. QoS prediction model using hybrid IOWA-ANFIS with fuzzy C-means, subtractive clustering and grid partitioning was proposed [76].

When all these fuzzy-based and inference-based systems are evaluated in detail, it is obviously seen that all of these models have important deficiencies. These facts can be explained from the aspect that fuzzy sets may fail to satisfy or characterize the uncertainty of the data in a comprehensive manner because they cannot depict the neutrality degree of time-series. This means that in the transaction of inference/prediction, they do not take into account the information belonging to non-membership degrees and the neutrality degrees of time-series. In these cases, the usage of intuitionistic fuzzy set-based prediction model makes time series prediction stronger. A couple of intuitionistic fuzzy set-based models have been proposed for the time series prediction. An intuitionistic fuzzy time series prediction model which uses intuitionistic fuzzy reasoning has been presented with the aim of predicting the enrolments of the University of Alabama and TAIEX [77]. An intuitionistic time series fuzzy inference system has been introduced to get better prediction results for TAIEX and IEX data sets [4]. In some studies, intuitionistic fuzzy functions are used to predict different real-world time series [78, 79]. A long-term intuitionistic fuzzy time-series has been used to predict network traffic [80]. A high-order forecasting model has been put forward based on intuitionistic fuzzy sets and artificial bee colony [81]. An intuitionistic fuzzy function approach has been proposed to forecast financial time series [82]. Moreover, an enhanced fuzzy time series forecasting model based on hesitant differential fuzzy sets has been introduced to forecast TAIEX, DJI, and SSEC data sets [83]. Although these limited intuitionistic fuzzy set-based prediction models manage to offer an effective approach to uncertainty by taking into account neutrality information, in these studies memberships and non-membership values are evaluated together as if both parts contribute the same to the system. And also, nonlinear relations need to be evaluated in detail to prevent the negative effects on prediction performance.

3 Intuitionistic fuzzy C-means

Intuitionistic fuzzy sets (IFS) provide a mathematical framework based on fuzzy sets to describe and deal with vagueness and uncertainty in data. They have some distinguishing advantage over fuzzy sets. While the fuzzy sets [84] only consider the membership function $\mu(x)$, $x \in X$, intuitionistic fuzzy sets take into account not only the non-membership function v(x) but also the membership function μ(x).

Let X be a universe of discourse, then

$$A = \{x, \mu_A(x), v_A(x)/x \in X\} \tag{1}$$

is called an IFS. Here $\mu_A(x)$ and $v_A(x)$ depict the membership degree and the non-membership degree for an element x, respectively.

$x \epsilon X \mu_A(x) \to [0,1]$ and $x \in X \to vA(x) \in [0,1]$ also they fulfil the condition;

$$0 \le \mu_A(x) + v_A(x) \le 1 \tag{2}$$

If $\pi A(x) = 1 - \mu A(x) - vA(x)$ then $\pi A(x)$ is called hesitation degree which represents the lack of knowledge in defining the membership and non-membership degrees and it is defined as a different measure from these degrees. It is clearly seen here, $0 \le \pi_A(x) \le 1$. If $\pi_A(x) = 0$ for all $x \epsilon X$, then intuitionistic fuzzy set becomes a fuzzy set; if $\mu_A(x) = v_A(x) = 0$ for all $x \epsilon X$, then the intuitionistic fuzzy set is called completely intuitionistic.

The objective function to be optimized in IFCM [85] includes two components: one of them is; modified objective function of FCM using IFS and the other one is; intuitionistic fuzzy entropy (IFE).

$$J_{IFCM} = \sum_{i=1}^{c} \sum_{k=1}^{n} u_{ik}^{*m} d_{ik}^2 + \sum_{i=1}^{c} \pi_i^* exp\left(1 - \pi_i^*\right) \tag{3}$$

Here u_{ik}^* represents the intuitionistic fuzzy membership and calculated by Eq. (4)

$$u_{ik}^* = u_{ik} + \pi_{ik} \tag{4}$$

And u_{ik} is the traditional fuzzy membership of the k th datum point in i th set and π_{ik} is the hesitation degree which can be given as:

$$\pi_{ik} = 1 - u_{ik} - \left(1 - u_{ik}^{\alpha}\right)^{1/\alpha}, \quad \alpha > 0 \tag{5}$$

and is obtained by intuitionistic fuzzy complement of Yager:

$$N(X) = (1 - x^{\alpha})^{1/\alpha}, \quad \alpha > 0 \tag{6}$$

Hereby, IFS becomes;

$$A_{\lambda}^{IFS} = \left\{ x, \mu_A(x), -(1-\mu_A(x)^{\alpha})^{1/\alpha} / x \in X \right\} \tag{7}$$

and

$$\pi_i^* = \frac{1}{N} \sum_{k=1}^{n} \pi_{ik}, k \in [1, N] \tag{8}$$

IFE, which forms the second term of the objective function, represents the amount of fuzziness or uncertainty in a set. While $\mu_A(x_i)$, $v_A(x_i)$, and $\pi_A(x_i)$ depict the membership degree, non-membership degree, and hesitation degree of the elements of the set $X = \{x_1, x_2, \ldots x_n\}$, in that case, IFE which represents the intuitionism degree can be given as:

$$IFE(A) = \sum_{i=1}^{n} \pi_A(x_i) exp(1 - \pi_A(x_i)), k \in [1, N] \tag{9}$$

and

$$\pi_A(x_i) = 1 - \mu_A(x_i) - v_A(x_i) \tag{10}$$

Modified cluster centres can be calculated via Eq. (11),

$$v_i^* = \frac{\sum_{k=1}^{n} u_{ik}^* x_k}{\sum_{k=1}^{n} u_{ik}^*} \tag{11}$$

Until the system reaches maximum iteration times $max_{ik}\left|U_{ik}^{*new} - U_{ik}^{*previous}\right|$ which is smaller then ε (ε is predefined value) both cluster centres and memberships are updated iteratively and then the process is stopped.

4 The proposed methodology

4.1 A new hybrid sigma-pi neural network

In the literature, while MLPs generally use the additive aggregation function, the single multiplicative neural network proposed by Yadav [8] uses the multiplicative aggregation function. Although a function of the product of the linear combinations of the inputs is obtained in a classical SP-NN as outputs, it is noteworthy that each linear combination in this multiplication transaction has a constant coefficient and they are all 1. This means that the signals moving from the hidden layer neurons to the output neuron are directly multiplied. However, these signals coming out of the hidden layer units may have different contribution levels in the formation of the output. Therefore, these signals representing linear combinations of inputs should be associated with different weights.

In this context, a new hybrid SP-NN structure is proposed in which these linear functions are subjected to the multiplication function with different coefficients. This new structure can be considered as a hybrid of the classical MLP and Single multiplicative neuron model. This new structure has not only the features of the classic SP but also has its own unique features. Hybrid SP-NN (HSP-NN), just like the classic sigma-pi neural networks, has a fast-learning ability that reduces network complexity by using efficient polynomials for many input layer variables.

The basic structure of HSP-NN has three layers as input, hidden and output layers. The number of linear combinations obtained from the sum of the units found in the hidden layer represents the degree of HSP-NN. Having both additive and multiplicative aggregation functions is the main feature of this model that makes it more flexible and adaptable.

An HSP-NN structure with N inputs and having K th order memory-based can be given by Fig. 1.

In this structure, $w_{ij}(i = 1, 2, \ldots, K; j = 1, 2, \ldots, N)$ represents the weight between jth input layer neuron and ith hidden layer neuron. b_i $(i = 1, 2, \ldots, K)$ is bias for ith hidden layer neuron. $w_i(i = 1, 2, \ldots, K)$ represents the weight between jth input layer neuron and the output layer neuron. Finally, b_o is the bias for the output layer neuron. In the classical SP-NN, the weight between hidden and output layers are not taking into account just accepted as a constant which equals 1. But, the proposed HSP-NN considers these weights differently rather than taking them 1. These different weights, representing different effects of the linear functions of the inputs on the outputs, are obtained through an optimization process. The outputs of hidden layer neurons are obtained via Eq. (12).

$$o_i = f_1\left(\sum_{j=1}^{N} w_{ij} X_j + b_i \right), i = 1, 2, \ldots, K \tag{12}$$

Here f_1 represents the linear activation function and $f_1(x) = x$. The outputs of the system are obtained, by using the weights between the hidden and output layers, as in the formula given Eq. (13).

$$o_{\mathrm{HSP-NN}} = \widehat{Y} = f_2\left((\prod_{i=1}^{K} o_i * w_i) + b_o \right) = \frac{1}{1 + \exp\left(-\left((\prod_{i=1}^{K} o_i * w_i) + b_o\right)\right)} \tag{13}$$

in Eq. (13), o_i is the output of ith hidden layer neuron, w_i is the weight between ith hidden layer neuron and output layer neuron, and b_o is bias for output layer neuron. f_2 is also the logistic activation function.

4.2 A new intuitionistic fuzzy time series prediction model

The prediction approaches of fuzzy time-series methods positive effect on prediction performance have become a known fact. However, on the other hand, in many cases,

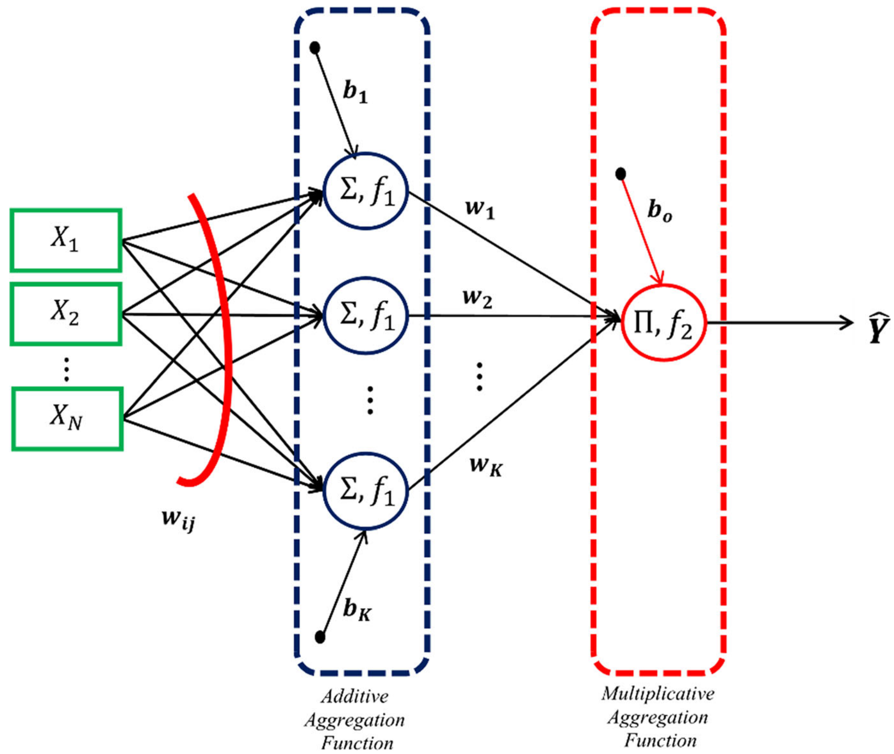


Fig. 1 The structure of HSP-NN designed for general modelling

fuzzy sets fail to satisfy or characterize the uncertainty of the data comprehensively. Because, in these studies, the degree of neutrality of time series is not taken into account. In this respect, our main purpose is to propose a new intuitionistic fuzzy time series prediction model in this study. This model, thanks to its own special analysis process, can produce remarkable predictive results by extracting important properties of time-series.

Compared to some current prediction models, the basic contributions of the proposed new intuitionistic fuzzy time series prediction model can be given as follows:

- This study is called as a new intuitionistic fuzzy time series model based on the combination of membership and non-membership values.
- HSP-NN is presented for the first time in the literature to determine nonlinear relationships between inputs and outputs of the proposed model.
- In this model, the intuitionistic fuzzy relations between membership, non-membership values and outputs are determined by the HSP-NN.
- In the study, the contribution of both membership values and non-membership values to the system has been evaluated separately considering the benefits of the intuitionistic structure.
- To be able to determine fuzzy clusters and membership values, I-FCM is performed.
- The optimal parameters of the HSP-NN are obtained through MPSO.
- The final outputs of the system are obtained by combining the results from two different HSP-NNs.

In the light of this information, the algorithm of the proposed method is summarized with some steps:

Step 1 All the necessity parameters of process are determined

pn	# particles of the swarm
$maxitr$	Maximum iteration number
(c_{1i}, c_{1f})	The intervals for possible values of cognitive coefficient (c_1).
(c_{2i}, c_{2f})	The intervals for possible values of social coefficient (c_2).
(w_1, w_2)	The intervals for possible values of inertia weight (w)
m_1	# lagged crisp variables for the first hybrid HPS-NN
m_2	# lagged crisp variables for the second HPS-NN.
K_1	Order of the first hybrid HPS-NN
K_2	Order of the second HPS-NN
t_{train}	Length of the training set
t_{test}	Length of the test set
c	# intuitionistic fuzzy sets
T	# observations of time series

Step 2 I-FCM clustering algorithm is performed

In this step, cluster centres, membership and non-membership values for training data set are obtained. And these obtained values are used for the inputs of HSP-NNs. Cluster numbers are represented by$L_l, (l = 1, 2, \ldots, c)$. At the end of the I-FCM process, the membership and non-membership values are stored S_1 and S_2 matrices. Cluster centres are laid up in C vector.

$$S_1 = [\mu_{lt}], l = 1, 2, \ldots, c; t = 1, 2, \ldots, t_{train} \tag{14}$$

$$S_2 = [v_{lt}], l = 1, 2, \ldots, c; t = 1, 2, \ldots, t_{train} \tag{15}$$

$$C = [c_1, c_2, \ldots, c_c] \tag{16}$$

Here S_1 and S_2 matrix depict the memberships $\left(\mu_{L_l}(y_t)\right)$ and non-membership $\left(v_{L_l}(y_t)\right)$ values of time series observations belong to L_l fuzzy set.

Step 3 Constituting the inputs (I_1, I_2) and targets (T) for the HSP-NNs

In this study, memberships and lagged variables of real observations and non-membership variables and lagged variables of real observations are considered as two separate input groups. These created inputs are also used separately for two different HSP-NNs. The inputs of HSP-NN can be given with two matrices ($I_1 and I_2$) as follow:

$$M_1 = \left[S_1, y_{t-1}, \ldots, y_{t-m_1}\right], t = m_1 + 1, \ldots, n - t_{train} \tag{17}$$

$$M_2 = \left[S_2, y_{t-1}, \ldots, y_{t-m_2}\right], t = m_2 + 1, \ldots, n - t_{train} \tag{18}$$

There is no separate target for the used two neural networks at this stage. There is only one target value corresponding to the final output to be obtained by combining the outputs produced by these two separate neural networks. And the target of the prediction model can be given as follows:

$$T = y_t, t = q + 1, \ldots, t_{train} \tag{19}$$

Step 4 Determining the structure of the particles

Some features of the relevant positions of the optimization process, belonging to the prediction model can be given in Table 1.

In this stage, particle structures are obtained by generating the initial positions and velocities. The Positions and the velocities of particles are randomly generated from Uniform$(-1, 1)$ distribution and they stored in P and V matrices:

$$P = [p_{rs}]; r = 1, 2, \ldots, pn; s = 1, 2, \ldots, p \tag{20}$$

$$V = [v_{rs}]; r = 1, 2, \ldots, pn; s = 1, 2, \ldots, p \tag{21}$$

Step 5 Calculation for the outputs of HSP-NN

Each of HSP-NN is performed and the outputs are obtained for each particle considering the particles' positions. The graphical abstract of the proposed new intuitionistic fuzzy time series prediction model can be presented with Fig. 2.

Linear combinations of inputs that represent the information in each of hidden layer's neuron are obtained by using weights (w_t^{ij} and w_t^{il}) and biases (b_t^i). While in the classical SP-NN, the weight between hidden and output layers are not taken into account just accepted as a constant which equals 1, in this proposed study to be able to obtain the output of the HSP-NN weights and biases are determined in a single optimization process. Then for HSP-NN 1 (for membership part), the outputs of hidden layer neurons are obtained via Eq. (22).

$$^{\mu}o_i^i = f_1\left(\left(\sum_{l=1}^{c} {}^{\mu}w_t^{il}\mu_{L_l}\right) + \left(\sum_{j=1}^{m1} {}^{\mu}w_t^{ij}Y_{t-j}\right) + b_t^i\right), \quad i = 1, 2, \ldots, K_1 \tag{22}$$

The outputs of HSP-NN 1 are obtained by multiplying all the hidden layers' outputs with the determined weights between hidden and output layers. The formula of the output belongs to the first HSP-NN can be calculated by Eq. (23)

$$o_t^{HSP-NN_1} = {}^{\mu}\widehat{Y}_t = f_2\left(\left(\prod_{i=1}^{K_1} {}^{\mu}o_i^{i\mu}w_t^i\right) + {}^{\mu}b_t^o\right) = \frac{1}{1 + \exp\left(-\left(\prod_{i=1}^{K_1} {}^{\mu}o_i^{i\mu}w_t^i\right) + {}^{\mu}b_t^o\right)} \tag{23}$$

Similar operations are performed for HSP-NN 2 (for non-membership part), and outputs ($o_t^{HSP-NN_2} = {}^{v}\widehat{Y}_t$) are obtained. The final output of the system, i.e. the prediction of the observation at time t, is obtained by weighting these two different HSP-NNs' outputs. This combined output is calculated as in Eq. (24).

$$\widehat{Y}_t = w_1 o_t^{HSP-NN_1} + w_2 o_t^{HSP-NN_2} = w_1{}^{\mu}\widehat{Y}_t + w_2{}^{v}\widehat{Y}_t \tag{24}$$

Table 1 The properties of optimization process

Neural networks	# Inputs	# Output	# Weights (input-hidden)	# Biases (input-hidden)	# Weights-(hidden-output)	# Biases (hidden-output)	# Combination
HSP-NN1	$(m_1 + c)$	1	$(m_1 + c) * K_1$	K_1	K_1	1	1
HSP-NN2	$(m_2 + c)$	1	$(m_2 + c) * K_2$	K_2	K_2	1	1
Total parameters	$p = ((m_1 + c) * K_1 + (m_2 + c) * K_2 + 2K_1 + 2K_2 + 1 + 1 + 2)$						

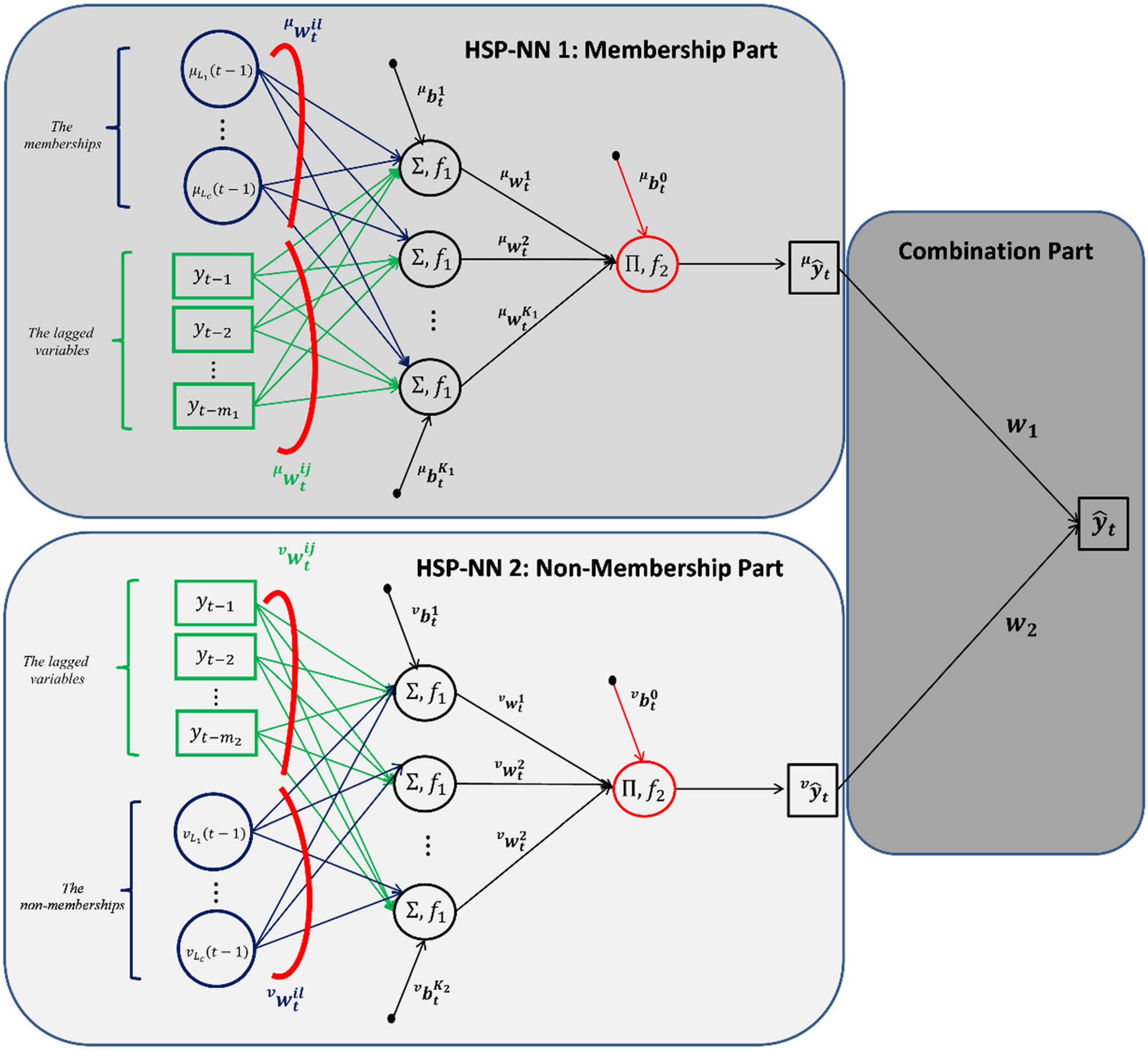


Fig. 2 The graphical abstract of IFTS prediction model

For both HSP-NNs, in the hidden layer neurons, linear activation function $f_1(x) = x$ is used and the logistic activation function is preferred in the output layer.

Step 6 Evaluation of the fitness function values

To be able to determine the best particles, each particles' fitness function values are obtained. In this study RMSE is preferred as an evaluation criteria or loss function. Its formula is given in Eq. (25).

$$RMSE = \sqrt{\sum_{t=max(m_1,m_2)+1}^{t_{train}} (Target_t - Output_t)^2} \tag{25}$$

Step 7 Determine Pbest and Gbest

After calculating RMSE values belonging to each of the particles, it is time to decide *Pbest* and *Gbest* values. *Pbest* and *Gbest* are the values which represent the best values from each particles and global best value belongs to all system, respectively. These values can be given in Eq. (26) and (27).

$$Pbest = [Pb_{rs}]; r = 1, 2, \ldots, pn; s = 1, 2, \ldots, p \tag{26}$$

$$Gbest = [Pg_s]; s = 1, 2, \ldots, p \tag{27}$$

Step 8 The process of updating the parameters of the MPSO

The parameters to be updated at this stage have been re-obtained by using the following formulas.

$$c_1 = \left(c_{1f} - c_{1i}\right)\frac{itr}{maxitr} + c_{1i} \tag{28}$$

$$c_2 = \left(c_{2f} - c_{2i}\right)\frac{itr}{maxitr} + c_{2i} \tag{29}$$

***Algorithm*: IFTS-PM**

Input : Observations of time series Y_t, $t = \overline{1, t_{train}}$
Output : Predictions of time series $\hat{Y}_t$, $t = \overline{1, t_{train}}$

BEGIN

Set the parameters

Normalize Y_t

Perform I-FCM

Establish inputs of HPS-NN $I_1: Y_{t-k_1}, \mu_l, k_1 = \overline{1, m_1}; l = \overline{1, c}$

$I_2: Y_{t-k_2}, v_l, k_2 = \overline{1, m_2}; l = \overline{1, c}$

$itr = 0$

Generate the positions of particles ${}_0p_{rs}$, $r = \overline{1, pn}$; $s = \overline{1, p}$ randomly from $Uniform(-1,1)$

Repeat

$itr = itr + 1$

Calculate the outputs $\hat{Y}_t$ $t = \overline{1, t_{train}}$

Calculate the cost function ${}_{itr}RMSE_r$, $r = \overline{1, pn}$

Determine ${}_{itr}Pbest_r$ and ${}_{itr}Gbest$

Update the positions of particles (Eq. 32)

Until $itr == maxitr$

Determine the best positions $Gbest = {}_{itr}Gbest$

Calculate the outputs for the best positions $\hat{Y}_t$ $t = \overline{t_{train} + 1, T}$

END

$$w = (w_1 - w_2)\frac{maxt - \mathrm{itr}}{maxitr} + w_2 \tag{30}$$

Here *itr* is the current iteration number.

Step 9 Update the positions and the velocities

In this stage to be able to determine the optimal parameters velocities and positions are needed to update by utilizing the formulas given in Eqs. 31 and 32.

$$v_{rs}^{itr+1} = w.v_{rs}^{itr} + c_1.rand_1^{itr}.\left(Pbest_{rs}^{itr} - P_{rs}^{itr}\right) + c_2.rand_2^{itr}.\left(gbest_s^{itr} - P_{rs}^{itr}\right) \tag{31}$$

$$p_{r,s}^{itr+1} = p_{r,s}^{itr} + v_{r,s}^{itr+1} \tag{32}$$

Step 10 Checking the stopping criteria

In the last stage of the prediction system, final outputs must be obtained. In order to realize this, it is checked whether the pre-determined iteration times have been reached. If the number of repetitions reaches the maximum iteration number $(maxitr)$ then stop the process, or else the system is repeated from Steps 5 to Step 9 until a pre-determined maximum iteration number $(maxitr)$ is reached. When *maxitr* is reached, G_{best} vector's elements are specified as the optimal values of weights and the biases of HSP-NNs, and the weights of combination part.

The working principle of IFTS-PM can be given by a pseudo-code for better understanding.

4.3 Experimental results and discussion

4.3.1 Data preparation

The prediction performance of the proposed IFTS-PM prediction model has been evaluated with the analysis of four different financial time series; TAIEX, DJI, SSCE and IEX and the findings based on prediction results are presented as concrete proof of the superior performance of the proposed model as well as its contribution to the literature. As the test set, the observations of the last two months have been taken for the TAIEX, DIJ and SSCE data sets. Moreover, two different test data sizes have been chosen as the out-of-samples (test set) for IEX data sets as last 7 and 15 observations. The properties of the data of all-time series are presented in Table 2.

Table 2 The properties of data organization

Series no.	Time series	Year	# Observations	Size of training set	Size of testing set
1	TAIEX	2000	271	224	47
2		2001	244	201	43
3		2002	248	205	43
4		2003	249	206	43
5		2004	250	205	45
6/17/28	TAIEX/DIJ/SSCE	2008	249/253/246	206/212/203	43/41/43
7/18/29		2009	247/252/244	203/201/200	44/42/44
8/19/30		2010	250/252/242	205/209/197	45/43/45
9/20/31		2011	247/252/244	203/210/200	44/42/44
10/21/32		2012	246/250/243	204/209/200	42/41/43
11/22/33		2013	244/252/238	201/211/195	43/41/43
12/23/34		2014	248/252/245	205/211/202	43/41/43
13/24/35		2015	244/252/244	200/210/200	44/42/44
14/25/36		2016	242/252/244	198/210/200	44/42/44
15/26/37		2017	243/251/244	200/210/201	43/41/43
16/27/38		2018	245/251/243	203/211/201	42/40/42
39/40	IEX	2009	103/103	96/88	7/15
41/42		2010	104/104	97/89	7/15
42/44		2011	106/106	99/91	7/15
45/46		2012	106/106	99/91	7/15
47/48		2013	106/106	99/91	7/15

4.4 Performance measure

The predictions of the proposed IFTS-PM have been examined with different evaluation measures. When the used various evaluation metrics have been considered to compare the performance of the models, it is seen that in most of the prediction studies Root-Mean-Square Error (RMSE) is preferred. RMSE is calculated as in Eq. (25).

Although the RMSE criterion is accepted as an effective evaluation criterion in comparing models, it cannot give clear information about the level of accuracy of the predictions produced by the model alone, i.e. whether it produces satisfactory predictive results.

Unlike the RMSE criterion, the mean absolute percentage error (MAPE) criterion is independent of the level of the data set values. This feature makes it superior in evaluating the performance of a prediction model alone. From this point of view, in this study to be able to evaluate the proposed model from different aspects we also choose MAPE, given in Eq. (33).

$$MAPE = \frac{1}{T}\sum_{t=1}^{T}\left|\frac{Target_t - Output_t}{Target_t}\right| \times 100\% \tag{33}$$

Although MAPE has the advantages outlined above, it is a measure of performance based on the average, therefore, produces biased values if any observation error is an outlier. In such a case, it is more appropriate to use the median absolute percentage error (MdAPE) criterion, which is not affected by outliers, instead of MAPE. This criterion, given in Eq. (34), has also been evaluated to take into account such situations.

$$MdAPE = median\left(\left|\frac{Target_t - Output_t}{Target_t}\right| \times 100\%\right), \quad t = 1, 2, \ldots, T \tag{34}$$

To evaluate the prediction results generated by the proposed IFTS-PM with a benchmark model, Median root absolute error (MdRAE), given in Eq. (35), has also been used. For this purpose, the naïve model has been taken as the benchmark model, as often in practice.

$$MdRAE = median\left(\left|\frac{Target_t - Output_t}{Target_t - Prediction_t^*}\right|\right), \quad t = 1, 2, \ldots, T \tag{35}$$

where $Prediction_t^*$ is the prediction results of the naïve model as a benchmark model, and $Prediction_t^* = Target_{t-1}$.

On the other hand, to be able to evaluate the results from a different perspective regression analysis is performed. This regression model which is created for the predictions and the target values can be given in Eq. (36).

$$Y_t = \beta \widehat{Y}_t + \varepsilon_t \tag{36}$$

For a superior estimation tool, the regression and specification coefficients of this model are expected to be quite close to 1.

The scatter diagrams, using Cartesian coordinates displaying the observations and prediction values, are used to visualize the superior prediction performance of the proposed IFTS-PM.

4.5 Alternative comparison models

In the study, in order to compare and make comments on the prediction ability of the proposed IFTS model, various methods which are known for their outstanding prediction performance are chosen. For the TAIEX data set, many studies' results have been evaluated with the proposed one. Moreover, the prediction tools used in comparison for IEX can be given below;

ARIMA	Autoregressive integrated moving average method [86] (Box and Jenkins).
ES	Exponential Smoothing [87]
MLP	Multilayer perceptron artificial neural network [88]
SC	Song and Chissom's fuzzy time series prediction model [19]
FF-T1	Type 1 fuzzy Regression function approach [89]
FTS-N	Fuzzy time series network [58]
ANFIS	Adaptive Neuro-Fuzzy Inference System [90]
MANFIS	Modified Adaptive Neuro-Fuzzy Inference System [1]
AR-ANFIS	Adaptive Neuro-Fuzzy Inference System with AR structure [91]
I-TSFIS	Intuitionistic time series fuzzy inference system [4]
I-FRF	Intuitionistic fuzzy regression functions approach [78]

4.6 Evaluation of the model performance

In the evaluation of the proposed model's performance part to be able to demonstrate the outstanding prediction ability of the model, different time series data sets are chosen. As mentioned in the data preparation part each of data set is divided into two parts as the training and test sets. The inputs of the proposed model are composed of, respectively, the memberships and the lagged crisp time series and the non-memberships with lagged crisp time series. Apart from that all the properties and hyperparameters of implementations can be summarized in Table 3.

4.6.1 TAIEX implementations—the daily data sets between 2000 and 2004 years

Firstly, daily TAIEX data belonging to five years are analysed through the proposed IFTS prediction model and obtained results are evaluated with various state-of-the-art models' results, together. In the analysis process, the observations of time series in the first ten months are taken as the training set, the last two months are taken as the test set for each year. The inputs of the proposed model are composed of, respectively, the memberships and the lagged crisp time series and the non-memberships with lagged crisp time series.

In the comparison process, the architecture that produces the best prediction performance among all architectures is used. Moreover, the results generated by the

Table 3 The properties of implementations

Abbreviation	Hyperparameter	
	Meaning	Value
pn	# particles of the swarm	30
$maxitr$	Maximum iteration number	100
(c_{1i}, c_{1f})	The intervals for possible values of cognitive coefficient (c_1)	$(2, 1)$
(c_{2i}, c_{2f})	The intervals for possible values of social coefficient (c_2)	$(1, 2)$
(w_1, w_2)	The intervals for possible values of inertia weight (w)	$(0.4, 0.9)$
m_1	# lagged crisp variables for the first hybrid HPS-NN	From 2 to 5
m_2	# lagged crisp variables for the second HPS-NN	From 2 to 5
K_1	Order of the first hybrid HPS-NN	From 2 to 5
K_2	Order of the second HPS-NN	From 2 to 5
t_{train}	Length of the training set	Data dependent
t_{test}	Length of the test set	Data dependent
c	# intuitionistic fuzzy sets	From 3 to 10
T	# observations of time series	Data dependent

Table 4 The prediction performances of the models for the TAIEX

Models	Time series/TAIEX data sets					RMSE's	
	1	2	2	4	5	Average	Median
[44]	293	116	76	77	82	129	82
[29]	225	116	76	77	82	115	82
[14][a]	473	359	234	247	384	339	359
[14][b]	473	810	116	308	384	418	384
[17]	133	124	82	62	85	97	85
[92]	152	130	84	56	116	108	116
[47]	154	124	93	66	72	102	93
[93]	131	130	80	58	67	93	80
[15]	168	120	76	58	63	97	76
[11]	129	113	67	54	60	85	67
[25]	124	115	71	58	58	85	71
[94]	120	114	67	52	52	81	67
[95]	131	113	66	52	54	83	66
[96]	255	130	84	56	116	128	116
[42]	227	102	66	51	55	100	66
[94]	126	114	65	54	53	82	65
[48]	125	115	65	53	53	82	65
[97]	132	113	60	52	50	81	60
[58]	140	120	77	60	59	91	77
[27]	126	113	63	51	54	81	63
[98]	180	134	81	77	55	105	81
[3]	128	106	65	52	54	81	65
[90]	137	115	66	57	61	87	66
[1]	124	112	63	52	54	81	63
[91]	123	111	66	52	54	81	66
[4]	209	73	22	43	54	80	54
[78]	122	110	54	51	50	77	54
[82]	120	111	64	52	52	80	64
The proposed IFTS-PM	**98**	**60**	**17**	**35**	**40**	**50**	**40**
Progress (%)	18.33	17.81	22.73	18.60	20.00	35.06	25.93

The bold values indicate the best results

[a]The average-based model

[b]The distribution-based model

proposed IFTS prediction model have been evaluated together and done a comprehensive comparison with the other state-of-the-art models' results. Table 4 represents the all results in terms of RMSE criterion.

When the findings given in Table 4 are evaluated, it is obviously seen that the proposed IFTS-PM has produced the best predictive results in comparison to other well-known prediction methods in all cases in terms of RMSE for out-of-samples. For time series 1 (TAIEX 2000), as conventional fuzzy time series forecasting models, [29] and [44] exhibit a really ordinary performance with the RMSE values of 225 and 293, respectively. For this data set, the proposed IFTS-PM shows the best performance with the RMSE value of 98. This performance is outstanding comparing the second successful model based on the intuitionistic fuzzy function approach (RMSE = 120). It means that the proposed model provides a 20% of improvement level. For this data set, as conventional fuzzy time series forecasting models, [29] and [44] exhibit a really ordinary performance with the RMSE values of 225 and 293, respectively. For time series 2 (TAIEX 2001), while conventional fuzzy time series forecasting models ([29] and [44]) produce prediction results with the RMSE values of 116, the proposed prediction model generates the prediction results with the RMSE values of 60. For this time series, with this performance, the proposed IFTS-PM makes about 18% progress comparing the second successful model, intuitionistic fuzzy inference system

(RMSE = 73). Similar comments can be made for time series 3 and 4 (TAIEX 2002 and 2003). For these implementations, the conventional fuzzy time series forecasting models ([29] and [44]) generate results with the RMSE values of 76 and 77, respectively. In these implementations, the proposed prediction model also produces the results with the RMSE values of 17 and 35, respectively. For these implementations, the proposed IFTS-PM makes about 23% and 19% progress comparing the second successful model, intuitionistic fuzzy inference system having RMSE values of 22 and 43.

For time series 5 (TAIEX 2004), as conventional fuzzy time series forecasting models, [29] and [44] produce ordinary results with the RMSE values of 115and 129, respectively. For this data set, the proposed IFTS-PM shows the best performance again with the RMSE value of 40. This performance is extraordinary comparing the second successful model based on the intuitionistic fuzzy function approach having an RMSE value of 50. These findings indicate that the proposed model provides a 20% improvement level. In addition, when the prediction results obtained for each year were examined in an entire perspective, for each year, the IFTS-PM provided about 20% improvement compared to the best one among other models. The main reason behind this situation is that the IFTS-PM which combines the results from two different HSP-NN trained by MPSO both has less risk of getting stuck in local optimums and also has the ability to extract the significant features from data sets. The average and median statistics of the RMSE criterion can be seen as further proof of the outcome that the proposed IFTS-PM produced superior performance. As an average rate, in fact, the improvement is about 35%.

Another way to reveal the superior prediction ability of a prediction model is to examine some properties of a linear regression model to be established between predictions and reel values. The regression coefficient ($\widehat{\beta}$) and also the coefficient of determination (R^2) of a linear regression model established as $Y_t = \beta \widehat{Y}_t + \varepsilon_t$ are desired to be 1 or quite close to 1. Table 5 summarizes all the findings for this regression analysis. In the analysis process while the observed values are taken into account as the dependent variable, predicted values are considered as the independent variable.

The findings given in Table 5 can be evaluated from three different angles. For all the time series, belonging to the different years, obtained beta coefficient estimates are quite close to 1 as expected for a successful prediction model. These results are the concrete indicator that the predictions produced by the IFTS-PM are very close to the reel observations. Moreover, the standard error of the beta can be considered as a measure of the obtained predictions' reliability. Because this value is the measure of the change that the regression coefficient can show from sample to sample. This value was obtained as very low, even very close to zero, for predictions regarding each TAIEX dataset. This is an indication that the predictions produced by the model are very reliable. In addition, having the determination coefficient very close to 1 is the other proof of the existence of a very high linear relationship between the predictions of the proposed IFTS-PM and the reel observations, for all data sets.

Besides all these statistical perspectives, scatter diagrams can also be interpreted in order to evaluate the performance of the proposed model. When Fig. 3 is investigated, it is observed that most of the points on the scatter plots are in proximity to the line segment as expected. These distribution graphs also show that the observations have a strong positive linear relationship with the predictions. So, it means that prediction values are quite close to the real observations.

In terms of MAPE and MdAPE criteria, it is seen that the proposed method produced prediction results with an error of around 1% for the years 2000 and 2001. Moreover, for 2002, 2003 and 2004 these values are below 1% and even around 0.5%.

In addition to all these findings, if Table 6, where analysis time is given as computational cost, is examined, it is seen that IFTS-PM reaches results within a very reasonable time.

Considering all these evaluation criteria and approaches, it can be said that the proposed I-FTS prediction model is a powerful and preferable candidate model for TAIEX prediction literature.

Table 5 Characteristics of a linear regression model $Y_t = \beta \widehat{Y}_t + \varepsilon_t$

Time series	Model estimation	Sig. of F	St. Error of $\widehat{\beta}$	Sig. of β	R^2
1	$Y_t = 1.014310\widehat{Y}_t$	2.15E−88	1.79E−03	4.17E−90	0.999857
2	$Y_t = 1.000736\widehat{Y}_t$	7.55E−80	1.935E−03	1.53E−81	0.999843
3	$Y_t = 1.000062\widehat{Y}_t$	2.87E−101	5.80E−04	1.75E−103	0.999986
4	$Y_t = 0.996021\widehat{Y}_t$	1.12E−98	6.68E−04	7.88E−101	0.999981
5	$Y_t = 0.997538\widehat{Y}_t$	1.30E−96	9.41E−04	1.32E−98	0.999961

Fig. 3 **a** The Scatter Plot for time series 1. **b** The Scatter Plot for time series 2. **c** The Scatter Plot for time series 3. **d** The Scatter Plot for time series 4. **e** The Scatter Plot for time series 5

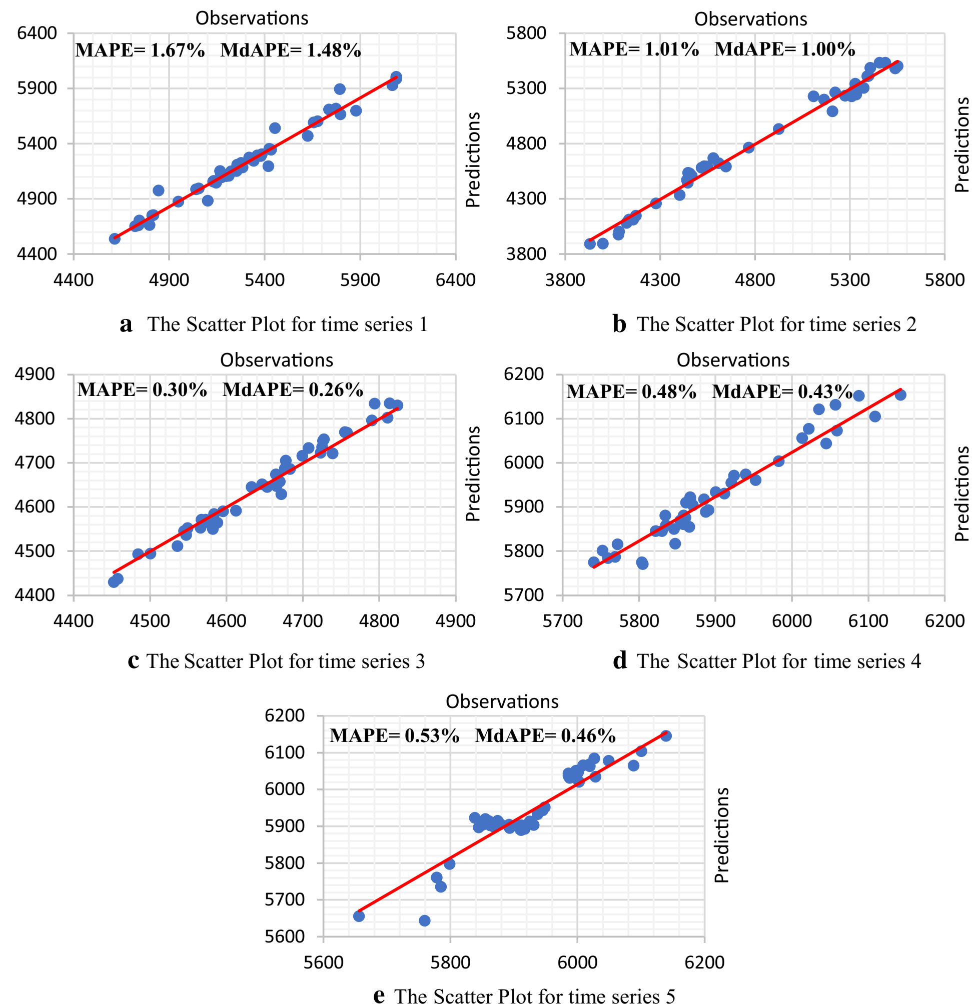


Table 6 Computational cost of IFTS-PM for TAIEX data sets

Time series	Computational cost as time (second)
1	5.134645
2	5.209962
3	4.971165
4	7.192677
5	6.900486

4.6.2 TAIEX-DJI-SSEC implementations—the daily data sets between 2008 and 2018 years

Besides the implementations of TAIEX datasets detailed in Sect. 5.4.1, the Dow Jones Industrial Average Index, Shanghai Securities Composite Index and TAIEX collected from 2008–2018. In implementations, the data set for each fiscal year has been considered an individual time series. As in previous studies ([83, 99–101]), the periods of the first ten months (from January to October) of data sets have been taken for training (in-sample); while the last two months (November and December) have been used as validation or testing set (out-of-sample) to reveal the performance of the proposed model.

While the results for TAIEX have been evaluated for each fiscal year, the results for DIJ and SSEC data sets have been evaluated on average, just as in [83]. Firstly, Tables 7, 8 and 9 present the comparison results between the proposed model and some of its current counterparts in the literature for the RMSE, MAPE, and MdRAE criteria, respectively. When the results given in Tables 7, 8 and 9 are analysed, it is obviously said that the proposed IFTS-PM shows the best prediction performance in terms of all three criteria. The superior performance of the proposed model can be emphasized once more, especially considering the level of progress delivered by the proposed method compared to the models with the second-best performance in the literature. In point of RMSE values given in Table 8, while these progression levels have been around 30–40% for some fiscal years (2009, 2011, 2016), they have exceeded the level of 50% in some fiscal years (2010, 2012, 2013, and 2014). In addition, these progress

Table 7 Performance evaluation for TAIEX test data in different years in terms of RMSE

Models	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	Average
[45]	225.38	204.11	151.42	237.43	81.08	82.3	98.64	176.6	122.71	91.69	173.36	149.52
[29]	186.48	207.42	211.21	215.13	78.37	184.83	308.72	100.75	316.57	902.18	492.89	291.32
[14]	105.10	78.25	103.84	117.78	59.14	49.59	87.52	90.72	80.98	63.20	186.21	92.94
[102]	138.94	71.30	72.92	115.54	62.32	49.53	69.60	78.73	82.22	62.98	102.19	82.39
[103]	142.38	110.88	100.76	180.79	63.30	63.16	71.62	104.4	88.35	89.02	231.35	113.27
[17]	131.18	71.48	74.07	119.34	59.55	50.09	65.72	79.93	82.10	61.67	101.99	81.56
[47]	144.23	72.16	62.23	114.25	59.94	50.97	64.79	79.27	82.29	62.15	107.08	81.76
[93]	129.48	69.91	67.11	123.03	58.37	50.39	69.15	78.22	80.51	64.25	103.74	81.29
[92]	140.49	70.34	93.08	118.89	62.35	49.63	66.55	78.46	82.73	62.21	109.39	84.92
[104]	142.37	112.45	99.83	164.97	63.31	64.76	74.85	94.33	86.59	88.93	161.78	104.92
[105]	114.03	70.91	68.59	117.02	60.50	49.92	70.44	77.67	80.86	62.61	106.75	79.94
[106]	129.29	70.27	75.46	111.45	63.20	50.84	67.67	78.33	84.39	61.09	103.26	81.39
[107]	137.44	71.76	74.52	117.38	61.59	51.37	69.23	79.89	83.53	62.37	103.24	82.94
[101][a]	108.52	121.46	75.33	128.46	60.43	51.17	79.44	94.92	80.39	78.76	88.23	87.92
[101][b]	108.57	68.57	52.15	113.38	58.84	48.87	65.90	80.22	82.24	64.31	74.52	74.32
[108]	140.19	73.22	80.37	124.38	62.53	52.39	66.26	82.78	80.14	61.98	99.03	83.93
[109]	126.29	73.77	143.53	122.49	60.73	55.38	66.15	79.47	83.18	65.54	203.72	98.20
[110]	140.48	70.63	67.46	121.27	61.10	50.23	67.08	80.65	81.12	66.34	98.13	82.23
[111]	113.03	72.30	62.82	113.69	60.45	50.17	68.28	78.65	82.43	62.49	104.49	78.98
[83][c]	101.91	105.64	62.93	123.79	82.30	65.62	46.38	102.82	81.94	61.83	101.56	85.16
[83][d]	91.95	57.43	44.61	90.32	43.58	33.83	46.23	58.79	53.15	36.99	70.67	57.05
The IFTS-PM	**31.25**	**40.10**	**22.23**	**55.27**	**20.35**	**13.79**	**22.63**	**16.03**	**35.11**	**30.89**	**28.24**	**28.717**
Progress (%)	66.01	30.18	50.17	38.81	53.31	59.25	51.04	72.74	33.94	16.50	60.03	49.66
Time (second)	10.70	7.16	7.18	5.87	7.01	8.99	7.03	5.97	5.64	7.37	6.92	

The bold values indicate the best results

The results of the current counterparts models are taken from [83]

[a]The model without ICA

[b]The model with ICA

[c]The model without AEL

[d]The model with AEL

levels have exceeded 60% for 2008 and 2018, and 70% for 2017. Considering all eleven fiscal years together, the rate of progress is also almost 50% on average.

As for the evaluation of the performance of the proposed IFTS-PM according to the MAPE criterion values shown in Table 8, it is observed that the proposed model produced predictions with a percentage error of even less than 0.5% for all years, except for 2008 when it gave predictions with a MAPE value of 0.58%. In terms of MAPE, the progress level of the proposed model has been over 50% for most of the fiscal years, and even over 60% and 70% in some years, as with the RMSE. In addition, it is seen from Table 9 that the proposed IFTS-PM produces extraordinary predictions in comparison to some of its current counterparts in the literature in terms of the MdRAE criterion, and similar evaluations and comments can be made for this criterion. The results in this table show that the proposed prediction model outperforms the naïve model as a benchmark model.

Also, when it comes to the issue what is the computational cost in time for each implementation of the TAIEX dataset, Table 7 also presents this information. The computational time has been approximately 5 to 10 s for each implementation, which can be considered a very reasonable computational cost.

In addition to all these evaluations for the TAIEX datasets, the three error criteria values obtained as a result of the implementations performed for the DIJ and SSCE datasets are summarized in Table 10 as an average. From Table 10, it is clearly seen that the proposed IFRS-PM has minimum RMSE, MAPE and the MdRAE values between all the counterpart models, as average. These implementation results show that, compared to the second-best counterpart model, the proposed model provides an improvement of over 40% on average for the DIJ and SSCE time series in terms of the RMSE criterion. For the MdRAE criterion, the improvement level has even over 60%.

Table 8 Performance evaluation for TAIEX test data in different years in terms of MAPE

Models	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	Average
[45]	3.87	2.04	1.53	2.65	0.89	0.78	0.86	1.78	1.10	0.71	1.29	1.59
[29]	3.49	2.41	1.90	2.61	0.86	1.77	3.31	0.97	3.20	8.45	4.73	3.06
[14]	1.66	0.73	1.09	1.24	0.58	0.49	0.79	0.86	0.63	0.47	1.71	0.93
[102]	2.55	0.68	0.71	1.27	0.61	0.48	0.61	0.79	0.62	0.47	0.80	0.87
[103]	2.57	1.06	0.98	1.99	0.68	0.57	0.63	1.04	0.72	0.67	1.96	1.17
[17]	2.36	0.67	0.72	1.35	0.59	0.49	0.58	0.79	0.65	0.46	0.78	0.86
[47]	2.65	0.68	0.59	1.25	0.60	0.48	0.57	0.79	0.63	0.46	0.82	0.87
[93]	2.34	0.66	0.64	1.35	0.59	0.50	0.61	0.77	0.63	0.47	0.81	0.85
[92]	2.55	0.67	0.93	1.33	0.60	0.49	0.58	0.78	0.64	0.47	0.87	0.90
[104]	2.57	1.07	0.97	2.01	0.68	0.61	0.66	0.98	0.70	0.67	1.42	1.12
[105]	1.96	0.70	0.67	1.32	0.60	0.49	0.63	0.77	0.61	0.46	0.84	0.82
[106]	2.33	0.67	0.74	1.22	0.63	0.52	0.60	0.78	0.63	0.46	0.81	0.85
[107]	2.43	0.68	0.72	1.23	0.59	0.52	0.61	0.80	0.63	0.46	0.81	0.86
[101][a]	1.73	1.33	0.74	1.43	0.59	0.51	0.74	0.90	0.61	0.58	0.92	0.93
[101][b]	1.75	0.65	0.50	1.20	0.57	0.47	0.58	0.78	0.62	0.48	0.76	0.76
[108]	2.57	0.70	0.78	1.41	0.61	0.52	0.58	0.80	0.62	0.45	0.76	0.89
[109]	2.31	0.71	1.39	1.31	0.60	0.56	0.59	0.76	0.68	0.51	1.90	1.03
[110]	2.55	0.68	0.65	1.34	0.59	0.48	0.59	0.81	0.62	0.47	0.76	0.87
[111]	2.00	0.70	0.60	1.24	0.59	0.50	0.59	0.77	0.62	0.47	0.82	0.81
[83][c]	1.87	1.01	0.60	1.42	0.91	0.62	0.39	0.98	0.78	0.50	0.92	0.91
[83][d]	1.57	0.55	0.43	0.94	0.42	0.33	0.44	0.59	0.43	0.27	0.53	0.59
The IFTS-PM	**0.58**	**0.40**	**0.19**	**0.48**	**0.14**	**0.14**	**0.17**	**0.15**	**0.21**	**0.18**	**0.20**	**0.26**
Progress (%)	63.14	27.78	56.82	49.13	65.98	58.53	57.66	74.22	51.02	33.36	63.09	56.51

The bold values indicate the best results

The results of the current counterparts models are taken from [83]

[a]The model without ICA

[b]The model with ICA

[c]The model without AEL

[d]The model with AEL

4.6.3 IEX implementation

To bring into open the prediction ability of IFTS-PM prediction model, secondly, daily IEX data observed in different 5 years from 2009 to 2013 are evaluated. In the analysis process, the observations of the last 7 and 15 days are taken as the test set for each year. Just like the implementations of TAIEX data sets, the results obtained from all models are comprehensively discussed by justifying the behaviours of all the models. Both fuzzy-based and ANN-based models have been used for comparison as well as traditional time series prediction methods. The accuracy errors of the models are represented in Table 11, in terms of RMSE.

In the light of Table 11, it is clearly seen that the proposed IFTS-PM can produce smaller RMSE values from other methods for all data sets except time series 9. Moreover, when the average statistic of RMSE relevant to all models is also considered, IFTS-PM produces superior prediction performance for all IEX time series.

On the other hand, when the proposed IFTS-PM is evaluated in terms of the improvement rates in prediction accuracy, the remarkable improvement rate of the proposed model needs to be highlighted. The proposed IFTS-PM for time series 10 and 12 has an outstanding improvement rate of 59.12% and 64.98% compared to other models suggested in the literature. For time series 13 and 14, the proposed model provides a performance improvement of more than 20% compared to the best model among other models. Moreover, in time series 6, 7, and 8, the proposed model produced a remarkable performance with a performance improvement of around 10% compared to the best model among other models. To corroborate the reached results and outstanding performance of the proposed model, the graphs, which present the error line, are given in Fig. 4.

Figure 4 shows that the lines between observations and predictions are fairly short for the all-time series. The fact that these lines, which are a measure of the error for each point of the time series, are quite short and the fact that the

Table 9 Performance evaluation for TAIEX test data in different years in terms of MdRAE

Models	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	Average
[45]	2.1448	3.0647	3.1887	2.4162	1.5793	1.5578	1.4383	2.5952	2.0856	1.5365	1.4623	2.0972
[29]	2.3294	5.1835	4.0107	2.9861	1.4906	3.6261	7.4309	1.4596	6.8543	20.8460	7.9646	5.8347
[14]	1.0254	0.9813	2.5643	0.9278	0.9387	1.0812	1.5661	1.4015	1.0697	1.1282	3.0914	1.4341
[102]	1.7855	0.9867	1.4093	1.0439	1.0565	1.0178	1.0377	0.9749	1.0053	1.0399	1.1754	1.1394
[103]	1.6624	1.3814	1.7419	1.7291	1.3326	1.1064	1.0937	1.4514	1.2785	1.3579	3.5893	1.6113
[17]	1.8415	1.0112	1.5120	1.1252	1.0032	1.0140	0.9828	1.0632	1.0527	0.9470	1.1059	1.1508
[47]	2.2061	0.9618	1.1587	1.0520	1.0805	1.0607	0.9824	1.0620	1.0020	1.0031	1.1271	1.1542
[93]	1.5375	0.9340	1.2959	1.1256	0.9341	1.1313	1.0279	0.9795	0.9720	1.0288	1.1297	1.0997
[92]	2.1805	0.9743	2.2596	1.1032	1.0622	1.0083	0.9963	1.0178	1.0164	1.0159	1.5370	1.2883
[104]	1.6624	1.4013	1.7267	2.4935	1.3261	1.1950	1.2662	1.3791	1.3113	1.3489	2.5870	1.6089
[105]	1.1617	1.0579	1.3690	1.0937	1.0442	1.0491	1.0518	0.9745	1.0116	1.0230	1.2559	1.0993
[106]	1.5489	0.9740	1.5073	1.0356	1.0546	1.1971	1.0333	0.9648	1.0158	0.9844	1.1862	1.1365
[107]	1.3179	0.9971	1.4121	1.0417	0.9861	1.1750	1.0431	1.1008	1.0303	0.9934	1.1790	1.1160
[101][a]	1.0000	2.6261	1.4819	1.4443	1.0000	1.0000	1.0000	1.0000	1.0000	1.3549	1.2929	1.3142
[101][b]	1.0499	0.8700	1.0397	0.9741	1.0261	1.0052	0.9875	0.9739	0.9991	1.1104	1.0042	1.0107
[108]	2.2481	1.0187	1.6757	1.1934	1.0406	1.1209	0.9537	1.0684	1.0034	0.9721	1.1430	1.2216
[109]	1.4611	1.0674	2.8796	1.0774	0.9531	1.3093	1.0119	0.8907	0.9581	1.1461	3.3213	1.4615
[110]	1.5865	1.0210	1.3938	1.1516	1.0469	1.0254	1.0291	1.0996	0.9989	1.0072	1.1203	1.1346
[111]	1.1915	1.0506	1.1582	1.0236	1.0164	1.0298	1.0011	0.9580	1.0377	1.0115	1.1432	1.0565
[83][c]	1.3628	1.2553	1.2433	1.4180	1.6564	0.9939	0.6507	1.2241	1.7314	0.9836	1.4136	1.2666
[83][d]	0.8938	0.7463	0.8529	0.8161	0.6368	0.7227	0.7275	0.7833	0.7738	0.5530	0.6912	0.7452
The IFTS-PM	**0.4638**	**0.6929**	**0.5354**	**0.4684**	**0.2062**	**0.3009**	**0.2588**	**0.2328**	**0.3435**	**0.3505**	**0.2724**	**0.3750**
Progress (%)	48.11	7.15	37.23	42.60	67.63	58.36	60.23	70.28	55.61	36.62	60.59	49.67

The bold values indicate the best results

The results of the current counterparts models are taken from [83]

[a]The model without ICA

[b]The model with ICA

[c]The model without AEL

[d]The model with AEL

observations and predictions are almost overlapped at many points is another indicator of the superior prediction performance of the proposed model. The values of the MAPE and MdAPE criteria presented in these graphs are also measures of the accuracy of the estimates produced by the model. Based on these measurements, it is clearly said that the proposed model produced predictions with an error of less than 0.5% for other time series except time series 7 and 15, where it produced predictions with an error of around 1%. The linear regression analysis findings, which reflect the existence of a very high linear relationship between predictions and observations, are also presented by these graphs. In these findings, the regression and determination coefficients for all-time series are quite close to 1 as expected for a successful prediction model.

Considering all these evaluation criteria and approaches, it can be said that the proposed I-FTS prediction model is a powerful and preferable candidate model for IEX prediction literature.

5 Conclusions

Intuitionistic fuzzy-based time series prediction models can produce satisfactory prediction results because they are capable of characterizing the uncertainty of the data in a comprehensive manner. In this study, a new intuitionistic fuzzy time series prediction model is proposed. The proposed model is based on the combination of predictions individually produced as nonlinear functions of memberships and non-memberships. Function approximation tool used in the model is a hybrid sigma-pi neural network, which is also proposed for the first time in this study. Totally, 48 implementations for TAIEX, DIJ, SSCE and IEX time series were carried out. The results showed that the proposed model is quite superior to other models in terms of different error criteria.

According to RMSE error criteria, approximately 20% progress has been achieved in prediction performance for the TAIEX time series in 2000–2004 years. As a result of

Table 10 Average evaluation of performance for DJI and SSEC test data

Models	DJI			SSES		
	RMSE	MAPE	MdRAE	RMSE	MAPE	MdRAE
[45]	533.41	3.12%	6.8827	150.76	5.33%	7.9442
[29]	371.15	2.19%	3.3221	132.04	4.39%	5.6223
[14]	163.33	0.90%	1.1089	37.35	1.08%	1.1180
[102]	166.53	0.93%	1.1662	38.12	1.10%	1.1452
[103]	268.60	1.58%	1.7911	67.96	1.92%	1.7404
[17]	165.50	0.92%	1.2929	36.90	1.05%	1.0959
[47]	159.84	0.87%	1.1347	37.82	1.08%	1.1530
[93]	169.34	0.94%	1.3238	38.78	1.14%	1.1516
[92]	168.96	0.94%	1.2985	38.38	1.09%	1.1563
[104]	174.96	1.01%	1.3253	38.60	1.11%	1.1551
[105]	160.67	0.86%	1.1383	36.54	1.05%	1.0698
[106]	163.39	0.89%	1.2271	36.99	1.06%	1.1114
[107]	166.04	0.92%	1.1897	37.37	1.08%	1.1583
[101][a]	177.63	0.96%	1.3015	42.81	1.26%	1.3354
[101][b]	152.48	0.81%	0.9759	36.30	1.01%	0.9981
[108]	160.21	0.86%	1.0847	37.41	1.07%	1.0654
[109]	170.66	0.94%	1.3609	38.95	1.13%	1.2981
[110]	165.77	0.93%	1.1767	37.20	1.06%	1.1182
[111]	133.17	0.46%	0.9949	31.95	0.89%	0.9272
[83][c]	164.73	0.88%	1.2563	38.79	1.17%	1.2335
[83][d]	126.74	0.68%	0.7997	23.24	0.69%	0.6951
The IFTS-PM	**73.74**	**0.28%**	**0.3328**	**12.9845**	**0.31%**	**0.2958**
Progress (%)	41.82%	38.05%	58.39%	44.13%	55.20%	57.45%

The bold values indicate the best results

The results of the current counterparts models are taken from [83]

[a]The model without ICA

[b]The model with ICA

[c]The model without AEL

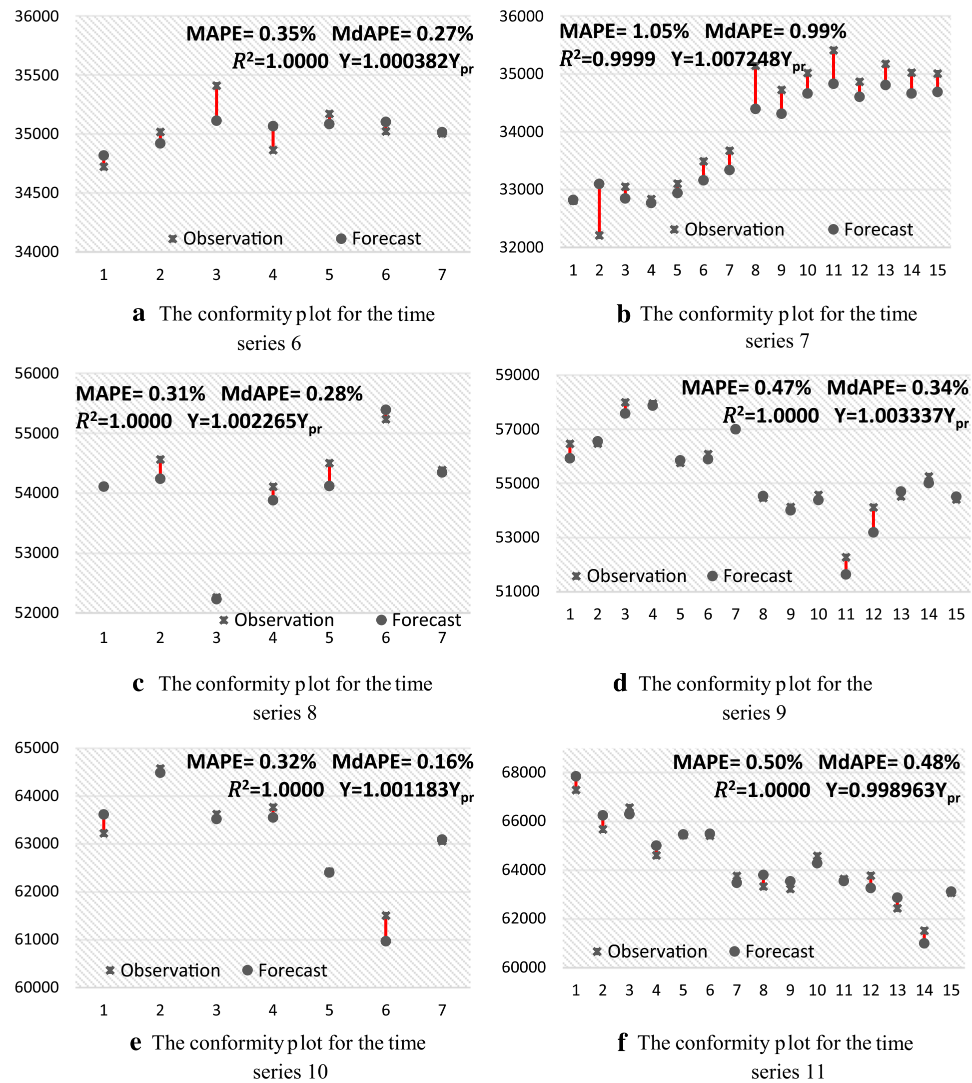
[d]The model with AEL

Table 11 The evaluation of the results in terms of RMSE for IEX data sets

	Time series/IEX data sets										RMSE's Average
	6	7	8	9	10	11	12	13	14	15	
ARIMA	345	540	1221	1612	1058	1130	651	621	1362	1269	981
ES	345	540	1208	1612	1057	1130	651	621	1362	1269	980
MLP	325	525	1077	1603	920	1096	775	783	1315	1233	965
SC	1402	1754	1128	1742	1396	1360	1292	1047	1450	1931	1450
FF-T1	446	534	1180	1852	1083	1146	1034	1038	1512	1279	1110
FTS-N	267	514	1050	1357	765	917	590	582	786	1208	804
ANFIS	405	647	1141	2033	1007	1134	634	938	1447	1413	1080
MANFIS	261	503	1144	1303	960	1009	634	629	1418	1264	913
AR-ANFIS	240	467	1136	1451	987	999	631	619	1362	1256	915
I-TSFIS	166	1046	250	**251**	817	384	277	228	451	1106	498
I-FRF	240	507	963	1390	658	994	296	530	690	1172	744
IFTS-PM	**152**	**425**	**217**	360	**269**	**373**	**97**	**173**	**356**	**1056**	**348**
Progress (%)	8.43	8.99	13.20	**NA**	59.12	2.86	64.98	24.12	21.06	4.52	30.1

The bold values indicate the best results

Fig. 4 **a** The conformity plot for the time series 6. **b** The conformity plot for the time series 7. **c** The conformity plot for the time series 8. **d** The conformity plot for the time series 9. **e** The conformity plot for the time series 10. **f** The conformity plot for the time series 11. **g** The conformity plot for the time series 12. **h** The conformity plot for the time series 13. **i** The conformity plot for the time series 14. **j** The conformity plot for the time series 15



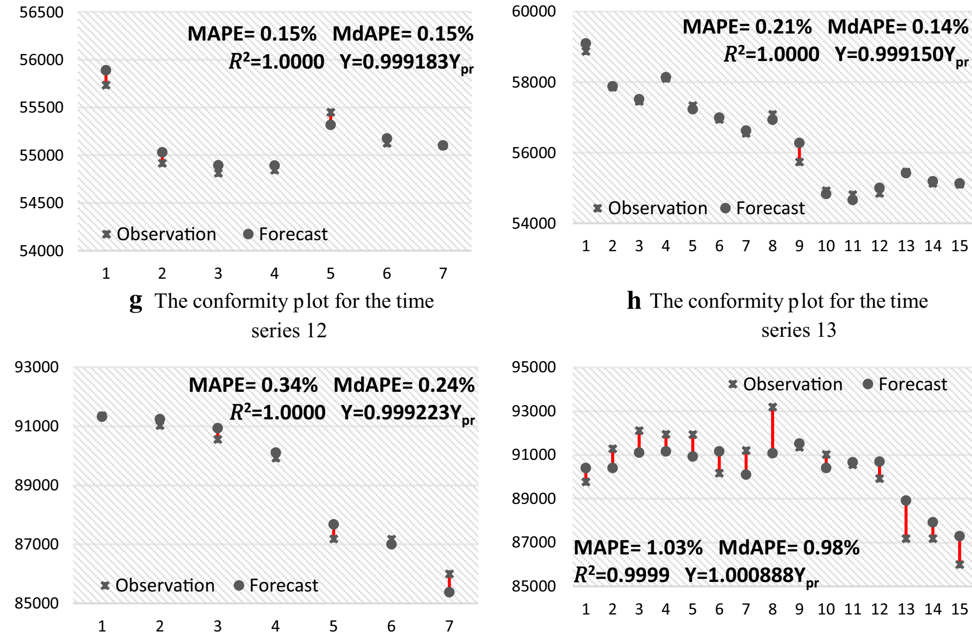
implementations realized for DIJ, SSCE, and TAIEX time series collected from 2008–2018, it is observed that the proposed IFTS-PM has outstanding performance in comparison to counterparts in the literature in terms of the RMSE, MAPE, and MdRAE. In most of the implementations, the improvement level exhibited by the proposed IFTS-PM has been around 50% and over.

Moreover, considering all the years, it has been observed that the average progress is over 30%. In 9 out of 10 implementations of the IEX time series, the proposed model improved prediction performance and even around 60% progress was made for two implementations (time series 10 and 12). When all of these 10 analyses are taken into account, an average of over 30% improvement in prediction performance is observed. Percentage error value measures, MAPE and MdAPE, for a total of 15 analyses, except for a few analyses with a value of around 1%, were obtained below 1% and even less than 0.5% in most of them. Even considering only the observation of such low MAPE and MdAPE values, the superior predictive performance of the proposed IFTS-PM becomes clear.

One of the two main elements of this study is the proposed intuitionistic fuzzy time series, prediction model. The other is the proposed hybrid sigma-pi neural network, which is introduced as a function approximation tool to model nonlinear relationships between inputs and outputs in this prediction model. The proposed prediction model considers the effects of membership and non-membership values on the predictions separately. Moreover, the proposed HSP-NN allows the linear functions of membership and non-membership values to have different weights in generating predictions. Thus, it provides a more flexible calculation and analysis process. This can be seen as the main reason for the proposed prediction model to exhibit such superior predictive performance.

In future studies, different neural network structures can be used as a function approximation tool. Moreover,

Fig. 4 continued



g The conformity plot for the time series 12

h The conformity plot for the time series 13

i The conformity plot for the time series 14

j The conformity plot for the time series 15

different functions of membership and non-membership values can be used as model inputs.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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