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The Analysis of Understanding Factorial Concept Processes of 7th Grade Students who have Low Academic Achievements with Pirie Kieren Theory

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Abstract

The purpose of this research is to analyze the understanding factorial concept processes of 7th grade students who have low academic achievements with Pirie-Kieren theory. The research participants are composed of two 7th grade students who are studying at a public primary school in the İnegol Province of the city of Bursa. This research has been designed according to case study design since it was carried out with two students who had low academic achievements in a constructive learning environment. The research participants were selected among voluntary students by considering their mathematics achievement test scores and their grades stated in the previous year' reports. Mathematics achievement test was composed of 15 questions that were covering pre-cognitive knowledge of the factorial concept. Educational games, group work, puzzles, worksheets and activities designed for the objectives in the factorial concept were used in the research. The data of the research was collected by through a semi-structured interview, participant observation and document analysis for each student in the study group. The obtained data was analyzed by using a descriptive analysis method. According to the findings, students who have low academic achievements were able to construct concepts difficulty by interpreting activities and they reached formalising level according to Pirie and Kieren theory. In the light of the findings of this study, the researchers have developed suggestions.

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Keywords: the dynamic model of Pirie-Kieren, low academic achievement, learning to learn, factorial

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1. Introduction

Even though the term of 'understanding' is freely used in literature, a search for a substantial definition of understanding is still on progress. According to Sierpinska since most of the writers used understanding under the assumption that it is a well-defined concept, they had difficulties in their efforts to examine its meaning through philosophy (Sierpinska, 1990b, p.24). It can be said that these difficulties are caused from the inability of mathematics educators to distinguish between understanding and knowledge before Skemp's (1987) definition of instrumental understanding against relational understanding. (Sierpinska,1990a, p.34). Pirie and Kieren (1994) developed a two-dimensional model which they named as 'the dynamic model of understanding' for understanding the concept of understanding and consisted of 8 intertwined layers in order to be able to monitor at which processes and how an individual structures a new subject or concept.

1.1. Piere Kieren's Understanding Model

Pirie and Kieren (1994) developed a two-dimensional model which they named as 'the dynamic model of understanding' and which was composed of eight intertwined layers in order to monitor at which processes and how an individual structures a new subject or concept for understanding 'understanding'. This model was presented in figure 1. In this model, there are eight potential layers that may occur on any matter for each individual at different levels in the developmental process of understanding (Pirie and Kieren, 1994, p.170). These layers are called as; primitive knowing, image making, image having, property noticing, formalizing, observing, structuring and inventizing. This theory claims that student moves into graded understanding layers at different levels by constructing new knowledge with his/her pre-cognitive knowledge (primitive knowledge) (Cavey, 2002, p.3). Each layer here implies a qualitative change in the development of the understanding of a learner (Meel, 2003, s.163).

The development process of understanding starts with the layer which is in the core position of the model and called as 'primitive knowledge'. Primitive knowledge does not mean low mathematics level. The primitive knowledge layer contains pre-cognitive knowledge which is necessary for students to construct new concept that he/she will learn (Pirie and Kieren, 1994, p.68). The second layer is expressed as 'image making'. In image having layer, it is expected from a person to create differences in his/her previous knowledge and to use them in the next understanding layer (Pirieand Kieren, 1994, p.170). At this layer, the learner engages in activities in order to have an idea regarding what the concept is about (Lyndon, 2008, p.65). The third layer is stated as 'image having'. This layer is the stage of creating an image in learner's mind about the subject. The learner needs a new activity for re-constructing the concept (Pirie and Kieren, 1994, p.170). The first 'no need to limits' stage in Pirie and Kieren's model occurs between image making and image having layers. The learner once reached the image having layer can use the knowledge that he/she obtained without the need to re-construct many times the itemsat the primitive knowledge layer or images at the image making layer (Pirie and Kieren, 1994, s.173).

The fourth layer was stated as the 'property noticing'. The learner at this layer can recognize the different features of the concept that he/she learnt through the images that he/she created in his/her mind. At the fifth layer of the understanding which is 'formalizing' the learner can generalize a newly recognized knowledge by using an image that he/she has before; besides he/she may state the reasons of the generalizations that he/she made through formalizing concepts by depending his/her previous images (Pirie and Kieren, 1994, p.171).

At the development process of understanding, the sixth layer is stated as 'observing'. At the observing layer, the learner can make a planning through organizing his/her personal thinking process by recognizing the branching in the thinking process (Pirie and Kieren, 1994, p.171).The person who is learning at this layer reached the position of expressing the coordination of theorems with each other and reflecting what he/she learnt. The seventh layer is expressed as 'structuring'. The people who are learning at this layer can express their thoughts as mathematical

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structures by postulating their previous experiences as a new theory (Pirie and Kieren, 1994, p.171). The outmost layer of the Pirie-Kieren's growth of mathematical understanding model is expressed as 'inventizing'. The person who is learning at this layer was completely structured understanding, for this reason he/she accessed a completely new understanding level and in a position to be able to generate new questions by expanding his/her previous mental structures (Pirie and Kieren, 1994, 172). The intertwined layers are showing that the development in understanding is not linear and unidirectional. Although the rings of the model are growing outwardly more abstract and more general, the development in understanding do not occur in that linear way.

It is not necessary to have visual or illustrated presentations at image making layer, the images can be expressed verbally or through activities (Lyndon, 2008, p.65). As a requirement for the philosophical dimension of the nature of mathematics, mathematical will be convenient to see this layer as an inventizing layer for the individuals whose meta-cognitive level is high at the process of structuring knowledge and when primary school level students are considered as discovery layer.

Instead of this, the development in understanding occurs in constant forward and backward movements through the layers of knowing' (Lyndon, 2008, p.65). The coming back movement for the purpose of organizing existing understandings and ideas about a mathematical concept at inner layers is called as fold back according to Pirie-Kieren Theory knowledge is inventizing for some and for others it is discovering.

It is thought that it Pirie-Kieren theory with its layered model; guides for observing and defining the process thoroughly in which knowledge is structured through repetitive organizations and about the perceptions of students, how they think and how they structure their thoughts with a suitable way. The fact that Pirie-Kieren accepts understanding as a process rather than sensing focuses our study to analyze the process of students' understanding any subject in their minds according to this theory. The purpose of this study is to analyze how the understanding factorial concept process of 7th grade students who have low academic achievements is developing according to Pirie-Kieren theory.

2. Method

2.1. Research Design and study group

The case study method was used in the study. This study is carried out with 2 student whom is a special case and who are grouped as low academic achievements in a constructive learning environment where designed activities regarding 'operations with integers' sub-learning field. As the purpose of this study is to deeply analyze understanding factorial concept processes of these students according to Pirie-Kieren theory, this study serves as an example for the 'multiple case' design among qualitative research methods. The study group is consisted of two volunteer 7th grade students who have low academic achievements by looking at their mathematics scores given in their previous term reports and applied achievement test results and who are studying in a primary school in Inegol district of Bursa city.

The pre-cognitive knowledge and mathematics achievement levels of the selected students are parallel. It was not paid attention to take equal number of female and male students since the purpose of the study did not include comparing female and male students. It was paid attention that students were able to explain how they solved the problems in the activities and to express their own thought since the purpose of this study is to deeply and multi-dimensionally analyze the mathematical understanding processes of students.

2.2. Data Collection Instruments

The achievement test, activities implemented with students and video recordings were used as data collection instruments in this study.

Achievement Test: It is a 20-question multiple choice test including subjects such as arithmetic four operations, sets, fractions, percentages, decimal numbers and linear equations for testing pre-cognitive knowledge of students to understand 'Permutation' and 'Factorial' concepts. The document analysis was used among qualitative research techniques for collecting qualitative data. The documents; provide real and easily accessible data and give

opportunities to researchers to produce solutions to problems (Yıldırım and Simsek, 2005). For this reason, the data consisted of the answers of the students to activities used within the scope of this study were used in order to support to data obtained through semi-structured interview and participant observation and to make alternative explanations to obtained results. For data analysis, the interviews were transformed into written forms.

2.3. Operations

The following objectives stated in the 7th grade teaching program for the subjects of numbers, probability and statistics according to new mathematics program (Ministry of Education, 2005) were taken into account within the scope of this study; i) Compares the basic principles of counting and uses them in problems; ii) Finds the factorials of natural numbers and iii) Explains the concept of Permutation and calculates. Since an evaluation only about the first two of these objectives was carried out in this study, the activities regarding these two objectives were included. The related literature was scanned before building up activities and the activities which are suitable to objective were used by the researcher through analyzing the previous studies carried out about this topic.

2.4. Data Analysis

The worksheets that students used for solving the activities and the dialogues between students during the activities were used as the source of data. All the collected data were analyzed by using descriptive analysis method.

Each student in the study group was coded with letters of S (with high academic achievement) and E (with high academic achievement) which are the initial letters of their own names. The data of semi-structured interview, participant observation and document analysis were taken into consideration as a whole respectively for each student whose academic achievement is low in the study group and the research question was answered by synthesizing them. So that the understanding processes of the students are presented by drawing a detailed map of their knowledge structuring processes according to Pirie-Kieren dynamic model.

3. Findings and Comments

In this part, the findings are included about at which layer Canan and Fatma with low academic achievements are in the process of constructing information according to the Pirie-Kieren dynamic model.

Primitive Knowing Layer

By looking at Canan (C) and Fatma's (F) achievement test results, it can be said that their primitive knowledge are inadequate for constructing permutation and factorial concepts in activities.

Image Making Layer

In the first activity entitled as 'Can you help Ayse?', for the question asking in how many different ways Ayse, the heroin of the event, can wear a shirt or a pair of pants among the clothes, Fatma by reading out loud the fictionalized story at the beginning of the activity and Canan by taking the material into her hand started to try on the clothes that Ayse can wear.

1 C: (Choosing a pair of pants and trying the shirts that can be worn on it) I.M.L

You can wear 1,2,3,4 different shirts on this pants, it was 4.

2 C: Five, six and seven…(While trying with the second pants, I.M.L skipping one of the shirts! 8, 9, 10, 11 (trying all the shirts that can be fit with the third pants) 12,13,14,15. I found 15 shirts!

3 F: (Choosing a pair of pants and I.M.L is starting to try on the shirts that can be worn on it) 1,2, 3.

In the conversations stated at the 1st, 2nd and 3rd lines of the above dialogues, it is seen that Canan and Fatma are using all the materials available for finding probable situations. It can be said that since two friends created an activity by studying together, they are at the image making layer according to Pirie and Kieren theory.

Later on in the activity, as a result of the image making activities, it is seen that Canan did not need to use the materials available for finding the probable situations.

4 C: There are 4 shirts and 4 pairs of pants. Are we going to try for all of them? I.H.L

4 different shirts can be worn with a pair of pants. The 4 times 4 is 16.

5 A: How many different ways can Ayse wear a shirt? (The teachers deliberate focused intervention)

6 C: 4 or this one or this one or that one (by pointing out the shirts) I.M.L

7 A: Well, then how many different ways can Ayse wear a pair of pants?

8 C: 4 I.M

By looking at the situation displayed on the 4th line, it can be accepted that as a result of making an image in mind, Canan moved on the next layer called as image having according to Pirie-Kieren theory. Later on, because of the erroneous image that Canan created in her mind about the question, the researcher was aimed to guide Canan to feedback from image having layer to image making layer by using the question form given on the 5th line. After the researchers' intervention, it can be said that students were able to make a new image at the end by effectively folding back to image making layer.

Image Having Layer

As a result of the activities at image making layer, is seen on the 32nd, 33rd, 35th and 36th lines that students moved into image having layer.

32 C: 4 types for all, in total it becomes 12 I.H.L

33 F: Yes, it becomes12. I.H.L

34 A: How can you express your result mathematically in short? Explain.

35 F: We multiply them as there are 4 types of sweet and 3 types of salty things. I.H.L

36 C: Yes, we multiply. I.H.L

37 A: If Ahmet's mother had said that you could have eaten one type of sweet food or one type of salty food,

How many different ways could Ahmet have chosen?

On the above dialogues, the purpose of the researcher is to guide students to image making in order for them to make more images by using the question form given on the 37th line. Later on, it can be said by looking at the situation on the 40th line that Canan moved into image making layer but a failure occurred in her understanding as a result of the activities carried out. Canan could not move into the next layer by effectively implementing fold back. The researcher intervened by using the question form on the 43rd line to Canan for repairing the failure in her understanding in order for her to be able to make a correct image in her brain. It can be said that students moved into image having layer according to Pirie-Kieren theory by looking at the situations on the 52nd and 53rd lines in the dialogues below.

38 F: because she said or…

38 C: There can be 3 kinds of salty food together with a sweet. It was one, two and three. I.M.L

There can be four types of sweet food together with a salty one. Hımmm..

40 C: (reading out loud the question again) One kind of sweet or I.M.L

one kind salty…8, uffff Is it 8? What about trying with a sweet one? (Failure in understanding)

There will 3 types, if we try with one salty, there will be 4 types, in total 7…

but the answer should be 3, I got confused?????

41 A: Why do you think that the answer should be 3?

42 C: For example, we chose one type of salty food; we will try the sweets that can be good with it.

43 A: How many different options do we have for the sweets?

44 F: 4 I.H.L

45 A: For the salty ones?

46 C: 3 I.H.

Property noticing Layer

The students in the study group formed for making them to feel that there is a correlation between general multiplication features and ordering. According to the story fictionalized in the fourth activity entitled as 'We are all colourful' Garfield the cartoon hero wonders about in how many different ways 3 boxes in different colours can be arrayed. After Fatma had read the first question which was asking from them to make all the possible orders by

changing the orders of the boxes and to list the situations that they founds, she started to order the boxes for solving the question.

120 A: Do you think that there is correlation between the numbers you found (4,3,2,1) and the order of four boxes?

121 C: The four boxes ordered in 24 different ways …Then… P.N.L

Yes, there is! 3 times two is 6. 6 times four is 24. If we multiply the numbers, we will find 24.

122 F: We will multiply the numbers P.N.L

In property noticing layer, students can analyze a mental image and determine features regarding the image. When the given situation on the 121st and 122nd lines of the dialogues above is analyzed, it is seen that Canan and Fatma can analyze the existing mental images about the order of the four boxes and can produce some knowledge depending on this.

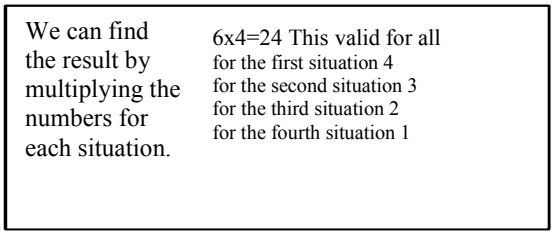

Figure 1. Fatma's worksheet

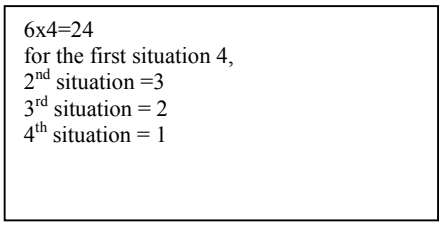

Figure 2. Canan's worksheet

Formalising layer

123 A: Well, if we had 5 boxes, then in how many different ways could we order them?

(The researcher added another box to the table)

124 C: There are 5 different colours for the first row P.N.L

4 for the second

125 F: Later 3,2,1. P.N.L

126 C: How come? 5, 4,3,2,1 (Gurbet takes a note to the worksheet). F.L

Let's multiply them. Five times four is 20. Twenty times three is 60.

Sixty times 2 is 120. The answer is 120.

127 F: hımmm, can't we find it by trying? P.N.L

128 C: It becomes more different in that way, we get confused F.L

The researcher uses the question form given on 123rd line of the dialogue stated above for students to advance their understanding by generalizing the image in their minds. After the researcher's intervention, it is seen form the 126th and 128th lines that students moved into formalising layer according to Pirie-Kieren theory since they could define the information that they just noticed by using an image that they had before.

After the students in the study group had completed the fourth activity, they moved on for the fifth activity entitled as 'Excited Numbers'. The fifth activity was designed to teach the factorial concept to the students and to have them find the results of operations with factorials. Since the students with whom the activity was carried out have seen the factorial concept for the first time, they are asked to make a definition of the concepts after providing them examples about the concept.

In our study, in parts till now, it was tried to analyze according to Pirie-Kiren theory the development of the understandings of students who have low academic achievements with activities designed under the name of constructing the concept of factorial.

4. Conclusion, Discussion and Implications

Fatma from the students who have low academic achievements could proceed to the property noticing layer of the Piere and Kieren's theory. Fatma's movement to the property noticing layer did not show a direct linear movement and continued as constant fold backs. Especially, the student before moving into the property noticing layer, often made fold backs from image having layer to image making layer. Moving into image having layer without making images could occur after two activities. The student needed to study with more materials for constructing her knowledge well through staying at this layer longer. According to Simon (2002, 2006) image making layer is the key for mathematical understanding. The understanding in this form is very important for acquiring conceptual learning skills of mathematics.

The student named as Canan was able to reach formalising layer through constant fold backs between Piere ve Kieren's first four layers of with respect to her friend. Also this student spent a lot of time at the transition between image having and image making layers as her friend. The transitions between these layers are not linear and Canan could move into property noticing layer by using the knowledge that she constructed at image having layer although there were failures in her understandings. The mathematical understanding at this layer requires from student to be able to use the concept that he/she constructed in his/her mind without needing his/her previous knowledge. At the end of this process, it is seen that the student transmitted her mathematical understanding to the formalising layer by generalizing the factorial concept which was presented to her after noticing the multiplicative feature of this concept. It is thought that focused-deliberate teacher intervention is effective on fold back situations between layers in moving into this layer. This finding obtained in the study is parallel with the studies carried out by Borgen and Manu (2002), Pirie and Kieren (1994).

Both students mostly folded back at 'image making' layer among layers stated in Pirie-Kieren's dynamic model. This process did not occur because of the students themselves but it occurred after teacher's focused intervention. For this reason, it can be said that primary school students do not lead themselves to fold back. This finding obtained in the study supports the studies carried out by Lyndon (2008), Pirie and Kieren (1994) and Pirie (2002).

It is observed that students strengthen the less developed understandings at the inner layers while folding back. In this case, it can be said that fold backs which are similar to types that students studied at a more inner layer by using their current understandings are effective on students' understanding levels. This finding obtained in the study is also supported by the studies carried out by Lyndon (2008), Pirie, Martin and Kieren (1996).

Most of the time students have difficulties in understanding the information that was presented to them due to lack of prior knowledge. For this reason, before starting to teach a new subject, it can be suggested to control whether students have necessary backgrounds and prior knowledge which is necessary for students to conceptualize the new information used at the subject matter. On the other hand, even though students had conceptualized the factorial concept at the end of the activity, they were unable to answer the questions given in worksheets due to lack of basic subjects (such as four arithmetic operations) in their prior knowledge. This finding obtained in the study is also supported by the study carried out by Borgen and Manu (2002).

References

Borgen, K.L., & Manu, S.S. (2002). What do students really understand? *Journal of Mathematical Behavior,* 21, 151-165.

Cavey, K.O. (2002). Growth in mathematical understanding while learning how to teach:a theoretical perspective. *North American Chapter of the İnternational Group for the Psychology of Mathematics education, 1*(4), 1078-1088.

Lyndon, C. M. (2008). Folding back and the dynamical growth of mathematical understanding: Elaborating the pirie-kieren theory. *Journal of Mathematical Behavior,* 27, 64-85.

MEB Talim ve Terbiye Kurulu Baskanlıgı (2005). *Ilkogretim matematik 6–8. sınıflar ogretim programı kitabı*, Ankara.

Meel, E.D. (2003). Models and theories of mathematical understanding: comparing Pirie and Kieren's Model of the growth mathematical understanding and Apos Theory. *Journal of Mathematical Education*, 12, 132-174.

Pirie, S, Martin, L. & Kieren, T. (1996). Folding back to collect: knowing what you need to know. In Proceedings of the 20th Annual Conference of the International Group for the Psychology of Mathematics Education. Valencia, Spain

Pirie, S., & Kieren, T. (1994). Growth in mathematical understanding: How can we characterise it and how can we represent it. *Educational Studies in Mathematics,* 26, 2-3.

Sierpinska, A. (1990b). Some remarks on understanding in mathematics. *For the Learning of Mathematics, 10*(3), 24-41.

Simon, M. A. (2002). Focusing on key developmental understandings in mathematics. *Learning, 24,* 990.

Simon, M. A. (2006). Key developmental understandings in mathematics: A direction for investigating and establishing learning goals. *Mathematical Thinking & Learning,8*(4), 359-371.

Skemp, R. R. (1987).1976 *The Psychology of Learning mathematics* (Expanded American Ed.). Hillsdale, NJ: Lawrence Erlbaum Associates.

Yıldırım, A., ve Simsek, H. (2008). *Sosyal bilimlerde nitel arastırma yontemleri.* Ankara: Seckin.