

WCES 2012

Using mathematics history to strengthen geometric proof skills

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Abstract

In recent years, there has been an increasing interest in integrating the history of mathematics in teaching-learning activities in the classes. The main aim of this study is to determine how such activities strengthen students' geometric proof skills and their spatial perception. This research is designed as a multiple case study. The working group is 15 students from 11th grade in a high school. Students have worked on four learning activities from the history of mathematics related the volume of solids. Then, they are answered to a questionnaire consisted of 7 open ended questions. Most of the students have accomplished geometric proofs by using dissection method. Using the smaller shapes whose volumes are easy to compute has strengthened students' geometric proof skills. Moreover, the chosen activities from the history of mathematics have attracted students' interests. To ask the volume different frusta of pyramids and prisms and to use the history of mathematics in classrooms can be considered as recommendations.

Keywords: *History of mathematics, Geometric proof skills, Solid's volumes*

1. Introduction

In daily life we see many of geometric shapes around us. To recognize them and to determine the relationship between them is important in development of geometric thought. The geometric thought takes part at the heart of the identification of the space and the development of spatial skills (drawing, producing modals and making chances on them, landscaping etc.) (Altun, 2008). Pierre and Dina Van Hiele Geldof made some important researches about how children learn geometry. In their research, they postulated five levels in order to describe how children learn to reason in geometry. They are labeled as 0, 1, 2, 3, 4. The third level, namely "Deduction", correspond to the high school grades. Students at this level construct geometric proofs by themselves and can see different applications of a theorem. At third level, properties of the shapes become an independent object of the shape and form. Being an abstract course, geometry is difficult for most of the students. Especially, students may have difficulties on the problems of 3D objects' volumes which require abstract thinking. Students can use the shapes whose volumes are known to compute, for calculating the asked volumes at this stage. This procedure to solve problems may excite them, further the inclusion of mathematics history may increase their motivation.

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There has been interest over several centuries in the relations between the history of mathematics and the teaching and learning of mathematics (Fauvel, J., Maanen, J. V., 1997). Integrating history of mathematics into classroom teaching has many benefits for both teachers and students. For example, it is an effective means of motivating students to learn (e.g., Fauvel, 1991; Swetz, 1994). In addition, integrating history of mathematics into mathematics teaching can help teachers and students explain many of the *why* questions that may arise in the classroom (Bidwell, 1993; Kelley, 2000). For example, students might ask questions about the origins of certain computational methods, notations, and words we currently use within the mathematical community (Bidwell, 1993; Kelley, 2000; Rubinstein & Schwartz, 2000; Tzanakis & Thomaidis, 2000).

Furthermore, the integration of history of mathematics into school mathematics is an excellent means of recognizing the many connections that exist among the various mathematical areas and between mathematics and other subjects (e.g., Furinghetti & Somaglia, 1998; Grugnetti & Rogers, 2000). Jankvist (2009) also mentions history as a pedagogical tool that can give new perspectives and insights into material and even can serve as a guide to the difficulties students may encounter as they learn a particular mathematical topic. For example, teachers can demonstrate the relationship among arithmetic, algebra, and geometry by drawing on the work of Euclid, Al-Khwarizmi, and Descartes, and the strong links among mathematics, geography, engineering, and astronomy can be (interestingly) found in the development of navigation instruments and calculation devices (Grugnetti & Rogers, 2000). In addition, students may realize the interaction between mathematics and society (Swetz, 1995), and how mathematics has influenced “the ways that people operate in, and think about the world” (Wilson & Chauvot, 2000), such as the new information and communication systems that are so central to today’s world life. In the report of NCTM (2003) Standards for Programs for Initial Preparation of Mathematics Teachers, it is emphasized that the historical developments of mathematics should take place in schools from different cultures.

Indeed, the history of mathematics is a living science that “creates a bridge from the past to the future” (Reimer & Reimer, 1995) and it shows “how the conjunction of the old and the new is a commonly occurring event in the development of mathematics today” (Grugnetti & Rogers, 2000). Hence, the history of mathematics can help students see how mathematical ideas were generated in the past (Furinghetti & Somaglia, 1998), and how many of the concepts they deal with in their classrooms and everyday life, as well as several techniques they use today to solve problems, have been around for quite a long time (Rice, 1998).

In other researches, it is revealed that the use of mathematics history in mathematics courses affected positively the attitudes of students towards mathematics (McBride & Rollins, 1977). Moreover, Jardine (1997) determined that if calculus students are interested more in subjects from history of mathematics, then their motivation about learning mathematics is increased. Fauvel (1991) suggest that in spite of integrating the history of mathematics into classroom activities is not result in excellent changes in student motivation, it can provide a new perspective about mathematics and students think mathematics is a human work. Furthermore, Furinghetti (1997) in her research about the relationship between history of mathematics, mathematics education and school activities, found that solving problems from history of mathematics also affect students those who are not interested in mathematics.

The importance of the use of the history of mathematics in teaching and learning mathematics has been emphasized in recent years (Favvel & Maanen, 1997; Marshall & Rich, 2000; Wilson & Chauvot, 2000). However, in our country, such studies merely exist. We encountered a few studies in the recent years’ literature. One of them is Bütüner’s (2008) application of instruction and other studies are Gönülateş’ (2004), Tözluyurt’s (2008), Gürsoy’s (2010), Albayrak’s (2011) master theses. Therefore, we did a study about the use of the history of mathematics in mathematics teaching in Turkey. Students, in geometry course, have worked on four learning activities from the history of mathematics related the volume of solid and have accomplished geometric proofs by using dissection method. For this study, a worksheet, namely “Construct the Volume”, is used as an in-class activity. From the ancients, Egyptians and Babylonians attained the volume formula of frustum of a square pyramid and a rectangular pyramid. Students are asked to attain same formula. Furthermore, in this study, to show students that many mathematical statements from history can still be used nowadays and that mathematics has a living and alive construct and it is related to its past.

2. Methodology

In this study, the general methodology used for gathering and analyzing information is the multiple case study procedure defined by Yin (2009). This procedure uses multiple case studies as replications, not as sampling procedure (Yin, 2009). “Multiple cases resemble multiple experiments” (Yin, 2009). The working group is 15 students from 11th grade in a high school in Turkey. 15 students are divided randomly into 4 groups.

2.1. Instruments

Students have worked on four learning activities from the history of mathematics related the volume of solids. Then, they answered a questionnaire consisted of 7 open ended questions to learn their thought about these activities.

The worksheet including four learning activities is distributed to each student in the class. In the first question a frustum of a square pyramid shape was given and said ancient Egyptians knew how to volume the shape. They were asked to achieve the following formula.

$$V = \frac{h}{3}(a^2 + ab + b^2)$$

The ancient Chinese built burial chambers that were similar in shape to the frustum of a pyramid. The diagram of such a chamber is demonstrated in worksheet. In the second question they were asked to achieve the following formula.

$$V = \frac{1}{6}[(2a + c)b + (2c + a)d]h$$

The ancient Babylonians also had a formula for the volume of a truncated square pyramid. In the third question they were asked to demonstrated that the following formula is equivalent to the one obtained by the Egyptians.

$$V = h \left[\left(\frac{a+b}{2} \right)^2 + \frac{1}{3} \left(\frac{a-b}{2} \right)^2 \right]$$

Finally there was an open-ended question in the worksheet: “Why do you think all these ancient societies were concerned with the volume of the frustum of a pyramid?”

2.2. Data Analysis

The qualitative measures were used while analyzing the data. The data obtained from worksheets were classified by the answers whereas activity papers and semi-structured interview forms.

3. Results

3.1. The results of learning Activities

For the first question, all of four groups used the dissection method. But one of the 4 groups also tried to complete the frustum of the pyramid to top. All students dissected the shape into nine pieces and obtained two square pyramids and one square prism. They computed the volume of the shapes separately by calculating sides of each shape and then by combining the formulas they attained the Egyptians’ one.

In the second question, all groups successfully achieved the asked formula by using the given hints. Students realized that unlike first question, there are two different rectangular prisms in the skeletons. After added up the volumes of the pieces, students got the perfect match of formula by factorisation.

For the third question, all groups except one answered the question. That group did not answer the question in given time. One other group redrew the frustum of the pyramid and tried to get the formula by themselves. They finally found the Egyptians formula after arranged the algebraic expression given in their square format.

Some of students’ answers to the open-ended question “Why do you think all these ancient societies were concerned with the volume of the frustum of a pyramid?” are given as follows: “Because the ancient sacred buildings of Babylonians and Egyptians are of the form of a frustum of a pyramid.”. “This type was used to

construct the buildings necessary for people in agriculture, religion, astronomy. It can be deduced that this form was seen in many buildings at that age.”, *“Since it had a sacred meaning, people from that age used it most of the time. Therefore, they were very interested in it.”*, *“Because of daily wants...”*, *“The frustum of pyramid exist in many parts of our life.”*, *“To calculate the quantity of materials and the resistance of buildings.”*

3.2. The results of questionnaire

After completing the worksheet the following evaluation questions which are in questionnaire were also asked to obtain their views about the activities. “Did you do an activity like this before?” is asked as the first question. While 14 students replied saying yes, 1 student said no.

Second question was “Did you like this activity? Do you think this activity is useful ? ” All of the students thought it was a beneficial activity. They were also said the following answers: *“I saw mathematics is working.”*, *“Yes, advantage is: It helped me to divide big shapes. I saw the formulas where came from”* *“Yes, it was nice. To find where formulas come from pleased me as if I found the formula firstly.”*

In 3th question, they are asked to identify the parts they had difficulty. *“To sum the formulas”*, *“To think 3D”*, *“Complex formulas”* were given as answers. For the next question “Do you want to do such activities in courses”, all students gave a positive answer except one said no, and other one said sometimes.

In the 5th question they asked to define what they gained from this activity. Each of them gave different answers. Some of the answers are *“I saw mathematics was working in daily life.”*, *“Thinking ability as a group”*, *“Mathematical reasoning”*, *“It gave me a chance to almost practically apply what I learned in class. Plus, it taught me a little about ancient civilizations.”*

In the 6th question, we asked them to associate this subject with daily life. All students except one made a association with real life. Some of them associated the subject with pyramids, other with the architecture. One student gave the following answer without any intervention: *“Real shapes → Situations → Life”*

In the last question, students are asked to give suggestions. Some of the answers are as follows:

“Some activities regarding the formulas used in technology can also be made”, *“I may have been better if in the 3rd question we learned how we Babilis came up with that formula in the first place. Perhaps then we could apply it in real life. It might even be easier than the formulas we have.”*, *“The volumes of shapes different than the frusta of pyramids can also be asked”*

4. Conclusions and Recommendations

Consequently, using the smaller shapes whose volumes are easy to compute has strengthened students’ geometric proof skills. Moreover, the chosen activities from the history of mathematics have attracted students’ interests. Students obtained formulas for the volume of geometric solids by dissecting the solids into simpler solids whose volumes could easily be computed then summed up the volumes of the individual pieces to obtain the volume of the whole.

Students were expected to construct their own knowledge through active investigation and group collaboration. We discuss benefits, objections, difficulties, and ways of integrating history of mathematics into classroom instruction. We firmly believe that the history of mathematics has much to contribute to the improvement of the teaching and learning of mathematics and it can become a tool for effective teaching (Fauvel, 1998; Wilson & Chauvot, 2000). Also it can help both teachers and students meet the challenges posed by the current reform in mathematics education. These results are considered as parallel with the studies of Fauvel (1998) and Wilson & Chauvot (2000).

In general, teachers have little knowledge and preparation from their teacher education programs about integrating the history of mathematics into their teaching. Therefore, there has been a worldwide increasing concern about educating both pre- and in-service mathematics teachers about history of mathematics and its integration into the classroom (Van Maanen, 1997). Therefore we deduce that this type of activities can stimulate and develop students’ mathematical communication skills and understanding of mathematical connections.

To ask the volume different frusta of pyramids and prisms and to use the history of mathematics in classrooms can be considered as recommendations. Different learning activities can be edited in teaching-learning process. If in mathematics lessons historical articles, images, tools is used it may be attracted more interest. At the same time examining the writings of famous mathematicians can provide to develop different ways of solution in problem solving.

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