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Correction

Correction to: On S-multiplication modules

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ABSTRACT
In this corrigendum, we give an example showing that the implication (2) ⇒ (1) of Proposition 4 is not true in general. Also, we provide the correct version of Proposition 4.

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The following example shows that the implication $(2) \Rightarrow (1)$ of [1, Proposition 4] is not true in general.

Example 1. Consider the $\mathbb{Z}$-module $E(p) = \left\{\gamma : \gamma = \frac{m}{p^n} + \mathbb{Z}, m \in \mathbb{Z}, n \geq 0\right\}$ and the multiplicatively closed subset $S = \{p^n : n \geq 0\}$ of $\mathbb{Z}$, where p is a prime number. Then $E(p)$ is an S-multiplication module (See, [1, Example 3]). Also, we know that all submodules of $E(p)$ is of the form $G_t = \left\{\gamma : \gamma = \frac{m}{p^t} + \mathbb{Z}, m \in \mathbb{Z}\right\}$ for some $t \geq 0$. Note that $(G_t : E(p)) = (0)$ is an S-prime ideal of $\mathbb{Z}$. However, G_t is not an S-prime submodule of $E(p)$. To see this, choose $s \in S$. Then $s = p^k$ for some $k \geq 0$. Since $p^{k+t+1}\left(\frac{1}{p^{k+t+1}} + \mathbb{Z}\right) \in G_t, sp^{k+t+1} \notin (G_t : E(p))$ and $s\left(\frac{1}{p^{k+t+1}} + \mathbb{Z}\right) = \frac{1}{p^{t+1}} + \mathbb{Z} \notin G_t$, it follows that G_t is not an S-prime submodule of $E(p)$.

Now, we give correct version of Proposition 4 as follows:

Proposition 1. *Let M be an S-multiplication R-module, where $S \subseteq R$ is a multiplicatively closed set, and N a submodule of M. Suppose that $(N : tM) \subseteq (N : uM)$ implies that $(N : t) \subseteq (N : u)$ for each $t, u \in S$.Then the following statements are equivalent.*

(i) *N is an S-prime submodule of M.*
(ii) *$(N : M)$ is an S-prime ideal of R.*

Proof. $(i) \Rightarrow (ii)$: Follows from [2, Proposition 2.9 (i)].

$(ii) \Rightarrow (i)$: Suppose that $(N : M)$ is an S-prime ideal of R. Then there exists an $s \in S$ so that $xy \in (N : M)$ implies $sx \in (N : M)$ or $sy \in (N : M)$ for all $x, y \in R$. First, we will show that $(N : tM) \subseteq (N : sM)$ for each $t \in S$. Let $b \in (N : tM)$. Then we have $bt \in (N : M)$, which implies that

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$sb \in (N:M)$ or $st \in (N:M)$. The latter case is impossible since $(N:M) \cap S = \emptyset$. Thus, we have $sb \in (N:M)$, that is, $b \in (N:sM)$. Then, we conclude $(N:tM) \subseteq (N:sM)$ for each $t \in S$. Suppose that $am \in N$ for some $a \in R$ and $m \in M$. Now, we will show that $sa \in (N:M)$ or $sm \in N$. Assume that $sa \notin (N:M)$. For every $x \in (Rm:M)$, we have $ax \in (N:M)$; so $sx \in (N:M)$, which implies that $s(Rm:M) \subseteq (N:M)$. Since M is an S-multiplication module, choose $t \in S$ such that $tRm \subseteq (Rm:M)M$. This gives that $stRm \subseteq s(Rm:M)M \subseteq N$, so we have $Rm \subseteq (N:st)$. Since $(N:stM) \subseteq (N:sM)$, by assumption, we conclude that $Rm \subseteq (N:st) \subseteq (N:s)$. Thus, we have $sm \in N$, which completes the proof. □

Remark 1. In the previous proposition, the condition "$(N:tM) \subseteq (N:uM)$ implies that $(N:t) \subseteq (N:u)$ for each $t, u \in S$" is necessary. Consider the $\mathbb{Z}$-module $E(p)$. Let S be the multiplicatively closed set as in Example 1. Take the submodule $N = G_m$ for some $m \geq 1$ as in Example 1. Then $(N:E(p)) = (0)$ is an S-prime ideal of $\mathbb{Z}$. On the other hand, put $t = p^{k+1}, u = p^k \in S$ for some $k \geq 1$. Then note that $(N:tE(p)) = (0) \subseteq (N:uE(p))$. Since $\frac{1}{p^{k+m+1}} + \mathbb{Z} \in (N:t) - (N:u)$, it follows that the aforementioned condition fails in S-multiplication $\mathbb{Z}$-module $E(p)$. Also, by Example 1, N is not an S-prime submodule of $E(p)$.

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