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Meta-cognitive Aspects of Solving Indefinite Integral Problems

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Abstract

The aim of this study is to analyze the role of meta-cognition while solving indefinite integral problems. In this study, there are particularly three objectives to be obtained. These are; i) observe meta-cognitive behaviors of prospective mathematics teachers while solving problems, ii) determine the importance of meta-cognition in problem solving and iii) determine challenges, typical errors and points to be considered for solving indefinite integral problems. The study is a qualitative research. Data collection tools are; i) Problems Form (PF) and ii) Meta-Cognitive Skills Questionnaire (MSQ). In this context, this study was completed by 60 prospective mathematics teachers. All prospective mathematics teachers are sophomore students at a university. Data collection tools were filled by the prospective mathematics teachers in 50 minutes. Collected data were analyzed descriptively. The analysis of the problem which were solved by the prospective mathematics teachers and the examination of completed questionnaires are to provide an understanding for the meta-cognitive aspects of solving indefinite integral problems. In conclusion, the challenges faced by prospective mathematics teachers that are subject to recall, choose incorrect solution and not knowing where to start in problem solving. The common mistakes are incorrect solutions and calculation errors. On the other hand, the point that should be considered in the problem solving is to choose a strategy.

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1. Introduction

Meta-cognition is defined as all the conscious, cognitive and emotional experiences which accompany a mental initiative (Flavel, 1979), individual's thinking about his/her own thoughts (Biryukov, 2004; Pierce, 2003), observing

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one's own mental activities (Pugalee, 2001), recognizing and observing your own cognitive system and functions (Panaoura & Philippou, 2005), realizing your own thoughts and ability to evaluate and edit them (Wilson, 2001), a person's ability to make correct inferences regarding how to implement a strategic information in a special case (Pierce, 2003), acquiring an information or awareness regarding that information (Rivers, 2001). Meta-cognition involves awareness regarding the learning process, planning, choosing a strategy, monitoring learning process, to be able to correct one's own mistakes, to be able to check whether the strategies that you are using is useful or not, to be able to change your learning method or strategy when necessary (Ozsoy, 2007). In addition to that meta-cognition plays an important role in fields such as communication, reading comprehension, learning a language, social cognition, attention, self-check, memory, self-teaching, writing and problem solving (Flavel, 1979).

In this sense, there are two meta-cognitive skills, which are self-monitoring and planning, in problem solving process (Biryukov, 2004). According to Tobias and Everson (2000) the basic component in problem solving process is meta-cognition and meta-cognitive skills ensure an awareness and control in problem solving process. In addition to this, they also state that meta-cognition connects cognitive behaviors with non-cognitive factors which exist at each phase of the problem solving. At this point, students' asking questions to themselves and monitoring their own behaviors in problem solving process are also considered as meta-cognitive skills (Gourgey, 1998). According to Costa (1984), the awareness regarding the preferred strategy and problem solving steps in problem solving process is also evaluated as a meta-cognitive skill.

At this point meta-cognition contributes problem solving as it provides strategic studies (Pugalee, 2001). Having meta-cognitive skills and knowing how to implement these skills will enable us to be successful in problem solving (Biryukov, 2004). According to Kapa (2001) meta-cognition can affect cognitive tasks in each phase of the problem solving as it is given in Table 1.

Table 1. Meta-cognitive functions classified according to the process phases

Solving-phase	The meta-cognitive function
Problem identification	Collecting data, coding and remembering
Problem representation	Analogy, inference, imaginativeness, selective comparison and combination
Planning how to solve	Integration, conceptualization, heuristic choosing, and formulating
Planning performance	Controlling and monitoring performance components of algorithmic mathematical knowledge and appropriate rules
Evaluation	Adjusting and contradicting a few possible solutions or suggesting alternative solution methods

In this sense, plenty of studies were carried out in order to determine the correlation between meta-cognition and problem solving (Culaste, 2011; Deseote, 2008; Deseote & Roeyers, 2002; Deseote, Roeyers & Buysee, 2001; Goldberg & Bush, 2003; Ifenthaler, 2012; Kapa, 2001; Panaoura & Philippou, 2005, 2007; Pugalee, 2001; Schurter, 2002; Sperling, Howard, Miller & Murphy, 2002; van der Stel & Veenman, 2008; Zan, 2000). When the studies stated above were analyzed, it was determined that there is a significant correlation between problem solving and meta-cognition, the problem solving skills are increased by improving meta-cognitive skills and successful problem solvers have meta-cognitive skills.

In this regard, the purpose of this study is to analyze the role of meta-cognition in solving indefinite integral problems. Besides, in accordance with this purpose, the aim is to; i) emphasize the importance of meta-cognition in problem solving process by observing prospective mathematics teachers' meta-cognitive behaviours in problem solving process and ii) to determine the difficulties, typical errors and points to be considered in solving indefinite integral problems.

2. Method

2.1. Research Design

This study which was carried out for demonstrating the results of a particular situation is a qualitative case study. The purpose here is to present results regarding a particular case (Yıldırım & Simsek, 2008). At this point, in this the

study, results about revealing the meta-cognitive aspects while solving indefinite integral problems are demonstrated.

2.2. Study Group

The study was carried out with 60 prospective primary school mathematics teachers who are studying in Kocaeli University, Education Faculty. 49 of the prospective teachers are female (81,67%), and 11 of them (18,33%) are male. All of the prospective mathematics teachers are sophomore students at the university.

2.3. Data Collection Tools and Collecting Data

Data collection tools are; i) Problems Form (PF) and ii) Meta-cognitive Skills Questionnaire (MSQ). The form regarding the problems used in this study involves two indefinite integral problems (Sengul, 2010). In the first problem, it is possible to get the result through making algebraic operations only by using change of variable feature. In the second one, the case is first to analyze the data of the given problem and then to adapt them into simple fractions. Later on, the obtained findings are transferred to the given problem and find the solution. The second problem involves more information and information transfer than the first one. For this reason, it requires more thinking. The meta-cognitive skill questionnaire was adopted from Biryukov (2004). The questionnaire consists of 14 questions which are defining cognitive and meta-cognitive behaviours. First of all, the problems form was distributed to the prospective teachers and they were given 40 minutes to complete this form. In accordance with the purpose of this study, after the prospective teachers had solved the given two problems, 'meta-cognitive skills questionnaire' was distributed. The prospective teachers were given 10 minutes to fill in this questionnaire. The study lasted 50 minutes in total. At the end of the time, all the distributed forms and questionnaires were taken back from the prospective teachers.

2.4. Data Analysis

The data collected as a result of the implementation were analyzed descriptively. In descriptive analysis, the data can be presented by considering the used questions or dimensions (Yıldırım & Simsek, 2008). In this sense, at the first phase of the conducted analysis, the problem solving of the prospective teachers were analyzed. After that, the items of the questionnaire that the prospective teachers filled in regarding their problem solving were analyzed one by one. The purpose of this kind of analysis is to present the obtained findings to the readers in an organized and interpreted way (Yıldırım & Simsek, 2008). For this reason, the findings were demonstrated clearly and systematically on the basis of frequency (f) and percentage (%).

3. Findings and Comments

At the first stage of the study, two indefinite integral problems were given to students. The following findings were observed after the students' responses had been analyzed within this context; 20 (33,33%) students solved both problems, 10 (16,67%) students solved only the first problem, 24 (40%) students solved only the second problem, 6 (10%) students could not solve any of the problems. Immediately after the students had solved the problems, they filled out a questionnaire, which defined their cognitive and meta-cognitive behaviours while solving problems. The results of the analysis of questionnaire and problems are provided below;

1 st Item of the Questionnaire	Yes	No	Not Sure
I read the problem more than once.	36	22	2

60% of the students answered this item as 'yes'. 11 students from among those who answered this items as 'yes' also solved both problems, while 3 of them could not solve any of the problems, 7 of them solved the first problem and 15 of them solved the second problem. After solution protocols, it was seen that 8 students out of those who answered this item as 'no' solved both problems, 3 of them solved only the first problem, 8 of them solved only the second problem and 3 of them could not solve any of the problems.

2 nd Item of Questionnaire	Yes	No	Not Sure
I checked what the problem was asking to me.	48	3	9

The expression in the second item demonstrates meta-cognitive and self-regulation behaviours. At the same time, it questions whether the problem is properly understood or not. At this point, 80% of the students answered this question with ‘yes’. 17 of the students who answered with ‘yes’ could solve both two problems. While 8 of them could solve only the first problem and 17 of them could solve only the second problem, 6 of them could not solve any of the problems. While only one student who answered with ‘no’ could solve both two problems, the other two could only solve the second problem.

3 rd Item of the Questionnaire	Yes	No	Not Sure
I evaluated how much time I need for solving this kind of problem.	14	30	16

50% of the students answered this item with ‘no’. It was seen that half of them did not consider any time limit for solving the problems. 12 of the students who answered with ‘no’ could solve both two problems. While 6 of them could solve the first problem and 9 of them could solve the second one, 3 of them could not solve any of the problems. 4 of the students who answered with ‘yes’ could solve both two problems. While 3 of them could solve only the first problem and 6 of them could solve only the second one, one student could not solve any of the problems.

4 th Item of the Questionnaire	Yes	No	Not Sure
I showed the problem schematically.	15	35	10

Creating models by individuals with their own experiences is an example for meta-cognitive behaviour and is helpful for understanding the relationship between the conditions and the problem to be solved. In this regard, as a result of the analysis, while 15 students said ‘yes’, 35 students said ‘no’ and 10 students said “not sure”. 5 of the students who answered with ‘yes’ solved both two problems. While one could only solve the first problem and 8 of them could solve only the second one, one of them could not solve any of the problems. 12 of the students who answered with ‘no’ solved both two problems. While 5 of them could solve only the first problem, 13 of them could solve the second one. 5 of them could not solve any of the problems.

5 th Item of the Questionnaire	Yes	No	Not Sure
I tried to recall whether I solved a similar problem before or not.	54	4	2

Four of the students answered this item as ‘no’. While one of them solved both problems correctly, one of them solved only the first problem, and one of them solved only the second problem successfully. One of them could not solve any of the problems. 19 students among those who answered this item with ‘yes’ solved both problems successfully. While 5 of them could not solve any of the problems, 9 of them solved only the first problem and 21 of them solved only the second problem successfully.

6 th Item of the Questionnaire	Yes	No	Not Sure
I developed a strategy for solving the problem.	53	1	6

Meta-cognitive analysis and control provide developing a strategy for achieving targets. In this regard, 20 of them who answered this item with ‘yes’ solved both two problems. While 9 of them could solve only the first problem and 19 of them could solve only the second problem, 5 of them could not solve any of the problems. The students who answered with ‘no’ could only solve the second problem.

7 th Item of the Questionnaire	Yes	No	Not Sure
I did not know where to start.	8	34	18

Two of the students out of those who answered this item as ‘yes’ solved both problems successfully, and while 2 of them solved only the first problem successfully, three of them solved only the second problem. The other one

could not solve any of the problems. 14 students out of those who answered this item as ‘no’ solved both problems successfully. While 4 of them solved only the first problem successfully, 12 of them solved only the second problem successfully. 4 of the students could not solve any of the problems.

8 th Item of the Questionnaire	Yes	No	Not Sure
I faced some difficulties while solving problem (If your answer is ‘yes’, define the features of the difficulty).	22	24	14

4 of the students who answered with ‘yes’ to the eight question solve both two problems. While one could only solve the first problem, 12 of them could solve only the second problem. 5 of them could not solve any of the problems. 22 students who answered with ‘yes’ also expressed the difficulties that they faced during problem solving process. They are as in the following, for instance; *“I had difficulties to remember what I know”, “I could not decide from where to start solving the problem”, “I could not solve the problem through the way I thought”, and “The reason why I had difficulties is that I could not remember the features of integral for a moment”*. 11 of the students who answered with ‘no’ could solve both two problems. While 6 of them could solve only the first problem, 6 of them could solve only the second one. Only one student could not solve any of the problems.

9 th Item of the Questionnaire	Yes	No	Not Sure
I found one of my mistakes while solving the problem and corrected it (If your answer is ‘yes’, define the mistake).	19	34	7

6 of the students who answered with ‘yes’ to the ninth item solved both two problems. 3 of them solved only the first problem and 8 of them solved only the second problem. 2 of them did not solve any of the problems. At this point, students expressed the difficulties that they faced as in below.

- “I chose the incorrect solution”,*
- “Operational mistakes”,*
- “I had an operational mistake. I corrected it”,*
- “I realized that I had made an operational mistake and I corrected it”.*

It was determined that only 12 students who answered with ‘no’ to this question could solve both two problems. While 5 of them could solve only the first problem, 13 of them could solve only the second problem. And 4 of them could not solve any of the problems.

10 th Item of the Questionnaire	Yes	No	Not Sure
I thought how the solution was going on.	55	2	3

It is important that 91,67% of the students answered this item as ‘yes’, which shows the cognitive evaluation towards a solution. 19 of the students out of those who answered ‘yes’ solved both problems. While 5 of them could not solve any of the problems, 9 of them solved only the first problem, and 22 of them solved only the second problem. While one of the students who answered with ‘no’ solved both two problems, the other one could not solve any of the problems.

11 th Item of the Questionnaire	Yes	No	Not Sure
I tried different approaches to solve the problem.	36	20	4

Eleven students out of those who answered this item with ‘yes’ solved both problems successfully. 5 of students could solve only the first problem and 15 of them could solve only the second one. 8 students from the ones who answered this question with ‘no’ solved both two problems. While 5 of them could solve only the first problem, 7 of the could solve only the second one.

12 th Item of the Questionnaire	Yes	No	Not Sure
I asked myself whether my answers were meaningful or not.	48	6	6

While solving mathematical problems, it is important to check whether the answers are meaningful or not. This is an example for a meta-cognitive behaviour. In this regard, 80% of the students answered this item as ‘yes’. 17 students out of those who answered this item with ‘yes’ solved both problems successfully. While 9 of them solved the first problem successfully, 19 of them solved the second problem successfully. 3 of them could not solve any of the problems. While 2 of the students who answered with ‘no’ could solve both two problems, one of them could not solve any of the problems. 3 of them could only solve the second problem. ‘

13 th Item of the Questionnaire	Yes	No	Not Sure
I checked my calculations to be sure that they were correct.	41	12	7

Fifteen students out of those who answered this item with ‘yes’ solved both problems successfully. Seven of them solved only the first problem and sixteen of them solved only the second problem successfully. 3 of them could not solve any of the problems.

14 th Item of the Questionnaire	Yes	No	Not Sure
I thought whether there was something that I should especially pay attention or not in the information given in the problem (If your answer is ‘yes’, define it).	19	19	22

7 of the students who answered with ‘yes’ to the fourteenth question solved both two problems. 3 of them could only solve the first problem and 6 of them could only solve the second one. 3 of them could not solve any of the problems. At this point, students expressed the points that they paid attention as in the following.

“I paid attention how I would find the solution”,

“I thought for finding a correct way to get the solution”,

“I thought whether the question could be solved by using a special formula”.

8 of the students who answered with ‘no’ solved both two problems. While one could only solve the first problem, 9 of them could solve only the second one. Only one student could not solve any of the problems.

4. Conclusion, Discussion and Implications

The analysis of the data about solving indefinite integral problems showed the importance of demonstrating cognitive experiences. Meta-cognition means planning, monitoring and evaluation of one’s own learning. In this regard, someone who has meta-cognitive skills has also a skill both to explain the information about cognition and to ensure to regulate this information (Flavel, 1979). Planning, understanding and observing how to approach a given task and evaluation for completing the task or to be able to make changes for new situations are all related with meta-cognition (Panaoura & Philippou, 2005; Pugalee, 2001). According to Hartman (2001) there are two general types of meta-cognition. The first one is executive management strategies which help someone to plan, monitor, evaluate and regulate the thinking processes. The second one, strategic knowledge includes knowing what information/strategies/skills you have, when and why to use them and how to use them. At this point, the person who has meta-cognitive skills makes a plan for finding which information is necessary for him/her. In addition to this, he/she solves problem, be aware of the strategies that are being used and uses self-in-depth thinking and evaluation processes actively with his/her performance (Costa, 1984). In this study, it was seen that 20 students who could successfully solve both two problems responded with ‘yes’ for the 2nd, 6th, 10th and 12th questions of the questionnaire. In this sense, it can be said that students utilized the meta-cognitive behaviours that make them successful while solving problems. According to Gagne, Briggs and Wager (1988) meta-cognition is internal processes which utilize cognitive strategies for monitoring and controlling learning process. These processes select and edit mental skills about problem solving and can create cognitive strategies about learning. At this point, the fact that students who solved both two questions successfully answered ‘yes’ for the 2nd and 13th questions can mean that they used their meta-cognitive skills. In the previous studies, it was found out that there was a significant correlation between meta-cognition and problem solving. Besides, it was seen that meta-cognition increased the success in problem solving and students who have meta-cognitive skills were able to organize their mental processes more actively (Culaste, 2011; Goldberg & Bush, 2003; Flavel, 1976; Schoenfeld, 1985; Ozsoy, 2011; Schurter, 2002). At

this point, it can be said that also in this study, students who solved both two problems successfully used their meta-cognitive skills. In this sense, the correlation between problem solving and meta-cognition was demonstrated once again in this study. It can be said that students who are using their meta-cognitive skills are more successful in problem solving.

There two important products of problem solving. The first one is the development of strategies and rules which are specific to the subject taught. The second one is the development of thinking processes that can be used for improving a rule or a formula (Olkun & Toluk, 2004). It is thought that the 6th question which is about developing a strategy at the questionnaire used in this study is important in terms of demonstrating meta-cognitive behaviours. In this sense, it was determined that all the 20 students who solved both two questions successfully responded this question with 'yes'. In this case, it can be said that these students who used their meta-cognitive skills are successful in problem solving. At this point, it can be said that the correlation between problem solving and meta-cognition is observed once again. In a study which was carried out before by Sengul and Katranci (2012), the meta-cognitive behaviours while solving problems about function were analyzed. Similarly, Biryukov (2004) analyzed the meta-cognitive behaviours while solving problems about combinatorials. At this point, it is suggested to choose different mathematical subjects and analyze meta-cognitive behaviours while problem solving processes about these subjects. This study was carried out with prospective teachers. When it is considered that prospective teachers will teach at middle school levels, it is thought that repeating this study by choosing a mathematics subject from middle school mathematics subjects will bring this study another perspective. Thus, it can be informative at the point that to what extent prospective teachers will focus on questioning processes about their students' meta-cognitive information and whether they are using their own meta-cognitive skills or not about the problems regarding mathematics subjects that they will encounter in their future professional life. In addition to this, it is predicted that repeating a similar study with high school and middle school students will be helpful.

Another purpose of this study is to determine the difficulties that prospective teachers have in the process of solving problems about indefinite integrals. In this sense, the difficulties of the prospective teachers are appeared as in the following; unable to remember the subject, choosing the wrong way for the solution and unable to know from where to start for solving the problem. The questions that are needed to be answered for having meta-cognition in an individual are; What do I know about this subject?, Did I understand what I saw, read and heard?, How can I review my plans if my expectations are not met? etc. (Soydan, 2001). In this regard, the fact that prospective teachers are aware of the difficulties that they had and be able to express them can mean that they questioned their meta-cognitive knowledge. In this case, the awareness of prospective teachers whether they question their meta-cognitive knowledge or not can be analyzed by focusing on this point. At this point, it is suggested to carry out a new study about this subject.

The other purpose of this study is to determine the errors made in problem solving process. In this sense, the errors are; preferring the wrong solution strategy and operational mistakes. Meta-cognition involves one's ability to control problem solving process (Biryukov, 2004). In addition to this, it provides problem solver the opportunity to reduce the number of repeated mistakes and to determine the failures about the strategy (Lipson, 1995). In this sense, the fact that prospective teachers noticed the mistakes that they had in problem solving process can mean that they used their meta-cognitive skills. Here, it can be questioned whether prospective teachers will recognize their own mistakes in the process of solving different mathematics problems by having more detailed studies. During this questioning process, whether they are aware of their meta-cognitive knowledge or not can be analyzed. The point that should be paid attention in solving problems about indefinite integrals was determined as choosing the solution strategy. The meta-cognition includes analyzing which strategy is used and whether the selected strategy will lead you to reach your goal or not. At the same time, it requires to think over about how to implement the solution method. There is a judgement process for the solution and for the process which leads you to the solution in meta-cognition (Desoete, 2001). It is outstanding in this study that prospective teachers focused on the solution strategy. Here, it can be said that prospective teachers are aware of the meta-cognition. However, it is not true to reach a definite conclusion. Therefore it is recommended to repeat this study with a different study group and to analyze the obtained results once again.

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