Expert Systems With Applications 215 (2023) 119336

Contents lists available at ScienceDirect

Expert Systems With Applications

journal homepage: www.elsevier.com/locate/eswa

A novel intuitionistic fuzzy time series prediction model with cascaded structure for financial time series

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ARTICLE INFO

Keywords:
Financial time series
Prediction
Cascade forward neural network
Intuitionistic fuzzy C-means

ABSTRACT

Financial time series prediction problems, for decision-makers, are always crucial as they have a wide range of applications in the public and private sectors. This study presents a cascaded intuitionistic fuzzy model for financial time series prediction. The proposed prediction model has the ability to jointly and simultaneously model linear and nonlinear relationships in financial time series. Thus, it can adapt itself to both linear and non-linear surfaces of the data and can produce satisfactory predictions for financial time series. Moreover, the other reason why to be produced better predictions, the proposed model reckons non-membership degrees in addition to membership degrees in the prediction process. With these aspects, the proposed prediction model is different and superior to all models in the literature. This superiority has been proven by the analysis of 48 different financial time series containing TAIEX, DIJ, SSEC, and IEX data sets. The results have been evaluated in terms of RMSE, MAPE, and MdRAE metrics and some other perspectives as well. The proposed prediction model has achieved progress in prediction performance, up to 80% for TAIEX 2000–2004 datasets, 60% for TAIEX 2008–2018 datasets, approximately 50% for DJI and SSEC, and up to 70% for IEX. All the discussed indicators demonstrated the outstanding prediction performance of the proposed cascaded intuitionistic prediction model compared to some other state-of-the-art prediction tools.

1. Introduction

The fact that it is of both theoretical and practical substantiality and affluence, in fact, makes time series forecasting models more amenable to use in broad research areas such as finance, health, environment, and energy. In prediction problems of financial time series, in order to reach accurate and sensible decisions about the future, obtaining an accurate picture of the future is a vital and main aim. The way to obtain satisfactory prediction results is only possible by using a suitable and competent prediction tool. Prediction models introduced in the literature can be classified under two subtitles: probabilistic and non-probabilistic models. The traditional statistical models constitute the probabilistic models. Moreover, the fuzzy-based systems and the computational-based systems containing machine learning and deep learning, etc. constitute the non-probabilistic models. Although statistical-based models are well-known and widely used for time series modelling, they need some strict assumptions to be satisfied. Non-probabilistic models do not require these strict assumptions. Some other prediction models as artificial neural networks (ANNs) and support vector machines are the non-probabilistic approaches that have been researched and used as viable alternatives to tackle complex dynamic processes. Also, fuzzy-based prediction models are effectively used as non-probabilistic prediction tools when the data sets are vague and have linguistic terms. Considering that almost all the daily life time-series datasets contain uncertainty, the interest in fuzzy time-series models is inevitable. On the other hand, the usage of fuzzy neural networks (FNNs), which combines the reasoning ability of the fuzzy-based systems with the capability of adaptively adjusting parameters of the neural networks, increases the efficiency and applicability of the prediction models.

Although fuzzy time series models have a large literature and unique features, they still have some problems to be solved. The elementary fuzzy time series model, based on Zadeh's fuzzy set theory (Zadeh, 1965), has been proposed by Song and Chisom (Song & Chissom, 1993a). Most fuzzy time series (FTS) models use Zadeh's fuzzy set to fuzzification the data. As a result of this process, only one attribute shows up: memberships which measure the subjection degree. This is neither objective nor comprehensive, and consequently limits the FTS

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https://doi.org/10.1016/j.eswa.2022.119336
Received 8 July 2022; Received in revised form 30 October 2022; Accepted 20 November 2022
Available online 24 November 2022

models in characterizing the uncertainty of the data in a comprehensive manner and improving prediction accuracy. This means that in the analyses process both the neutrality (hesitation) degree and the non-membership degree of the time series are not taken into consideration. As it is known, in the decision-making process, judgments are commonly under the influence of different factors, so hesitations always exist. From this point of view, obtaining negatively affected prediction results will be inevitable if the model is incapable of dealing with this situation. So, based on this information a new fuzzy set definition called an intuitionistic fuzzy set, was presented (Atanassov, 1986). The intuitionistic fuzzy set consists of memberships and hesitation degrees together. In this way, the usage of information relevant to both membership and non-membership degrees would cause to get more realistic results. In the limitedly intuitionistic fuzzy time series literature, in most of the studies, the relations between inputs and outputs are determined according to fuzzy rules or with the help of ANN models. ANN types used to determine fuzzy relationships in these models only reveal the nonlinear relationship between input and output. However, the relationship between input and output may not always be nonlinear. So, this situation is a gap in fuzzy literature. This study focuses on filling this gap.

In this study, we aimed to model the linear and nonlinear relationships between inputs and outputs together. So, combining these two relationships will provide a more flexible computational approach. And, as expected, it will lead to get better prediction performance. From this point of view, we have utilized a cascade forward neural network in the determination of intuitionistic fuzzy relations. A cascaded forward neural network can accommodate the nonlinear relationship between input and output without eliminating the linear relationship between the two. Thus, both linear and non-linear relationships that may occur have been taken into consideration. To adapt the model to the solution surfaces of the data, including both linear and nonlinear relationships, we propose a cascade-based intuitionistic fuzzy time series prediction model (CIPM). The distinguished aspects of the proposed model are given below:

- Approach to uncertainty: It reckons with non-memberships and hesitation degree.
- Model structure: Holds a cascading model where each layer uses the information available in the previous layers.
- Modelling: Considers both linear and non-linear relationships between inputs and outputs and is capable of modelling them together.

A comprehensive analysis has been performed to show the effectiveness and performance of the proposed model on financial time series data sets. In the analysis process, various financial time series have been evaluated. The estimation performance of the proposed model has been shown over various financial data sets. These data sets are the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX), Dow Jones Industrial Average Index (DIJ), Shanghai Securities Composite Index (SSEC), and Istanbul Stock Exchange (IEX). The obtained results have been evaluated thoroughly by comparing the current state-of-the-art prediction methods. In the comparative evaluation part, different error measures, some graphs, and the basic properties of the linear regression analysis have been employed.

The remaining sections of the paper are organized as follows: Section 2 presents a thorough review of the related literature. Section 3, as the preliminary concepts, consists of some basic definitions, the working principles of intuitionistic fuzzy C-means clustering algorithm, and the main structure of the cascade forward neural network. Section 4 introduces the methodology of the proposed prediction model. Section 5 summarizes the characteristics of data sets, the scenarios of implementations, and the evaluation perspectives. Section 6 presents the compilations of implementation results and elaborate discussions related to them. Finally, in the last section, the conclusions are summarized together with perspectives on the future.

2. Literature review

Time series prediction has a grand scope containing the areas such as financial market prediction, electricity service load prediction, and weather and environmental prediction. Given all these, time series prediction is quite essential for setting policies and strategies for all institutions, operations, and even governments of countries. In terms of determining policies and strategies, the underlying system models and time-series data processing processes need to be known in order to provide logical perspectives going forward. So many prediction models have been introduced in the literature to determine the right strategies. These well-known traditional time series models do not always manage to produce satisfactory predictions. In these conditions, the prediction process requires more advanced time series prediction algorithms such as fuzzy-based and computational-based models. Among these advanced models, fuzzy time series models have become attractive models. The ability to characterize both the uncertainty of the data and the used linguistic terms makes FTS models the frequently preferred estimation tool.

In most of the studies on the FTS model, interval lengths are determined as constant. On the other hand, in some studies, equal interval lengths are determined arbitrarily (Chen, 1996; Chen, 2002; Song & Chissom, 1993b; Song & Chissom, 1993a; Song & Chissom, 1994). Also, in later studies, the average and distribution-based methods (Huarng, 2001a) and the optimization-based methods (Egrioglu et al., 2010) have been used. Non-constant interval lengths have been determined with a ratio-based approach (Huarng & Yu, 2006a) and with the idea of optimization of the ratio (Yolcu et al., 2009). The usage of heuristic optimization algorithms has a significant effect on prediction results (Cheng et al., 2016; Chen & Chen, 2011; Gangwar & Kumar, 2012; Kuo et al., 2010; Lee & Chou, 2004; Lee et al., 2007; Lee et al., 2008). For the high-order prediction models, in some studies, entropy-based and other methodologies have been chosen to specify the interval lengths (Chen & Chen, 2015; Chen, 1996; Liu, 2007; Zhao & Yang, 2009). Especially in recent years, fuzzy C-means (FCM) (Bezdek, 1981) and some other clustering techniques have been utilized to be able to realize fuzzification in a more systematic way (Aladag et al., 2012; Cheng et al., 2016; Cheng & Li, 2012; Chen et al., 2012; Dos Santos & De Arruda Camargo, 2013; Eğrioğlu, 2012; Li & Cheng, 2010; Wei et al., 2014; Yolcu et al., 2013). While in early studies fuzzy logic relation matrix has been used to determine the fuzzy relations (Song & Chissom, 1993a; Song & Chissom, 1993b; Song & Chissom, 1993c)subsequently, the use of transition matrices has been preferred (Sullivan & Woodall, 1994) After a while, ANNs became popular and attractive as fuzzy relation determination tool in the FTS modelling process (Aladag et al., 2009; Aladag et al., 2010; Chen & Chen, 2015; Chen & Chen, 2015; Egrioglu, 2014; Egrioglu et al., 2009a; Egrioglu et al., 2009b; Lee & Hong, 2015; Wang & Xiong, 2014; Wei et al., 2014; Yu & Huarng, 2008).

On the other hand, in some following research, all stages of the FTS model have been evaluated together. In order to reduce the stock price prediction error, a study that simultaneously evaluates the stages in a single process has been presented (Cagcag Yolcu & Alpaslan, 2018), while a hybrid model has also been put forward (Cagcag Yolcu & Lam, 2017). Moreover, for the analyses of the TAIEX data set, a new fuzzy prediction model has been introduced (Kocak et al., 2020).

Like the FTS models, adaptive network-based fuzzy inference system (ANFIS) models are the other most preferred prediction model. An ANFIS-based model has been used to predict automobile sales (F.-K. Wang et al., 2011). A hybrid ANFIS has been put forward to predict the TAIEX data sets (Chang et al., 2011). An ANFIS has been introduced for the prediction of the stock market return (Boyacioglu & Avci, 2010). A study has been presented which evaluates the performance of ANN and hybrid ANFIS model for the prediction of energy consumption (Li et al., 2011). A new fuzzy inference system based on ANFIS has been introduced for time series prediction (Egrioglu et al., 2015) Iran's annual energy consumption has been predicted using the ensemble approach of

ARIMA and ANFIS (Barak & Sadegh, 2016).

Moreover, there are lots of studies using different kinds of ANNs have been used in time series prediction problems (Cao et al., 2019; Jin et al., 2022) and a systematic review for time series prediction using ANNs methodologies was given by Tealab (Tealab, 2018).

It is also possible to talk about some shortcomings of all these models. In all these models, the neutrality (hesitation) degree of the time series is not considered. And also, in this case, used fuzzy sets may fail to characterize the uncertainty of the data in a comprehensive manner. These models basically depend on classical fuzzy set terms in which only the membership degrees are evaluated. Not considering the membership and the non-membership degrees together also adversely affects the prediction performance. In these cases, the creation of the intuitionistic fuzzy set-based prediction model will lead to a significant improvement in time series prediction performance. Because such models also consider the non-memberships based upon neutrality (hesitation) degree.

In the literature, a limited number of intuitionistic fuzzy set-based models have been proposed for financial time series prediction. In a few studies, based on intuitionistic fuzzy sets, non-membership degrees, as well as memberships, have been used. An intuitionistic fuzzy time series prediction study has been proposed to predict the University of Alabama enrolments and the TAIEX data sets which are the most preferred data sets in time series prediction (Wang et al., 2016). On the other hand, for stock market prediction, a fuzzy inference system based on intuitionistic time series has been introduced (Egrioglu et al., 2019a). In some studies, different real-world time series have been analyzed by utilizing intuitionistic fuzzy functions (Cagcag Yolcu et al., 2020; Kizilaslan et al., 2020). Besides these approaches, to predict network traffic long-term intuitionistic fuzzy time series has been presented (Fan et al., 2020). An intuitionistic fuzzy set-bases prediction model using an artificial bee colony algorithm has been proposed (Egrioglu et al., 2019b). With the aim of predicting financial time series, a fuzzy regression function approach using intuitionistic fuzzy sets has been introduced (Bas et al., 2021). In addition, to predict TAIEX, DJI, and SSEC data sets, an advanced fuzzy time series prediction model has been proposed (Dong & Ma, 2021).

Almost all of the limited numbers of intuitionistic fuzzy sets-based models provide an effective tool for modelling uncertainty by considering both the membership and the non-membership degrees together. However, these studies only deal with nonlinear relationships between inputs and outputs and reveal them. None of these considers linear relationships between inputs and outputs. It is almost inevitable to face time series with both linear and nonlinear structures and relationships. In these cases, evaluating both relations, which the data sets possess, will provide better modelling and prediction performance. Using the cascade forward neural network (Demuth & Raele, 2009) as a prediction tool ensures this improvement and progress. Because cascade forward neural network (CFNN) has the ability to model both linear and non-linear relationships thanks to its architectural structure. In the literature, the CFNN has been used in some areas. A hybrid model has been presented that integrates the CFNN with Elman Neural Network to take advantage of both networks with signals moving in both directions (Alkhasawneh & Tay, 2018). For time series prediction, a CFNN has been introduced in which the optimal architecture is computationally determined by using the incremental search method in both input and hidden units (Warsito et al., 2018). On the other hand a simplified cascade model has been developed to predict the critical flow for the critical mass flux and critical pressure (An et al., 2020). To predict the water and ammonium-based ionic liquids' density and speed of sound, a CFNN has been evaluated (Zimmermann & Mattedi, 2020). A hybrid model combining CFNN and PSO has been proposed to cover the energy demand. (Chiñas-Palacios et al., 2021).

From the literature on the CFNN, although it seems that researchers profited from the CFNN in various scientific and academic disciplines, there are not enough studies on financial time series prediction. Furthermore, with the aim of financial time series prediction, there is no study using CFNN in fuzzy time series or intuitionistic fuzzy time series. Therefore, one of our aims in this study is to fill this gap in the literature. In this context, a novel cascaded intuitionistic fuzzy time series prediction model has been introduced and the prediction performance of the CFNN has been evaluated in the prediction of different financial time series.

3. Preliminary concepts

In this section, the preliminary concepts of the study are given, such as the intuitionistic time series definitions and prediction models of them, and the clustering algorithm used to determine the degrees of membership and non-membership. In addition, a cascade forward neural network is introduced that allows the prediction model to determine linear and non-linear relationships simultaneously.

Definition 1. (*Intuitionistic Fuzzy Time Series*) *Let Y_t be a time series with real observations. $IFS_1, IFS_2, \cdots, IFS_c$ are intuitionistic fuzzy sets identified upon a universal set. So intuitionistic fuzzy time series $IFTS_t$ can be specified as a multivariable time series and can be given by Eq. (1). The components of this $IFTS_t$ are composed of real observations and membership and non-membership values for each intuitionistic fuzzy set.*

$$IFTS_t = \{Y_t, \mu_{IFS_1}(t), \mu_{IFS_2}(t), \cdots, \mu_{IFS_c}(t), \nu_{IFS_1}(t), \nu_{IFS_2}(t), \cdots, \nu_{IFS_c}(t)\} \tag{1}$$

where $\mu_{IFS_l}(t)$ and $\nu_{IFS_l}(t)$ are the degree of membership and non-membership values of the t^{th} observation in the l^{th} intuitionistic fuzzy set, respectively. $\mu_{IFS_l}(t)$ and $\nu_{IFS_l}(t)$ can be obtained via intuitionistic fuzzy C-means (IFCM) clustering algorithm.

Definition 2. (*Intuitionistic Fuzzy Time Series Prediction Model*) *Let $IFTS_t$ be an intuitionistic fuzzy time series, $IFS_1, IFS_2, \cdots, IFS_c$ are intuitionistic fuzzy sets on the universal set, and $\mu_{IFS_l}(t)$, $\nu_{IFS_l}(t)$ are the degree of membership and non-membership values of the t^{th} observation in the l^{th} intuitionistic fuzzy set, respectively. The high order single variable intuitionistic fuzzy time series prediction model can be defined as below*:

$$Y_t = F\begin{pmatrix} Y_{t-1} Y_{t-2} \cdots, Y_{t-m}, \\ \mu_{IFS_1}(t-1), \mu_{IFS_2}(t-1), \cdots, \mu_{IFS_c}(t-1), \\ \nu_{IFS_1}(t-1), \nu_{IFS_2}(t-1), \cdots, \nu_{IFS_c}(t-1) \end{pmatrix} + \varepsilon_t \tag{2}$$

where, F is a linear or non-linear function that can be estimated by an estimation tool.

3.1. Intuitionistic fuzzy C-means

Intuitionistic fuzzy sets (IFSs) provide a mathematical framework based on fuzzy sets to describe and deal with vagueness and uncertainty in data. They have some distinguishing advantages over fuzzy sets. While the fuzzy sets (Zadeh, 1965) only consider the membership function $\mu(x)$, $x \in X$, intuitionistic fuzzy sets consider not only the non-membership function $\nu(x)$ but also the membership function $\mu(x)$. Let X be a universe of discourse, then

$$A = \{x, \mu_A(x), \nu_A(x)/\mathrm{x} \in \mathrm{X}\} \tag{3}$$

is called an IFS. Here $\mu_A(x)$ and $\nu_A(x)$ depict the membership degree and the non-membership degree for an element x respectively.

$x \in X \mu_A(x) \rightarrow [0,1]$ and $x \in X \rightarrow \nu_A(x) \in [0,1]$ also they fulfil the condition;

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1 \tag{4}$$

If $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ then $\pi_A(x)$ is called hesitation degree which represents the lack of knowledge in defining the membership and non-membership degrees and it is defined as a different measure from these degrees. It is clearly seen here, $0 \leq \pi_A(x) \leq 1$. If $\pi_A(x) = 0$ for all

$x \in X$, then intuitionistic fuzzy set becomes a fuzzy set; if $\mu_A(x) = \nu_A(x) = 0$ for all $x \in X$, then the intuitionistic fuzzy set is called completely intuitionistic.

The objective function to be optimized in IFCM (Chaira, 2011) includes two components: one of them is; modified objective function of FCM using IFS and the other one is; intuitionistic fuzzy entropy (IFE).

$$J_{IFCM} = \sum_{i=1}^{c}\sum_{k=1}^{n} u_{ik}^{*m} d_{ik}^2 + \sum_{i=1}^{c} \pi_i^* exp(1-\pi_i^*) \tag{5}$$

Here u_{ik}^* represents the intuitionistic fuzzy membership and calculated by Eq. (6)

$$u_{ik}^* = u_{ik} + \pi_{ik} \tag{6}$$

And u_{ik} is the traditional fuzzy membership of the k th datum point in i th set and π_{ik} is the hesitation degree which can be given as:

$$\pi_{ik} = 1 - u_{ik} - \left(1 - u_{ik}^{\alpha}\right)^{1/\alpha},\ \alpha > 0 \tag{7}$$

and is obtained by intuitionistic fuzzy complement of Yager:

$$N(X) = (1 - x^{\alpha})^{1/\alpha},\ \alpha > 0 \tag{8}$$

Hereby, IFS becomes;

$$A_{\lambda}^{IFS} = \left\{x,\ \ \mu_A(x),\ (1-\mu_A(x)^{\alpha})^{1/\alpha} \Big/\ \ x \in X\right\} \tag{9}$$

and

$$\pi_i^* = \frac{1}{N}\sum_{k=1}^{n}\pi_{ik}, k \in [1, N] \tag{10}$$

IFE, which forms the second term of the objective function, represents the amount of fuzziness or uncertainty in a set. While $\mu_A(x_i)$, $\nu_A(x_i)$, and $\pi_A(x_i)$ depict the membership degree, non-membership degree, and hesitation degree of the elements of the set $X = \{x_1, x_2, \cdots x_n\}$, in that case, IFE which represents the intuitionism degree can be given as:

$$IFE(A) = \sum_{i=1}^{n}\pi_A(x_i) exp(1 - \pi_A(x_i)), k \in [1, N] \tag{11}$$

and

$$\pi_A(x_i) = 1 - \mu_A(x_i) - \nu_A(x_i) \tag{12}$$

Modified cluster centres can be calculated via Eq. (13),

$$v_i^* = \frac{\sum_{k=1}^{n} u_{ik}^* x_k}{\sum_{k=1}^{n} u_{ik}^*} \tag{13}$$

Until the system reaches maximum iteration times $max_{ik}\left|u_{ik}^{*new} - u_{ik}^{*previous}\right|$ which is smaller then ε (ε is pre-defined value) both cluster centres and memberships are updated iteratively and then the process is stopped.

3.2. Cascade forward neural network

Cascade Forward Neural Network is a kind of multilayer feedforward neural network. Demuth (Demuth & Raele, 2009) proposed a CFNN based on the cascade correlation approaches which was proposed Fahlman (Fahlman, 1990), and Fahlman and Lebiere (S. E. Fahlman & Lebiere, 1992). Although the architecture of the CFNN, just like the other traditional feed-forward multilayer neural networks, consists of input, output, and hidden layer(s) (Alkhasawneh, 2019), it also has some distinguished aspects. The basic distinguishing feature of CFNN is that each layer of neurons is linked to all previous layers of neurons. Another feature that distinguishes and makes CFNN superior to others is that it can model linear and non-linear relationships between inputs and outputs together and simultaneously, thanks to the sigmoid activation function it uses in the hidden layer and the linear activation function it uses in the output layer. A hypothetical architecture of the CFNN with two hidden layers can be illustrated in Fig. 1.

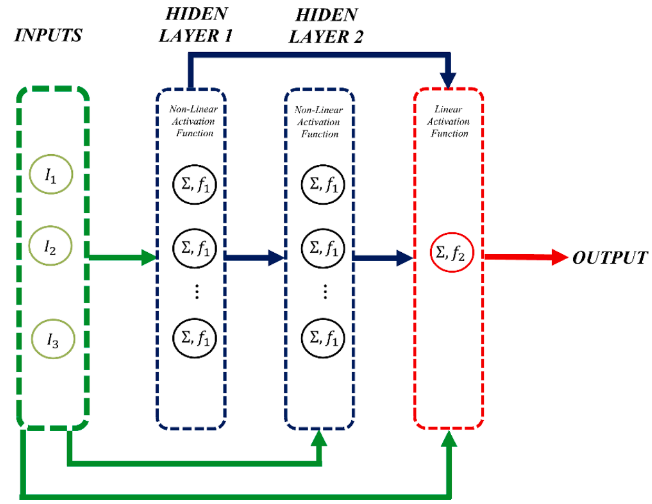


Fig. 1. A hypothetical architecture of CFNN.

4. The proposed methodology

4.1. An intuitionistic fuzzy time series model: The cascaded prediction

In fuzzy time series literature, there are two basic hang-ups. The first one is current fuzzy time series prediction models do not consider hesitation degree which is another aspect of uncertainty of the data. Thanks to considering hesitation degree, the non-membership values can be obtained, and they can be reckoned in the analysis process and thus the prediction performance can be improved. This progress can be actualized by using intuitionistic fuzzy sets which consider the hesitation degree. The second issue is that all the current fuzzy time series prediction methods discuss the relationships between inputs and outputs either a linear manner or a non-linear manner. However, most of the time series contain both linear and non-linear relationships together. To eliminate these two issues mentioned in this study, a novel time series prediction model has been proposed. The proposed model is based on intuitionistic fuzzy sets, and it considers non-memberships in the analysis process. Moreover, by using CFNN, linear and non-linear relationships between inputs and outputs can be modelled by the proposed approach. Thus, the innovations brought by the proposed methodology compared to the existing methods can be summarized as follows:

- CIPM allows a more realistic approach to the uncertainty by considering the degree the neutrality (compared to existing fuzzy-based models)
- CIPM uses the non-membership values as well as memberships as inputs of the model (compared to existing fuzzy-based models)
- CIPM, thanks to its cascading structure, enables linear and non-linear relationships between inputs and outputs to be modeled together (compared to existing fuzzy-based models and neural network-based models)

So, the proposed method is called as cascaded intuitionistic fuzzy time series prediction model. A definition given below form the basis of the proposed CIPM.

Definition 3. *(Cascaded Intuitionistic Fuzzy Time Series Prediction Model) Let $IFTS_t$ be an intuitionistic fuzzy time series, $IFS_1, IFS_2, \cdots, IFS_c$ are intuitionistic fuzzy sets on the universal set, and $\mu_{IFS_l}(t)$, $\nu_{IFS_l}(t)$ are the degree of membership and non-membership values of the t^{th} observation in the l^{th} intuitionistic fuzzy set, respectively. The Cascaded Intuitionistic Fuzzy Time*

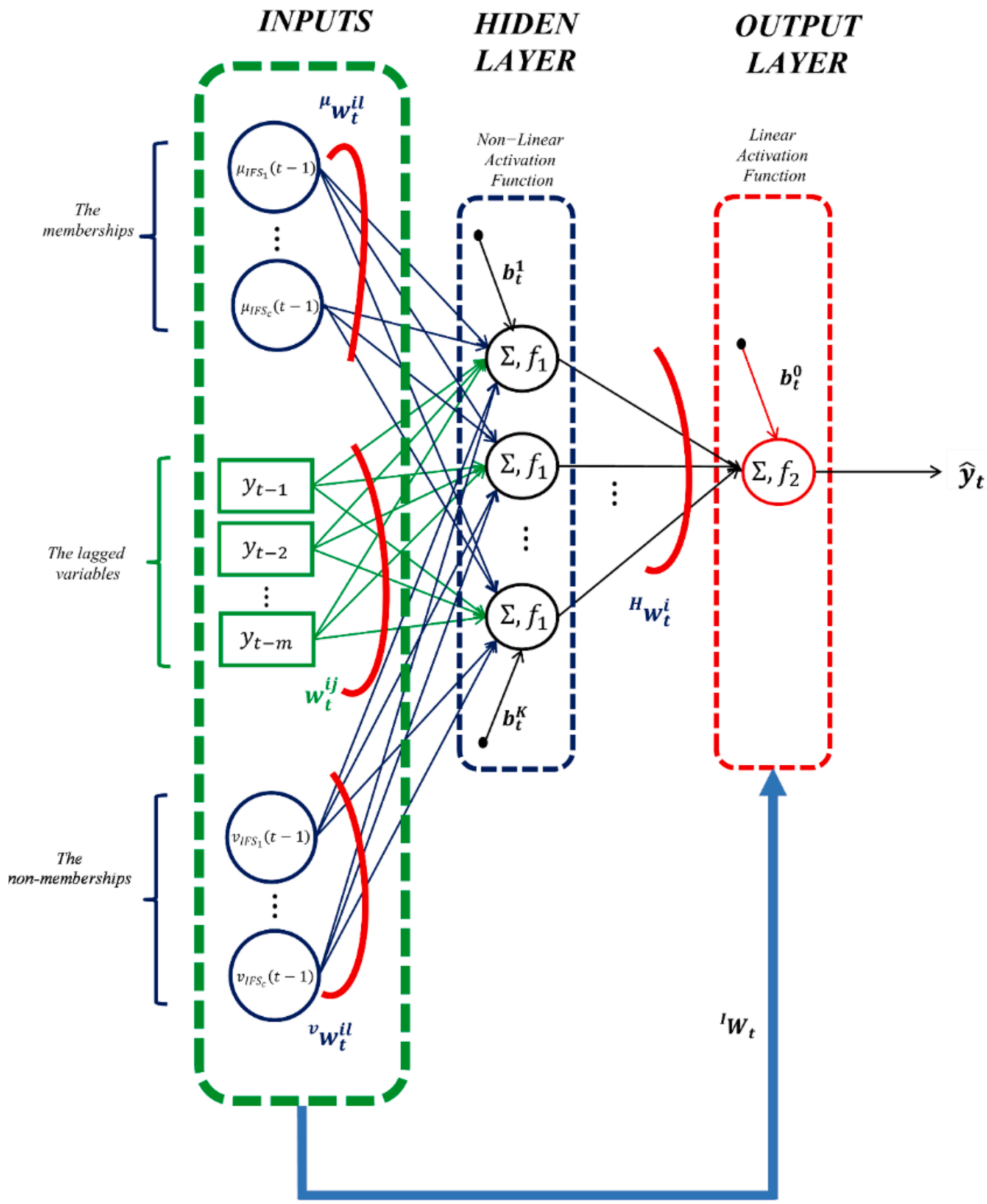


Fig. 2. The illustration of the CIPM.

Series Prediction Model (CIPM) can be defined as in Eq. (14).

In Eq. (14), F and F_1 are the linear functions and F_2 is a non-linear function that can be estimated by an estimation tool. This estimation tool can be chosen as a linear or non-linear regression analysis, or it can be an artificial neural network as in this study. ε_t is an error term. Thanks to the cascaded structure of the model, both linear and non-linear relationships can be modelled synchronously at the same time.

$$Y_t = F\left(\begin{array}{c} F_1\left(\begin{array}{c} Y_{t-1}Y_{t-2}\cdots, Y_{t-m}, \\ \mu_{IFS_1}(t-1), \mu_{IFS_2}(t-1), \cdots, \mu_{IFS_c}(t-1), \\ v_{IFS_1}(t-1), v_{IFS_2}(t-1), \cdots, v_{IFS_c}(t-1) \end{array} \right), \\ F_2\left(\begin{array}{c} Y_{t-1}Y_{t-2}\cdots, Y_{t-m}, \\ \mu_{I_{FS1}}(t-1), \mu_{IFS_2}(t-1), \cdots, \mu_{IFS_c}(t-1), \\ v_{IFS_1}(t-1), v_{IFS_2}(t-1), \cdots, v_{IFS_c}(t-1) \end{array} \right) \end{array} \right) + \varepsilon_t \tag{14}$$

An illustration of the CIPM can be given by Fig. 2. In the structure given by Fig. 2;

$\mu_{IFSc}(t-1)$: The degree of belonging to c^{th} intuitionistic fuzzy set for time series observation at the $t-1$ time (membership inputs).

$v_{IFSc}(t-1)$: The degree of non-belonging to c^{th} intuitionistic fuzzy set for time series observation at the $t-1$ time (non-membership inputs).

y_{t-j}: The j^{th} lagged variable for crisp time series $(j = 1, 2, \cdots, m)$.

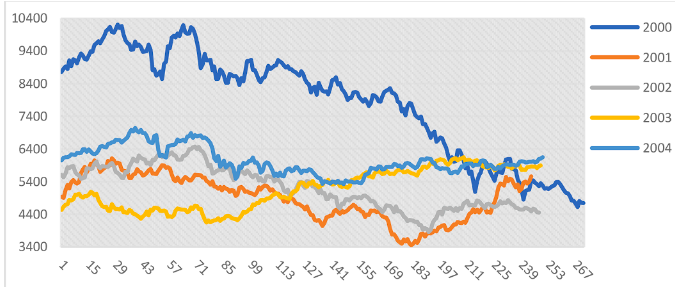
$\hat{y}_t$: The output of the CIPM for the t time.

μW_T^{IL}: The weight between l^{th} input layer neuron (memberships) and i^{it} hidden layer neuron (i = 1,2 k; l-1,2,c).

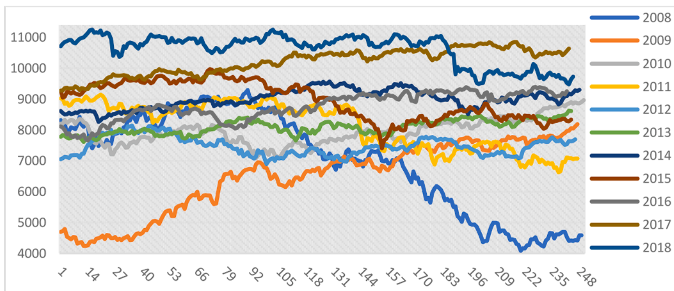
$V W_T^{IL}$: The weight between l^{th} input layer neuron (non-memberships) and i^{th} hidden layer neuron $(i = 1, 2, \cdots, K; l = 1, 2, \cdots, c)$.

w_t^{ij}: The weight between j^{th} input layer neuron (lagged variables) and i^{th} hidden layer neuron $(i = 1, 2, \cdots, K; j = 1, 2, \cdots, m)$.

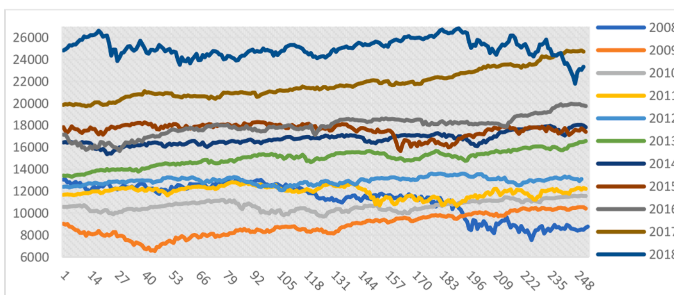
b_t^i: The bias for i^{th} hidden layer neuron $(i = 1, 2, \cdots, K)$.



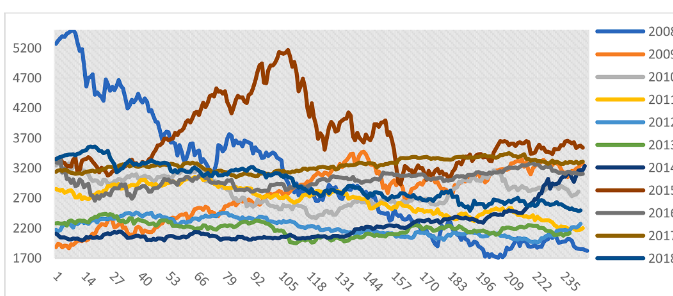
(a) TAIEX data sets 2000-2004.



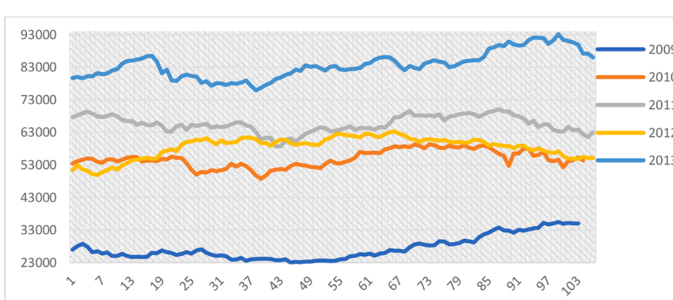
(b) TAIEX data sets 2008-2018.



(c) DIJ data sets 2008-2018.



(d) SSEC data sets 2008-2018.



(e) IEX data sets 2009-2013.

Fig. 3. Data sets.

HWIT: The weight between i^{th} hidden layer neuron and the output layer neuron $(i = 1, 2, \cdots, K)$.

IW_T: The vector of weights between input layer neurons and the output layer neuron$({}^{I}W_T = IW1_T\ {}^{I}W_T^{2XL+M})$.

b_t^o: The bias of output layer neuron.

The outputs of hidden layer neurons are obtained by Eq. (15).

$$o_i = f_1\left(\left(\sum_{l=1}^{c\mu} w_t^{il}\mu_{IFS_l}(t-1)\right) + \left(\sum_{l=1}^{cv} w_t^{il}v_{IFS_l}(t-1)\right) + \left(\sum_{j=1}^{m} w_t^{ij}y_{t-j}\right) + b_t^i\right),\ i = 1, 2, \ldots, K \tag{15}$$

Here f_1 represents the sigmoid activation function and $f_1(x) = \frac{1}{1+\exp(-x)}$. The outputs of the system are obtained, by using the weights between the hidden and output layers, as in the formula given Eq. (16).

$$o_{C-IFTS-PM} = f_2\left(\left(\sum_{l=1}^{cH} w_t^i o_i\right) + \left(\sum_{l=1}^{cI} W_t INPUTS(t)\right) + b_t^0\right),\ i = 1, 2, \ldots, K \tag{16}$$

f2, in Eq. (16), is the linear activation function and $INPUTS(t)$ is a vector which is composed of inputs for the t time $(\mu_{IFS_l}(t-1), v_{IFS_l}(t-1), y_{t-j})$.

All the parameters of process can be given as:

maxitr: Maximum iteration number.

m: # of lagged crisp variables.

K: # of hidden layer neuron.

T_{train}: Length of the training set.

T_{test}: Length of the test set.

c: # of intuitionistic fuzzy sets.

T: # of observations of time series.

4.2. Working principle of the cascaded prediction

The working principle of the cascaded intuitionistic fuzzy time series prediction model can be given by a pseudo-code for better understanding.

Algorithm: CIPM.

Input: Observations of time series.$Y_t, t = \overline{1, T_{train}}$

Output: Predictions of time series.$\widehat{Y}_t, t = \overline{1, T_{train}}$

Begin

Set the parameters.

Scale YT

Perform I-FCM.

Establish inputs of the model.

I : $Y_{t-j}, \mu_{IFS_l}(t-1), v_{IFS_1}(t-1), j = \overline{1, m}; l = \overline{1, c}$ (CIPM)

$itr = 0$

Generate the weights and biases randomly from.$Uniform(-10, 10)$

Repeat

$itr = itr + 1$

Calculate the outputs.$\widehat{Y}_t t = \overline{1, T_{train}}$

Calculate the cost function itr RMSE.

Update the weights and biases.

Until itr==matrix

Determine the best weights and biases.

Calculate the outputs for the best weights and biases.$\widehat{Y}_t t = \overline{T_{train} + 1, T}$

End.

Table 1
The data and implementations properties.

Series No	Time Series	Year	# of Observations	Size of Training Set	Size of Testing Set
1	TAIEX	2000	271	224	47
2		2001	244	201	43
3		2002	248	205	43
4		2003	249	206	43
5		2004	250	205	45
6/17/28	TAIEX/DIJ/SSEC	2008	249/253/246	206/212/203	43/41/43
7/18/29		2009	247/252/244	203/201/200	44/42/44
8/19/30		2010	250/252/242	205/209/197	45/43/45
9/20/31		2011	247/252/244	203/210/200	44/42/44
10/21/32		2012	246/250/243	204/209/200	42/41/43
11/22/33		2013	244/252/238	201/211/195	43/41/43
12/23/34		2014	248/252/245	205/211/202	43/41/43
13/24/35		2015	244/252/244	200/210/200	44/42/44
14/25/36		2016	242/252/244	198/210/200	44/42/44
15/26/37		2017	243/251/244	200/210/201	43/41/43
16/27/38		2018	245/251/243	203/211/201	42/40/42
39/40	IEX	2009	103/103	96/88	7/15
41/42			104/104	97/89	7/15
42/44			106/106	99/91	7/15
45/46			106/106	99/91	7/15
47/48			106/106	99/91	7/15

Table 2
The properties of implementations.

Hyperparameter		
Abbreviation	Meaning	Value
maxitr	Maximum iteration number	100
m	# of lagged crisp variables	from 2 to 5
K	# of hidden layer neuron	from 4 to 10
T	# of observations of time series	data dependent
T_{train}	Length of the training set	data dependent
T_{test}	Length of the test set	data dependent
c	# of intuitionistic fuzzy sets	from 5 to 7

5. Data and scenarios of implementation

5.1. Data preparation

The prediction capability of the proposed CIPM has been tested with the implementations of four different financial time series; TAIEX (2000–2004 and 2008–2018), DIJ (2008–2018), SSEC (2008–2018), and IEX (2009–2013) given in Fig. 3. The features of the data sets and implementations are summarized in Table 1. In the implementations, each data set is divided into two parts: training and testing sets. The memberships, the non-memberships and the lagged crisp time series constitute the inputs of the proposed model. Moreover, the other properties of the implementations can be summarized in Table 2.

5.2. Performance measure

Root mean square error (RMSE), given in Eq. (17), is the most used evaluation function in the time series literature. In addition to this, mean absolute percentage error (MAPE), given in Eq. (18), is another widely used error measure since it is a proportional error measure regardless of

Table 3
The properties of implementations.

# of intuitionistic fuzzy sets	# of lagged crisp variable (*m*)	# of hidden layer neuron (*K*)	# of target
from 5 to 7	from 2 to 5	from 4 to 10	1
# of implementation	84		

the size of the data.

$$RMSE = \sqrt{mean\left((Target_t - Predicted_t)^2\right)}; t = 1, 2, \cdots, T \tag{17}$$

$$MAPE = mean\left(\left|\frac{Target_t - Predicted_t}{Target_t}\right| \times 100\%\right); t = 1, 2, \cdots, T \tag{18}$$

A measure of relative error is the median relative absolute error (MdRAE) and it can also be utilized in comparison with a reference model. The MdRAE is based on the relative errors given in Eq. (19), and it is calculated as in Eq. (20).

$$r_t = \frac{Target_t - Predicted_t}{Target_t - Predicted_t^*} \tag{19}$$

$$MdRAE = median|r_t|; t = 1, 2, \cdots, T \tag{20}$$

where $Predicted_t^*$ is the predicted values produced by the reference or benchmark model. In this study as in most of the studies, the naïve model is preferred as the reference model. In the naive model $Prediction_t^*$ is equal to $Target_{t-1}$.

Evaluation of the results, obtained from another point of view, can also be carried out with a regression analysis. Such a regression analysis consists of estimating a regression model given by Eq. (21), interpretation of coefficients and some properties.

$$Y_t = \beta \widehat{Y}_t + \varepsilon_t \tag{21}$$

For a remarkably successful prediction tool it is expected that the regression (β) and determination (R^2) coefficients of the model are quite close to 1. Furthermore, the scatter diagrams are used as visual measure of the level of the fit of predicted and observed data points.

5.3. Implementation scenarios

For all data sets, various implementations are carried out under different scenarios based on different parameters of the analysis process. The basic characteristics of these scenarios are given in Table 3.

For TAIEX (2000–2004) analysis, the results of 84 implementations are evaluated via mean, minimum, maximum, and standard deviation statistics. In this manner, the results of scenarios that the worst and best prediction performances are produced also compared to the results of other models available in the literature.

For the analysis of TAIEX (2008–2018) and IEX (2009–2013) datasets, the test performances corresponding to the best training performance are compared with the performances of other models in the literature. Finally, for the analysis of DIJ and SSEC datasets, averages of eleven test performances corresponding to the best training performances are compared with the average performances of some other models in the literature in terms of three error measures. Thus, the performance of the proposed CIPM has been extensively examined.

All implementations have been carried out by software MATLAB R2022b -with *Marmara University Academic License*-.

5.4. Comparison to the state-of-the-art models

Various prediction models known with superior prediction performance are preferred to compare and interpret the results. While the results of the TAIEX, DIJ and SSEC data sets have been evaluated with many studies' results, the prediction models used in comparison for IEX are given below;

ARIMA: Autoregressive integrated moving average method (Geurts et al., 1977) (Box and Jenkins).

ES: Exponential Smoothing (Brown, 1957).

MLP: Multilayer perceptron artificial neural network (Rumelhart et al., 1986).

SC: Song and Chissom's fuzzy time series prediction model (Song & Chissom, 1993b).

FF-T1: Type 1 fuzzy Regression function approach (Türkşen, 2008).

FTS-N: Fuzzy time series network (Bas et al., 2015).

ANFIS: Adaptive Neuro-Fuzzy Inference System (Jang, 1993).

MANFIS: Modified Adaptive Neuro-Fuzzy Inference System (Egrioglu et al., 2015).

AR-ANFIS: Adaptive Neuro-Fuzzy Inference System with AR structure (Sarıca et al., 2018).

I-TSFIS: Intuitionistic time series fuzzy inference system (Egrioglu et al., 2019a).

I-FRF: Intuitionistic fuzzy regression functions approach (Cagcag Yolcu et al., 2020).

5.5. Consistency/reliability and validity of predictions

For a time-series prediction tool, reliability and validity are the two crucial desired properties. A reliable model is a model having an internal consistency. It is out of the question that a prediction model is completely reliable because reliability is a graduated principle. It is inevitable for time series prediction tools to vary in performance since the chance and random effects are always present in a prediction problem. However, for a prediction tool with a sufficient level of reliability, it is expected that this variation should be as low as possible from one implementation to another. On the other hand, validity is a measure of accuracy in a prediction process. Reliability and validity are directly related to each other and there is a strong interaction between them. This relationship and interaction have been exhibited through a visual in

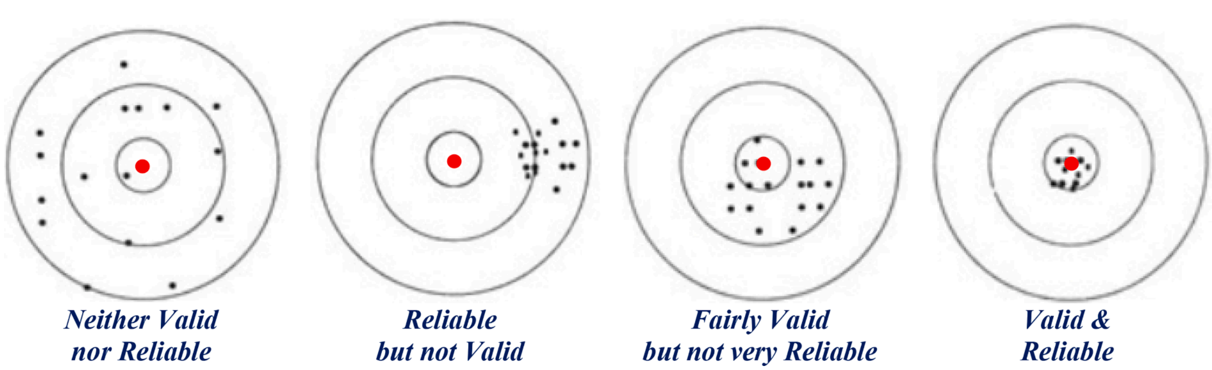


Fig. 4. The interaction between reliability and validity.

Table 4
The statistics on RMSE for 84 implementations.

Statistic	Data Set	Time Series/TAIEX Data Sets				
		1	2	2	4	5
Min	Training	18.3125	8.1653	9.4138	10.1112	5.3937
	Test	31.7838	11.4751	8.4822	15.7865	5.5054
Max	Training	67.9776	23.0833	32.1178	23.5339	22.9726
	Test	109.2285	31.7246	21.9357	39.5764	14.2475
Average	Training	34.2965	15.3861	15.5055	13.6448	10.9281
	Test	73.1066	19.5286	15.6739	25.6505	9.2087
Standard Deviation	Training	9.2252	3.7757	4.5438	2.4776	3.1948
	Test	18.8246	3.7607	3.1627	5.6793	1.6450

Fig. 4. The first dartboard presents the hits which are not close to the target and are quite scattered. For this kind of model, it can be said that there is an error with a high margin, and its consistency cannot be relied upon. The hits in the second dartboard are reliable but do not have the feature of valid. Such a model can produce outputs (hits) with fairly low variability, but these are far from targets. The hits on the third dartboard are valid, unlike the second, but have no reliability feature. Because this kind of model can produce close outputs (hits) to the targets. However, these outputs have fairly high variability. The final dartboard shows the outputs (hits) expected to be produced by a satisfactory prediction model. The outputs, in this case, have fairly low variability and position fairly near to the targets.

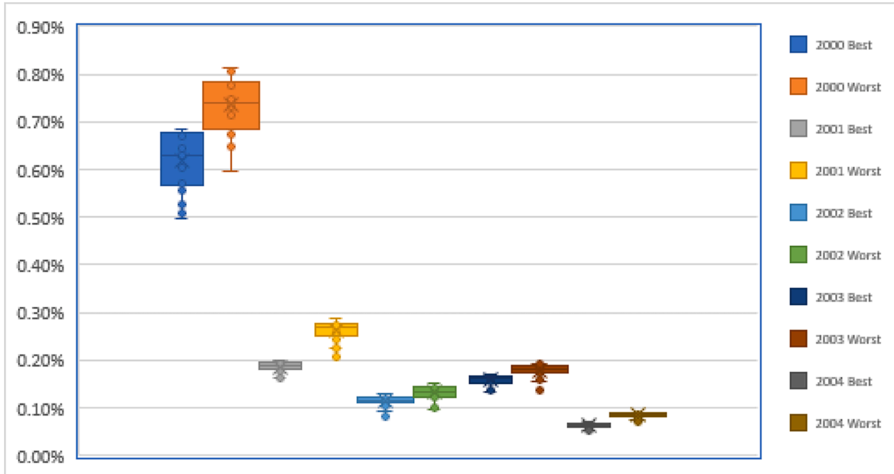
This information regarding the interaction between reliability and validity can also be used to evaluate the proposed prediction model in this study. In this direction, the proposed novel model was run 30 times for architectural structures that produce the best and the worst results, and the outputs evaluated in terms of reliability and validity.

6. Results and discussions

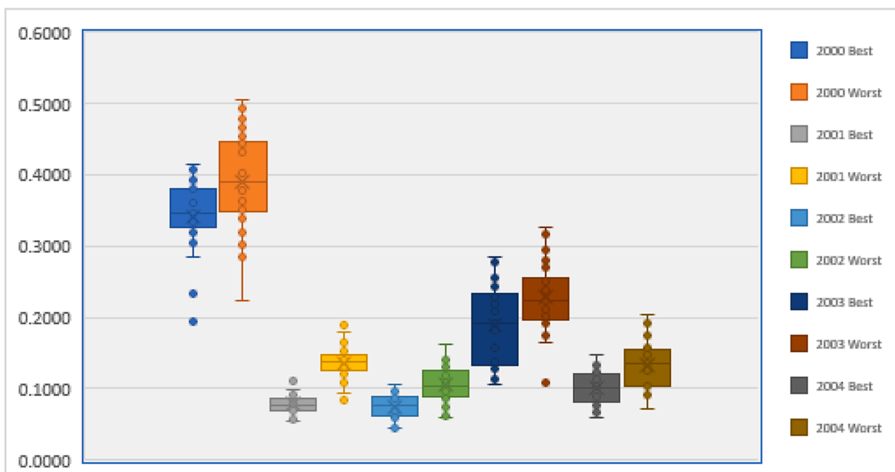
To reveal the efficiency and prediction capability of the proposed CIPM, the results are considered from different perspectives.

Table 5
The prediction performances of the models for the TAIEX 2000–2004 data sets.

Models		Time Series/TAIEX Data Sets					RMSE's	
		2000	2001	2002	2003	2004	Average	Median
(Song & Chissom, 1993a)		293	116	76	77	82	129	82
(Chen, 1996)		225	116	76	77	82	115	82
(Huarng, 2001a)[1]		473	359	234	247	384	339	359
(Huarng, 2001a)[2]		473	810	116	308	384	418	384
(Huarng & Yu, 2006b)		133	124	82	62	85	97	85
(Hsu et al., 2010)		152	130	84	56	116	108	116
(Huarng et al., 2007)		154	124	93	66	72	102	93
(Yu and Huarng, 2008)		131	130	80	58	67	93	80
(Aladag et al., 2010)		168	120	76	58	63	97	76
(Chen & Chang, 2010)		129	113	67	54	60	85	67
(Chen & Chen, 2011)		124	115	71	58	58	85	71
		120	114	67	52	52	81	67
(Chen et al., 2013)		131	113	66	52	54	83	66
(Egrioglu et al., 2010)		255	130	84	56	116	128	116
(Yolcu et al., 2013)		227	102	66	51	55	100	66
(Chen et al., 2014)		126	114	65	54	53	82	65
(Chen & Chen, 2015)		125	115	65	53	53	82	65
(Cai et al., 2015)		132	113	60	52	50	81	60
(Bas et al., 2015)		140	120	77	60	59	91	77
(Cheng et al., 2016)		126	113	63	51	54	81	63
(Chen et al., 2016)		180	134	81	77	55	105	81
(Tak et al., 2018)		128	106	65	52	54	81	65
(Jang, 1993)		137	115	66	57	61	87	66
(Egrioglu et al., 2015)		124	112	63	52	54	81	63
(Sarıca et al., 2018)		123	111	66	52	54	81	66
(Egrioglu et al., 2019a)		209	73	22	43	54	80	54
(Cagcag Yolcu et al., 2020)		122	110	54	51	50	77	54
(Kirisci & Cagcag Yolcu, 2022)		105	110	60	51	50	75	60
LSTM from (Kirisci & Cagcag Yolcu, 2022)		136	101	89	92	70	108	92
CIPM - The Best Scenario		**32**	**11**	**8**	**16**	**6**	**15**	**11**
CIPM – The Worst Scenario		**109**	**32**	**22**	**40**	**14**	**43**	**32**
Progress (%)	**The Best**	**70 %**	**84 %**	**61 %**	**63 %**	**88 %**	**80 %**	**80 %**
	The Worst	**NAN**	**57 %**	**0 %**	**7 %**	**72 %**	**43 %**	**41 %**



(a) MAPE values



(b) MdRAE values

Fig. 5. The box-plots of obtained error metrics by running 30 times.

6.1. TAIEX implementations

6.1.1. TAIEX (2000–2004) implementations

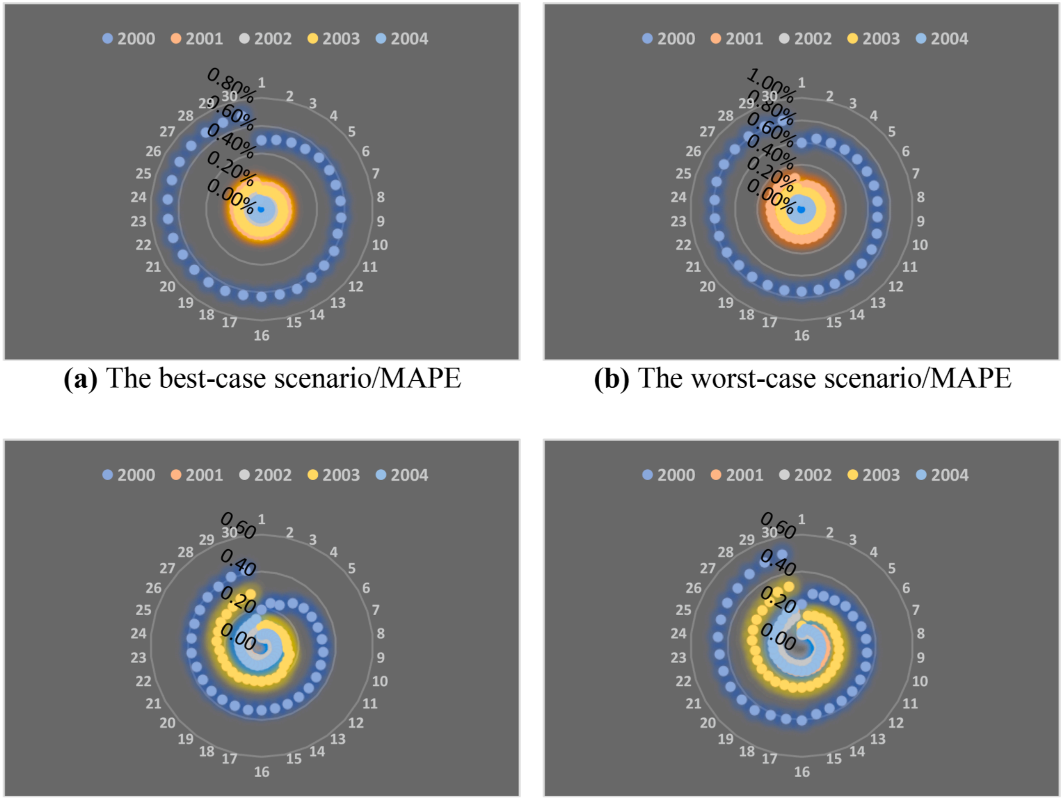
As mentioned before, for TAIEX data sets, 84 implementation scenarios are established. The basic statistics for both training and test sets of the RMSE measure obtained for these 84 implementations are given in Table 4.

From Table 4,

- The CIPM has produced predictions with a minimum RMSE value of 31.78 for the test set of TAIEX data in the 2000 year.
- For the test set of TAIEX data in the 2001 year, the predictions produced by the proposed CIPM has a minimum RMSE value of 11.48.
- For the other years, related figures have been obtained as 8.48, 15.79 and 5.51, respectively.
- As can be seen from the comparative results given by Table 4, these values are quite low when we consider ***the state-of-the-art models*** in the literature.
- It is seen that the proposed model also obtains reasonable prediction results even in cases where maximum RMSE values are obtained, in addition to the cases where the minimum RMSE values are obtained for the test set in 84 alternative implementations.
- Considering the variation in the error criteria regarding the results obtained in 84 alternative applications, it is seen that the change is quite low for both training and test sets in all 4 years, especially with the exception of 2000.
- This situation can be accepted as an indication that the proposed method is strong in terms of consistency/reliability, besides the validity.

Out of the 84 analyses, the minimum RMSE (the best cases with minimum RMSE) values in the prediction of the test set for best-performing cases were obtained as approximately 31.78, 11.48, 8.48, 15.79, and 5.51 respectively. For the worst-performing cases, the RMSE values are approximately 109.22, 31.72, 21.94, 39.58 and 14.25, respectively. Especially, considering the standard deviation values, except for the standard deviation values obtained for the 2000 TAIEX dataset and not very large (9.22; for the training set and 18.82; for the test set), for the TAIEX datasets, very small standard deviation values have been obtained. This is an indication that all 84 alternative implementations produce predictions with RMSE values that are not far from each other and do not contain a large variation.

The details of the comparison in terms of the RMSE error measures are given in Table 5. From Table 5,



(a) The best-case scenario/MAPE **(b)** The worst-case scenario/MAPE

(c) The best-case scenario / MdRAE **(d)** The worst-case scenario / MdRAE

Fig. 6. The Scatter plots of relative metrics.

- The proposed CIPM shows the superior prediction performance for all 5 data sets in both the best and the worst scenarios.
- For the best-case scenario, the progress level of the proposed CIPM, over the second-best of the other models, is 61 % and 63 % for 2002 (*with RMSE value of 8*) and 2003 (*with RMSE value of 16*); 70 % for 2000 (*with RMSE value of 32*); and even more than 80 % for 2001 (*with RMSE value of 11*) and 2004 (*with RMSE value of 6*).
- For the best-case scenario, the average and median of the RMSE metric produced by the CIPM for five years are 15 and 11, respectively. It means that the CIPM shows more than 80 % progress over five years, on averagely.

Table 6
Convergence times (in seconds) of the CIPM.

Year of Data	Case	# of Lagged Variables	# of IFS	# of Hidden Layer Neuron	Time in averagely second
2000	Best	3	6	6	3.52
	Worst	4	5	5	2.78
2001	Best	2	7	7	3.94
	Worst	5	5	4	2.90
2002	Best	2	5	10	2.82
	Worst	5	5	7	2.93
2003	Best	2	7	8	3.85
	Worst	4	6	7	2.35
2004	Best	2	5	9	2.93
	Worst	2	5	4	2.91

- Even for the worst-case scenario, the CIPM exhibits remarkable improvement with a ratio of more than 40 % on averagely over five years.
- As all these findings support, according to the RMSE metric, the proposed CIPM outperforms other state-of-the-art models, producing outstanding predictions even in the worst-case scenario.

Moreover, the results produced from the best and the worst scenarios of the proposed CIPM are discussed with the prediction results of various state-of-the-art models.

Additionally, the proposed CIPM has been run 30 times for best-case and worst-case scenarios. The distributions of MAPE and MdRAE relative error metrics are visually examined to evaluate the reliability and validity of the CIPM's performance. Fig. 9 presents the box-plots for TAIEX data sets from 2000 to 2004. From these box-plots, for all repetition, it is clearly seen that the proposed CIPM produces outstanding predictions with MAPE metric values of under 0.30 % for both the best-case and the worst-case scenarios for all data sets except the 2000 year. Even for the 2000 year, CIPM produces the superior predictions with MAPE metric values of under 1 % for both the best-case and the worst-case scenarios for all duplications. These visual findings related to distributions of MAPE values indicate that the differences regarding relative error are pretty limited between the best-case scenario and the worst-case scenario. Moreover, the variations of relative errors are also rather low within each scenario. Similar comments can be made for MdRAE, another relative error metric that provides a comparative perspective with the naive model as a benchmark model.

The findings achieved from Fig. 5 can be seen as evidence that the proposed CIPM generates reliable and consistent predictions. Scatter plots of the same metrics are another proof of the reliability and validity of the predictions.

From these scatter plots given in Fig. 6, it can be seen that both relative measures vary within narrow ranges from one replicate to the

Table 7
Characteristics of the linear regression model ($Y_t = \beta\widehat{Y}_t + \varepsilon_t$).

Time Series	Model Estimation	Sig. of$F$	St. Error of $\widehat{\beta}$	Sig. ofβ	R^2
1	$Y_t = 0.997785\widehat{Y}_t$	2.28E-103	8.17E-04	2.05E-105	0.999,969
2	$Y_t = 0.999814\widehat{Y}_t$	2.05E-109	3.67E-04	7.91E-112	0.999,994
3	$Y_t = 0.999205\widehat{Y}_t$	5.0897E-116	2.53E-04	1.35E-118	0.999,997
4	$Y_t = 1.000241\widehat{Y}_t$	2.1424E-107	4.11E-04	9.25E-110	0.999,993
5	$Y_t = 0.999779\widehat{Y}_t$	8.79E-133	1.36E-04	1.29E-135	0.999,999

next, as would be expected for a reliable predictor. For the best-case scenario, the MAPE values of the predictions produced by the CIPM range from 0.05 % to 0.20 % for all years except 2000. Even for 2000, these values scatter between 0.50 % and 0.63 %. In the case of the worst scenario, the MAPE values of the predictions of the CIPM range from 0.07 % to 0.29 % for all years except 2000. Even for 2000, these values scatter between 0.60 % and 0.81 %. All these MAPE values show that the variations of the predictions fall in a narrow range and prove the reliability of the CIPM. Even in worst-case scenarios, the MAPE values are so small that it can be said with certainty that the CIPM can be used as a valid prediction tool in any case. The drawn scatter plots for the MdRAE relative error metric also support these findings.

One of the important qualities that a superior prediction tool should have is a reasonable convergence time. In this respect, the average convergence time over 30 repetitions are determined in seconds for both the best and worst cases. These average convergence times are presented in Table 6 with the info on the properties of the scenarios. For these cases and all data sets, the convergence times are on average between 2 and 4 s, and such convergence times are remarkably reasonable and applicable. Particularly, also considering the predictions having outstanding accuracy obtained for all scenarios, it is obviously said that these durations are outstanding for decision-makers who work on time series prediction problems.

Examining some of the features of a linear regression model to be designed between predictions and targets is another good and effective way to present the outstanding predictive ability of the CIPM. The basic expectation here is that the regression coefficient ($\widehat{\beta}$) and also the co-

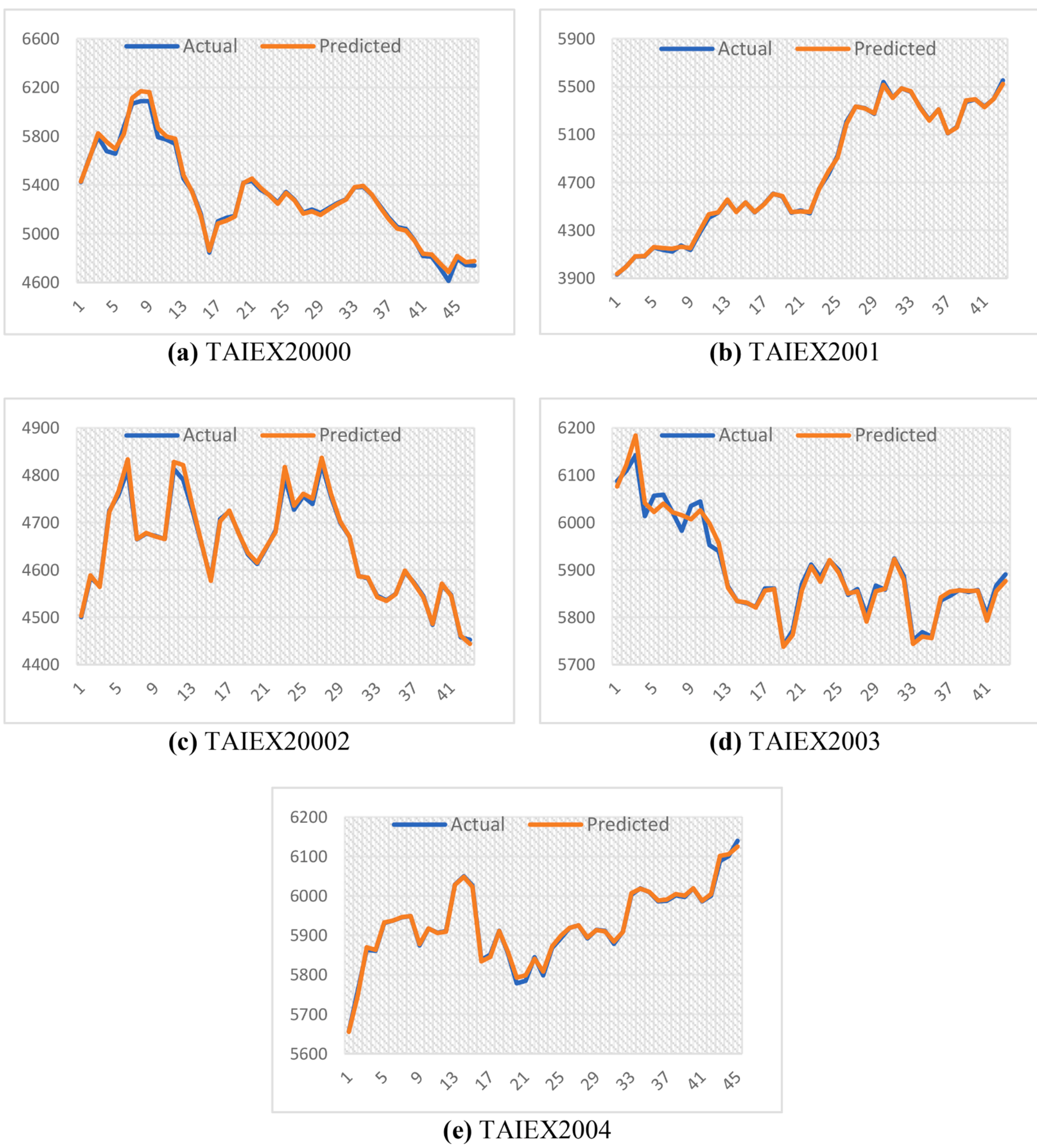


Fig. 7. The illustration of the harmony of predicted and observed values.

Table 8
The comparative prediction results in terms of RMSE for TAIEX (2008–2018).

Models	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	Average
(Song & Chissom, 1994)	225.38	204.11	151.42	237.43	81.08	82.3	98.64	176.6	122.71	91.69	173.36	149.52
(Chen, 1996)	186.48	207.42	211.21	215.13	78.37	184.83	308.72	100.75	316.57	902.18	492.89	291.32
(Huarng, 2001b)	105.10	78.25	103.84	117.78	59.14	49.59	87.52	90.72	80.98	63.20	186.21	92.94
(Huarng, 2001c)	138.94	71.30	72.92	115.54	62.32	49.53	69.60	78.73	82.22	62.98	102.19	82.39
(Yu, 2005)	142.38	110.88	100.76	180.79	63.30	63.16	71.62	104.4	88.35	89.02	231.35	113.27
(Huarng & Yu, 2006a)	131.18	71.48	74.07	119.34	59.55	50.09	65.72	79.93	82.10	61.67	101.99	81.56
(Huarng et al., 2007)	144.23	72.16	62.23	114.25	59.94	50.97	64.79	79.27	82.29	62.15	107.08	81.76
(Yu and Huarng, 2008)	129.48	69.91	67.11	123.03	58.37	50.39	69.15	78.22	80.51	64.25	103.74	81.29
(Hsu et al., 2010)	140.49	70.34	93.08	118.89	62.35	49.63	66.55	78.46	82.73	62.21	109.39	84.92
(Chen & Tanuwijaya, 2011)	142.37	112.45	99.83	164.97	63.31	64.76	74.85	94.33	86.59	88.93	161.78	104.92
(Aladag et al., 2014)	114.03	70.91	68.59	117.02	60.50	49.92	70.44	77.67	80.86	62.61	106.75	79.94
(Askari et al., 2015)	129.29	70.27	75.46	111.45	63.20	50.84	67.67	78.33	84.39	61.09	103.26	81.39
(Cagcag Yolcu et al., 2016)	137.44	71.76	74.52	117.38	61.59	51.37	69.23	79.89	83.53	62.37	103.24	82.94
(Sadaei et al., 2016)[1]	108.52	121.46	75.33	128.46	60.43	51.17	79.44	94.92	80.39	78.76	88.23	87.92
(Sadaei et al., 2016)[2]	108.57	68.57	52.15	113.38	58.84	48.87	65.90	80.22	82.24	64.31	74.52	74.32
(Ye et al., 2016)	140.19	73.22	80.37	124.38	62.53	52.39	66.26	82.78	80.14	61.98	99.03	83.93
(Wan & Si, 2017)	126.29	73.77	143.53	122.49	60.73	55.38	66.15	79.47	83.18	65.54	203.72	98.20
(Cheng & Yang, 2018)	140.48	70.63	67.46	121.27	61.10	50.23	67.08	80.65	81.12	66.34	98.13	82.23
(Wu et al., 2019)	113.03	72.30	62.82	113.69	60.45	50.17	68.28	78.65	82.43	62.49	104.49	78.98
(Dong & Ma, 2021)[1]	101.91	105.64	62.93	123.79	82.30	65.62	46.38	102.82	81.94	61.83	101.56	85.16
(Dong & Ma, 2021)[2]	91.95	57.43	44.61	90.32	43.58	33.83	46.23	58.79	53.15	36.99	70.67	57.05
CIPM	**26.13**	**26.41**	**23.47**	**40.32**	**15.19**	**10.78**	**20.52**	**14.58**	**25.77**	**20.00**	**27.55**	**22.79**
Progress (%)	**71.58**	**54.02**	**47.39**	**55.35**	**65.14**	**68.13**	**55.61**	**75.20**	**51.51**	**45.93**	**61.02**	**60.05**

Note: The results of the current counterparts models are taken from (Dong & Ma, 2021).

Table 9
The comparative prediction results in terms of MAPE for TAIEX (2008–2018).

Models	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	Average
(Song & Chissom, 1994)	3.87	2.04	1.53	2.65	0.89	0.78	0.86	1.78	1.10	0.71	1.29	1.59
(Chen, 1996)	3.49	2.41	1.90	2.61	0.86	1.77	3.31	0.97	3.20	8.45	4.73	3.06
(Huarng, 2001b)	1.66	0.73	1.09	1.24	0.58	0.49	0.79	0.86	0.63	0.47	1.71	0.93
(Huarng, 2001c)	2.55	0.68	0.71	1.27	0.61	0.48	0.61	0.79	0.62	0.47	0.80	0.87
(Yu, 2005)	2.57	1.06	0.98	1.99	0.68	0.57	0.63	1.04	0.72	0.67	1.96	1.17
(Huarng & Yu, 2006a)	2.36	0.67	0.72	1.35	0.59	0.49	0.58	0.79	0.65	0.46	0.78	0.86
(Huarng et al., 2007)	2.65	0.68	0.59	1.25	0.60	0.48	0.57	0.79	0.63	0.46	0.82	0.87
(Yu and Huarng, 2008)	2.34	0.66	0.64	1.35	0.59	0.50	0.61	0.77	0.63	0.47	0.81	0.85
(Hsu et al., 2010)	2.55	0.67	0.93	1.33	0.60	0.49	0.58	0.78	0.64	0.47	0.87	0.90
(Chen & Tanuwijaya, 2011)	2.57	1.07	0.97	2.01	0.68	0.61	0.66	0.98	0.70	0.67	1.42	1.12
(Aladag et al., 2014)	1.96	0.70	0.67	1.32	0.60	0.49	0.63	0.77	0.61	0.46	0.84	0.82
(Askari et al., 2015)	2.33	0.67	0.74	1.22	0.63	0.52	0.60	0.78	0.63	0.46	0.81	0.85
(Cagcag Yolcu et al., 2016)	2.43	0.68	0.72	1.23	0.59	0.52	0.61	0.80	0.63	0.46	0.81	0.86
(Sadaei et al., 2016)[1]	1.73	1.33	0.74	1.43	0.59	0.51	0.74	0.90	0.61	0.58	0.92	0.93
(Sadaei et al., 2016)[2]	1.75	0.65	0.50	1.20	0.57	0.47	0.58	0.78	0.62	0.48	0.76	0.76
(Ye et al., 2016)	2.57	0.70	0.78	1.41	0.61	0.52	0.58	0.80	0.62	0.45	0.76	0.89
(Wan & Si, 2017)	2.31	0.71	1.39	1.31	0.60	0.56	0.59	0.76	0.68	0.51	1.90	1.03
(Cheng & Yang, 2018)	2.55	0.68	0.65	1.34	0.59	0.48	0.59	0.81	0.62	0.47	0.76	0.87
(Wu et al., 2019)	2.00	0.70	0.60	1.24	0.59	0.50	0.59	0.77	0.62	0.47	0.82	0.81
(Dong & Ma, 2021)[1]	1.87	1.01	0.60	1.42	0.91	0.62	0.39	0.98	0.78	0.50	0.92	0.91
(Dong & Ma, 2021)[2]	1.57	0.55	0.43	0.94	0.42	0.33	0.44	0.59	0.43	0.27	0.53	0.59
CIPM	**0.37**	**0.24**	**0.24**	**0.45**	**0.16**	**0.10**	**0.18**	**0.14**	**0.22**	**0.15**	**0.22**	**0.22**
Progress (%)	**76.31**	**56.54**	**45.11**	**52.04**	**61.01**	**69.73**	**54.13**	**75.69**	**49.63**	**44.54**	**58.32**	**61.93**

Note: The results of the current counterparts models are taken from (Dong & Ma, 2021).

efficient of determination (R^2) of a linear regression model established as $Y_t = \beta\widehat{Y}_t + \varepsilon_t$ is equal to 1 or quite close to 1. The results produced by the linear regression analysis are summarized in Table 7. In the regression analysis process, the observed and the predicted values are taken as the dependent and the independent variables, respectively. The estimates of the beta coefficients are quite close to 1, as would be expected from a prediction model that produces satisfactory predictions. All the determination coefficients are also very close to 1. These findings are a substantial indicator that the predictions produced by the CIPM are very close to the targets.

The outstanding prediction performance of the CIPM can also be verified by another visual evidence showing high harmony of predicted and observed values given by Fig. 7.

6.1.2. TAIEX (2008–2018) implementations

Additionally, TAIEX datasets collected between 2008 and 2018 have been analysed by considering each fiscal year as an individual time series to provide a wider and deeper inspection of the prediction performance of the proposed CIPM. Tables 8-10 present the results comparatively for RMSE, MAPE, and MdRAE error measures. From these three tables, it is clear that the proposed CIPM has a remarkable superior prediction performance compared to some other state-of-the-art prediction models.

In terms of the RMSE criterion, the proposed model provides over 70

Table 10
The comparative prediction results in terms of MdRAE for TAIEX (2008–2018).

Models	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	Average
(Song & Chissom, 1994)	2.1448	3.0647	3.1887	2.4162	1.5793	1.5578	1.4383	2.5952	2.0856	1.5365	1.4623	2.0972
(Chen, 1996)	2.3294	5.1835	4.0107	2.9861	1.4906	3.6261	7.4309	1.4596	6.8543	20.8460	7.9646	5.8347
(Huarng, 2001b)	1.0254	0.9813	2.5643	0.9278	0.9387	1.0812	1.5661	1.4015	1.0697	1.1282	3.0914	1.4341
(Huarng, 2001c)	1.7855	0.9867	1.4093	1.0439	1.0565	1.0178	1.0377	0.9749	1.0053	1.0399	1.1754	1.1394
(Yu, 2005)	1.6624	1.3814	1.7419	1.7291	1.3326	1.1064	1.0937	1.4514	1.2785	1.3579	3.5893	1.6113
(Huarng & Yu, 2006a)	1.8415	1.0112	1.5120	1.1252	1.0032	1.0140	0.9828	1.0632	1.0527	0.9470	1.1059	1.1508
(Huarng et al., 2007)	2.2061	0.9618	1.1587	1.0520	1.0805	1.0607	0.9824	1.0620	1.0020	1.0031	1.1271	1.1542
(Yu and Huarng, 2008)	1.5375	0.9340	1.2959	1.1256	0.9341	1.1313	1.0279	0.9795	0.9720	1.0288	1.1297	1.0997
(Hsu et al., 2010)	2.1805	0.9743	2.2596	1.1032	1.0622	1.0083	0.9963	1.0178	1.0164	1.0159	1.5370	1.2883
(Chen & Tanuwijaya, 2011)	1.6624	1.4013	1.7267	2.4935	1.3261	1.1950	1.2662	1.3791	1.3113	1.3489	2.5870	1.6089
(Aladag et al., 2014)	1.1617	1.0579	1.3690	1.0937	1.0442	1.0491	1.0518	0.9745	1.0116	1.0230	1.2559	1.0993
(Askari et al., 2015)	1.5489	0.9740	1.5073	1.0356	1.0546	1.1971	1.0333	0.9648	1.0158	0.9844	1.1862	1.1365
(Cagcag Yolcu et al., 2016)	1.3179	0.9971	1.4121	1.0417	0.9861	1.1750	1.0431	1.1008	1.0303	0.9934	1.1790	1.1160
(Sadaei et al., 2016)[1]	1.0000	2.6261	1.4819	1.4443	1.0000	1.0000	1.0000	1.0000	1.0000	1.3549	1.2929	1.3142
(Sadaei et al., 2016)[2]	1.0499	0.8700	1.0397	0.9741	1.0261	1.0052	0.9875	0.9739	0.9991	1.1104	1.0042	1.0107
(Ye et al., 2016)	2.2481	1.0187	1.6757	1.1934	1.0406	1.1209	0.9537	1.0684	1.0034	0.9721	1.1430	1.2216
(Wan & Si, 2017)	1.4611	1.0674	2.8796	1.0774	0.9531	1.3093	1.0119	0.8907	0.9581	1.1461	3.3213	1.4615
(Cheng & Yang, 2018)	1.5865	1.0210	1.3938	1.1516	1.0469	1.0254	1.0291	1.0996	0.9989	1.0072	1.1203	1.1346
(Wu et al., 2019)	1.1915	1.0506	1.1582	1.0236	1.0164	1.0298	1.0011	0.9580	1.0377	1.0115	1.1432	1.0565
(Dong & Ma, 2021)[1]	1.3628	1.2553	1.2433	1.4180	1.6564	0.9939	0.6507	1.2241	1.7314	0.9836	1.4136	1.2666
(Dong & Ma, 2021)[2]	0.8938	0.7463	0.8529	0.8161	0.6368	0.7227	0.7275	0.7833	0.7738	0.5530	0.6912	0.7452
CIPM	**0.2679**	**0.3626**	**0.4222**	**0.4557**	**0.3070**	**0.2340**	**0.3547**	**0.1656**	**0.2861**	**0.3280**	**0.2893**	**0.3157**
Progress (%)	**70.03**	**51.42**	**50.50**	**44.16**	**51.79**	**67.63**	**45.49**	**78.86**	**63.02**	**40.70**	**58.15**	**57.64**

Note: The results of the current counterparts models are taken from (Dong & Ma, 2021).

Table 11
Comparison of average performance for DJI and SSEC data sets.

Models	DJI			SSEC		
	RMSE	MAPE	MdRAE	RMSE	MAPE	MdRAE
(Song & Chissom, 1994)	533.41	3.12 %	6.8827	150.76	5.33 %	7.9442
(Chen, 1996)	371.15	2.19 %	3.3221	132.04	4.39 %	5.6223
(Huarng, 2001b)	163.33	0.90 %	1.1089	37.35	1.08 %	1.1180
(Huarng, 2001c)	166.53	0.93 %	1.1662	38.12	1.10 %	1.1452
(Yu, 2005)	268.60	1.58 %	1.7911	67.96	1.92 %	1.7404
(Huarng & Yu, 2006a)	165.50	0.92 %	1.2929	36.90	1.05 %	1.0959
(Huarng et al., 2007)	159.84	0.87 %	1.1347	37.82	1.08 %	1.1530
(Yu and Huarng, 2008)	169.34	0.94 %	1.3238	38.78	1.14 %	1.1516
(Hsu et al., 2010)	168.96	0.94 %	1.2985	38.38	1.09 %	1.1563
(Chen & Tanuwijaya, 2011)	174.96	1.01 %	1.3253	38.60	1.11 %	1.1551
(Aladag et al., 2014)	160.67	0.86 %	1.1383	36.54	1.05 %	1.0698
(Askari et al., 2015)	163.39	0.89 %	1.2271	36.99	1.06 %	1.1114
(Cagcag Yolcu et al., 2016)	166.04	0.92 %	1.1897	37.37	1.08 %	1.1583
(Sadaei et al., 2016)[1]	177.63	0.96 %	1.3015	42.81	1.26 %	1.3354
(Sadaei et al., 2016)[2]	152.48	0.81 %	0.9759	36.30	1.01 %	0.9981
(Ye et al., 2016)	160.21	0.86 %	1.0847	37.41	1.07 %	1.0654
(Wan & Si, 2017)	170.66	0.94 %	1.3609	38.95	1.13 %	1.2981
(Cheng & Yang, 2018)	165.77	0.93 %	1.1767	37.20	1.06 %	1.1182
(Wu et al., 2019)	133.17	0.46 %	0.9949	31.95	0.89 %	0.9272
(Dong & Ma, 2021)[1]	164.73	0.88 %	1.2563	38.79	1.17 %	1.2335
(Dong & Ma, 2021)[2]	126.74	0.68 %	0.7997	23.24	0.69 %	0.6951
CIPM	**45.72**	**0.22 %**	**0.4208**	**10.72**	**0.30 %**	**0.3100**
Progress (%)	**63.93**	**52.17**	**47.38**	**53.87**	**56.52**	**55.40**

Note: The results of the current counterparts models are taken from (Dong & Ma, 2021).

% improvement in prediction performance for the datasets of 2008 (*71.58 %*) and 2015 (*75.20 %*) compared to the second-best performing model. While this progress is over 60 % for 2012 (*65.14 %*), 2013 (*68.13 %*) and 2018 (*61.02 %*), it is over 50 % for 2011 (*55.35 %*), 2014 (*55.61 %*) and 2016 (*51.51 %*). Even for the rest of the years, the CIPM makes progress over 40 %. Considering the average performance for all years, about 60 % progress is obtained, meaning that the proposed CIPM can be considered as an outstanding prediction tool for TAIEX's relevant datasets.

The MAPE metric values given in Table 9 also mark that the CIPM generates spectacular predictions for related TAIEX data sets. These results show that CIPM produced the predictions with an error level of under 5 per thousand in all years. The progress levels of the proposed model are over 60 % for 2012 (*61.01 %*) and 2013 (*69.73 %*), and even over 70 % for the fiscal years of 2008 (*76.31 %*) and 2015 (*75.69 %*). For other datasets, these figures are over 40 % and 50 %. Moreover, about 60 % progress in terms of the average performance for all years is obtained. Similar conclusions and arguments can be drawn from the comparative results for MdRAE given in Table 10.

Table 12
RMSE metric values for IEX data sets.

Prediction Model	TIME SEIES/IEX DATA SETS/# of Testing Set										RMSE'sAverage
	2009		2010		2011		2012		2013		
	7	15	7	15	7	15	7	15	7	15	
ARIMA	345	540	1221	1612	1058	1130	651	621	1362	1269	981
ES	345	540	1208	1612	1057	1130	651	621	1362	1269	980
MLP	325	525	1077	1603	920	1096	775	783	1315	1233	965
SC	1402	1754	1128	1742	1396	1360	1292	1047	1450	1931	1450
FF-T1	446	534	1180	1852	1083	1146	1034	1038	1512	1279	1110
FTS-N	267	514	1050	1357	765	917	590	582	786	1208	804
ANFIS	405	647	1141	2033	1007	1134	634	938	1447	1413	1080
MANFIS	261	503	1144	1303	960	1009	634	629	1418	1264	913
AR-ANFIS	240	467	1136	1451	987	999	631	619	1362	1256	915
I-TSFIS	166	1046	250	251	817	384	277	228	451	1106	498
I-FRF	240	507	963	1390	658	994	296	530	690	1172	744
CIPM	**80**	**309**	**56**	**89**	**66**	**89**	**72**	**128**	**213**	**397**	**150**
Progress (%)	**52 %**	**34 %**	**78 %**	**65 %**	**90 %**	**77 %**	**74 %**	**44 %**	**53 %**	**64 %**	**70 %**

6.2. DIJ and SSEC implementations

The DIJ and the SSEC time series, observed from 2008 to 2018, the other financial datasets are analysed. The results are compiled as an average of fiscal eleven years in terms of three error metrics, and the results are summarized in Table 11 as comparatively. From Table 11, it is obviously observed that the proposed CIPM provides excellent predictive performance for both DIJ and SSEC datasets. CIPM for both financial time series provides a remarkable improvement in prediction performance in terms of all average RMSE (63.93 %; 53.87), MAPE (52.17 % 56.52 %) and MdRAE (47.38 %; 55.40 %) metrics.

6.3. IEX implementations

The prediction performance of the proposed CIPM is finally tested on IEX data sets as another financial time series, and the results are compiled and examined in detail. The spectacular prediction performance of the CIPM is proven by the findings inferred from Table 12, once more. The CIPM produces superior predictions for all IEX datasets compared to other models. The prediction performance improvements ensured by CIPM are over 30 % in one implementation (IEX2009/15 = 34 %), over 40 % in another implementation (IEX2012/15 = 44 %), and over 50 % for two implementations (IEX2009/7 = 52 %; IEX2013/7 = 53 %). Moreover, it is observed that improvement levels are observed over 70 % in some other implementations. Even for IEX2011/7, it is reached 90 %. Besides very low RMSE and very high progression rates, Fig. 8 represents the visualizing of the high harmony between observed and predicted values as another indicator of superior prediction performance. The red lines in Fig. 8 represent the error size for each point of the time series, i.e., the differences between the observed and predicted values. The fact that these lines are quite short and the observed and predicted values almost overlap at many time points is another indicator of the respectable prediction performance of the CIPM. In addition, the MAPE and MdRAE values given in these graphs are also quite low, proving that CIPM would be a preferable prediction tool for IEX datasets.

7. Conclusions

Intuitionistic fuzzy sets consider hesitation degree and also produce non-membership values as well as membership values in the modelling process. In this respect, they offer a deeper and more sensible approach in the characterization of the uncertainty compared with conventional fuzzy sets. In this direction, time series prediction models based on intuitionistic fuzzy sets give successful results in financial time series prediction problems. However, a notable problem for intuitionistic fuzzy-based time series prediction models is that almost all of them just focus on modelling linear relationships, while some only model non-linear relationships. Considering that most of the financial time series may contain both linear and non-linear relationships, this may cause serious shortcomings and defects in the prediction process. To address these shortcomings, we created a new prediction tool that can co-model linear and nonlinear relationships between inputs and outputs in a time series prediction problem. From this point of view, we performed cascade neural network in the determination of intuitionistic fuzzy relations stage. A cascade neural network can accommodate the non-linear relationship between input and output without eliminating the linear relationship between the two.

To observe and test the prediction performance on financial time series of the proposed CIPM, 48 individual implementations are realized on TAIEX (2000–2004), TAIEX, DIJ, and SSEC (2008–2018) and IEX (2009–2013, with two different testing set size) time series.

For TAIEX (2000–2004) data sets, for the best-case scenarios, the progress level of the proposed CIPM, between 60 % and 80 %, over the second-best of the other prediction models. The CIPM shows more than 80 % progress over five years, on averagely. Even for the worst-case scenario, the CIPM exhibits a marked improvement with a ratio of more than 40 % on averagely over five years. The variation of error metrics has been obtained very low in 84 alternative implementations for both training and test sets, proving that the proposed CIPM is quite strong in terms of consistency/reliability, besides the validity. Moreover, convergence times are between 2 and 4 s, and such low computation time can be seen as a significant advantage, particularly in financial time series problems where accurate and fast decision making is crucial.

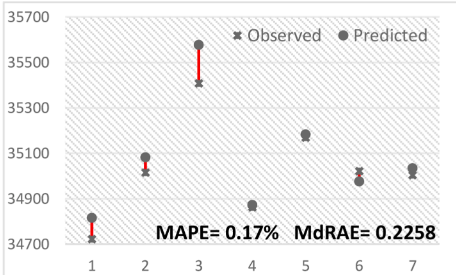
For TAIEX (2008–2018) data sets, the CIPM, in terms of the RMSE criterion, provides between 40 % and 70 % improvement in prediction performance over the second-best one among the other prediction models. Similar results have been obtained in terms of MAPE and MdRAE metrics from comparative evaluations.

For DIJ and SSEC data sets, the CIPM, in terms of each of three error metrics, as an averagely, provides a remarkable improvement in prediction performance with the progress level around and over 50 %.

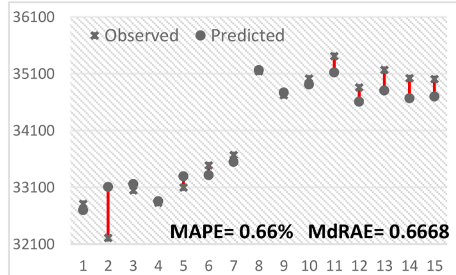
For IEX data sets; The CIPM, in ten implementations, provides a progress level in prediction performance from 30 % to 90 %.

Eventually, as a result of the analysis of 48 financial time series evaluated separately for each fiscal year, CIPM showed an outstanding prediction performance well above the satisfactory level compared to state-of-the-art prediction models. The main reasons for this situation;

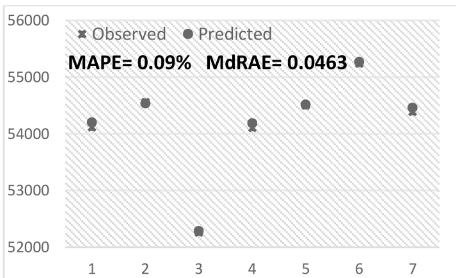
- The proposed CIPM uses a deeper approach for uncertainty and thus uses more information.



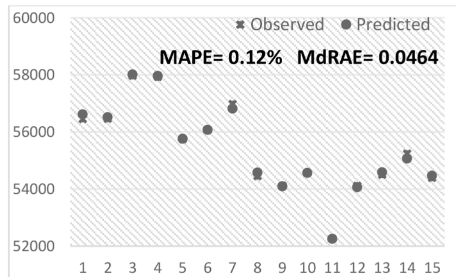
(a) IEX20009 / Test Size 7



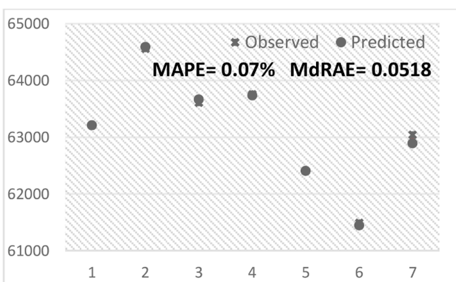
(b) IEX20009 / Test Size 15



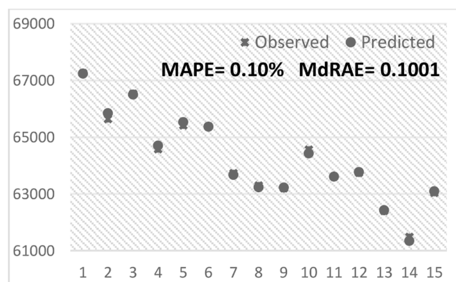
(c) IEX20010 / Test Size 7



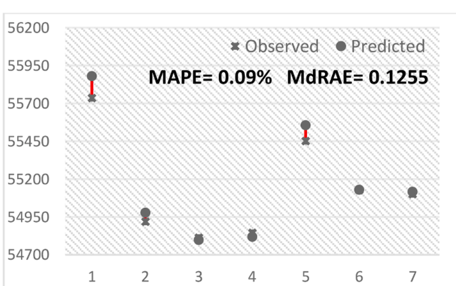
(d) IEX20010 / Test Size 15



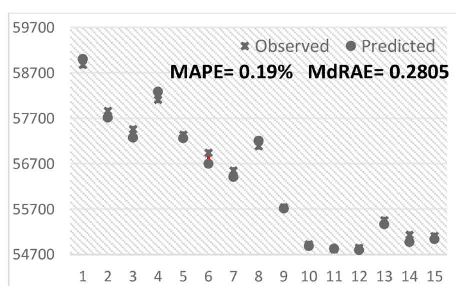
(e) IEX20011 / Test Size 7



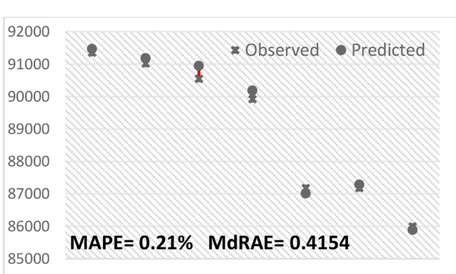
(f) IEX20011 / Test Size 15



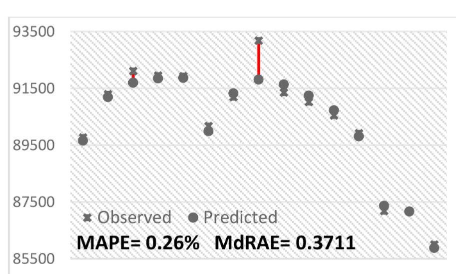
(g) IEX20012 / Test Size 7



(h) IEX2012 / Test Size 15



(i) IEX2013 / Test Size 7



(j) IEX2013 / Test Size 15

Fig. 8. A visual presentation of the predicted and the observed values for IEX.

- The proposed CIPM, thanks to its cascading structure where each layer uses the information available in the previous layers, can model both linear and non-linear relationships simultaneously while executing the prediction process and is able to easier adapt to the solution surface.

Future research can focus on the use of neural networks with different structures, and fuzzy sets with different perspectives in order to enhance prediction performance further.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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