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The examination of elementary mathematics pre-service teachers' spatial abilities

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Abstract

The purpose of this study is to analyze the spatial abilities of pre-service mathematics teachers according to SOLO taxonomy. For this purpose, three students with different spatial skills were selected and clinical interviews were carried out with these students. The data was evaluated qualitatively and analyzed descriptively. A section was created for each of the answers given by students. In addition to that direct quotations from the dialogues were also used. The data collected shows that as the spatial skills of pre-service teachers increase their reasoning skills also increase.

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1. Introduction

According to Sutton and Williams' (2007) definition, spatial skills include rotating objects in the mind, understanding how objects look like from different perspectives, imagining the relationship of objects with others in space and understanding forms in three-dimension. Therefore, spatial skills and geometrical thinking is interrelated. In order to understand how geometrical thinking develops in individuals, it will be useful to analyze geometrical thinking levels created by a Dutch couple Pierre and Dina Van Hiele Geldof (1957). According to their research, each individual passes through from the given levels in the same order even if not in the same age. These levels are

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accepted as sequenced and hierarchical. That is, when individuals fulfil the necessities of lower levels, they pass on another Van Hiele level or higher levels (Clements and Battista, 1992).

Jurdak (1991) in his study made a comparison between Van Hiele geometrical thinking skills and SOLO model used in this study. SOLO levels created for determining the levels of responses provided by students not for determining the levels of students. Therefore in this comparison the aim was generally to determine at which Van Hiele level the students were according to their own responses. In the table below, a comparison was made between Van Hiele geometrical thinking levels and level of responses in each thinking phases of SOLO model.

Table 1. The Comparison between Van Hiele Geometrical Thinking Levels and SOLO Levels

Van Hiele Levels	SOLO Levels
	Pre-structural
Visualization	Uni-structural
Analysis	Multi-structural
Informal deduction	Relational
Formal deduction	Extended abstract
Rigor	

Jurdak (1991) was stated in his study that when the SOLO Model which is valid for all disciplines is applied to geometry, the SOLO thinking levels and Van Hiele geometrical thinking levels which were created particularly for geometry are similar to each other. There are two situations which are not match to each other. First, pre-structural SOLO level is not matching with any of van Hiele levels. This is an understandable situation as the pre-structural SOLO level is either refusing or the insufficiency in focusing. Secondly, van Hiele level ‘Rigor’ is not matching with any of the SOLO levels. This is because there are individuals who are more specifically studying mathematics in the fourth level.

All the generalizations may not be suitable for all the students, thus SOLO model is created in accordance with this thought but it is important in terms of giving some ideas about how teachers on the average should behave. One of the important points is the reality that while passing on a higher level from a lower level, teachers are playing an important role. Providing education by preparing necessary learning environments makes transition between levels easier and provides us to achieve expected level.

According to Halloway (2010) SOLO Model consists of thinking levels. Each thinking level stated in SOLO model includes five sub-levels of response. These levels are also called as ‘SOLO Taxonomy’. These are pre-structural, uni-structural, multi-structural, relational and extended abstract levels. As the complexity of the responses increases the level is getting increase. Besides, as the level increases the skills such as making consistent explanations, creating relations, and thinking by considering more than one situation also increase (Biggs and Collis, 1991; Çelik, 2007).

In accordance with this information, the purpose of this study was to analyze spatial abilities of elementary mathematics pre-service teachers. In the part of analyzing spatial skills, spatial visualization skill among the components of spatial skill was considered and the evaluation and the determination of the student responses were made by using SOLO model.

2. Methods

The study is a qualitative study and the data was obtained through clinical interviews. In order to analyze students’ thoughts deeply, interactive interviews with students are called as clinical interviews (Güven, 2006; Zazkis and Hazzan, 1999). Clinical interviews are one of the most important techniques in analyzing what students are thinking about on a particular subject and eliciting the scope of their knowledge thoroughly (Zazkis and Hazzan, 1999; Goldin, 1998). The most important features which distinguish clinical interviews from others are being semi-structured, clinical and subject field is in the centre. The meaning of the word clinical comes from the fact that interviews take place in environments where students can feel themselves more comfortable than other teaching and learning environments. The researcher decided on the questions but he/she can also ask different questions in order

to analyze students' thoughts in details. For this reason it is semi-structured. It subject filed centred as the main interest is especially some particular mathematics subjects and concepts (Zazkis and Hazzan, 1999).

Goldin (1998) indicates that one of the purpose of using clinical interviews is to have information about individual's mathematical behaviours through problem solving. Thus, recently clinical interviews started to be used in teaching mathematics researches (Güven, 2006; Dindyal, 2003; Çelik and Baki, 2007; Bishop, 1997; Money, 2002). In accordance with this information, it was chosen to use clinical interviews during this research.

2.1. Study group

First of all in order to determine pre-service teachers' spatial ability levels, Purdue Spatial Visualization Test (PSVT) was carried out. According to the results of this test, from each student group that were classified in three levels (bad, average, good), clinical interviews were carried out with three pre-service teachers by selecting one student from each group. Three classrooms were determined by dividing the total score into three according to maximum and minimum scores obtained from the general of the test. While selecting three pre-service teachers among 81 students, it was paid attention to have pre-service teachers who were representing their own classrooms. The selection was made on the basis of willingness. Students were selected from A1 low, A2 average, A3 high levels. R shows researchers' answers in the dialogues.

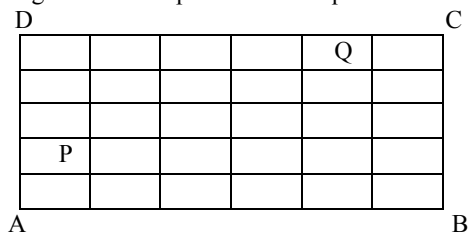
2.2. Data collection tools

The Purdue Spatial Visualization Test which was used in the research for determining the spatial skill levels of pre-service teachers was created by Guay in 1997. The test has 3 sections and 36 questions. In each section, there are 12 multiple choice questions (Bodner and Guay, 1997). The sections are as Developments, Rotations, and Views. These parts were adopted to Turkish by Uygan (2011) and used within the permission of the writer. Besides a permission was taken from George Bodner who worked with Guay to develop the original version of the test. In this study, Purdue Spatial Visualization Test was carried out with 81 pre-service primary mathematics teachers and alpha reliability coefficient was found as 0,834 according to KR-20 reliability analysis. Since the coefficient is higher than 0, 80, as stated by Kalaycı (2010) the scale is highly reliable.

'Geometrical Achievement Test' which tested spatial abilities of students at different dimensions was prepared by the researcher for using during clinical interviews. The test was finalized after the pilot study. Geometrical Achievement Test has 16 questions which is testing the spatial skills. The questions are open ended, multiple-choice and requiring to make drawings. Besides, the questions were designed in a form which requires switching in and between dimensions. There are questions which require thinking in two and three dimension and besides switching from two-dimension to three-dimension and from three-dimension to two-dimension. In this study, there is one question which require switching from two-dimension to three-dimension in Geometrical Achievement Test. The answers provided for this question by each student from three levels were considered in details and the solutions of the students were given. The responses of the students were placed in a suitable level by evaluating according to the SOLO taxonomy.

The question is as in the following:

A Cylinder was obtained by attaching the AB and CD sides of the below cartoon which is in the shape of rectangular and composed of unit squares.



When an ant in the P point of this cylinder advanced to the Q point by using the shortest way, how many units has this ant advanced?

2.3. Clinical Interviews

The clinical interviews were made by using a recorder in the seminar room of the school. Each interview lasted approximately 60 minutes. During the interviews, the students were asked to answer questions and explain how they found the solution loudly by giving the questions one by one. In addition to that, questions such as ‘What kind of a generalization do you make about this question?’ and ‘What is your conclusion as a result of the asked rotation action?’ were posed to students. According pre-service teachers’ responses, the spatial visualization skills of them was determined according to SOLO taxonomy by asking different questions.

2.4. Data Analysis

During the analysis of the data obtained through clinical interviews, interviews which were recorded with the permission of the pre-service teachers were transcribed. The personal notes taken by the researcher also added to this text.

The data obtained through clinical interviews were analyzed qualitatively and the place of spatial visualization skills of pre-service primary schools mathematics teachers in SOLO taxonomy was searched.

The data collected in descriptive analysis are interpreted and summarized according to the levels specified before. In descriptive analysis, direct quotations are stated frequently in order to reflect dramatically the opinions of the individuals interviewed or observed. The purpose of this analysis is to present the data to the reader after the data was organized and interpreted. First, the data collected in this purpose is described systematically and openly. Later, these descriptions are explained, cause and effect relations are analyzed and finally some conclusions are inferred (Yıldırım and Şimşek, p.224).

In this study, how pre-service primary mathematics teachers think while solving questions which require thinking from two to three-dimension of spatial visualization components, ways of understanding, direct quotations from the dialogues between the researcher and the pre-service teacher for expressing solution strategies were included.

3. Findings

In this part, three students’ problem solutions and the dialogues having direct quotations were mentioned.

The Problem Solution of A1

A1 got a cylinder by attaching the AB and CD sides of the rectangular. While drawing the figure of a cylinder, she pointed the places where P and Q points will coincide. But we do not think that she used this figure for solution. She formed the hypotenuse of a right triangle by combining P and Q points on a rectangular at two-dimension. She stated that the shortest length would be found by using Pythagorean relation and pointing side length of this triangle in horizontal and vertical. For this, she expressed that hypotenuse length therefore the shortest length would also be calculated immediately by using 3-4-5 triangle. She did not think that there could another way for the shortest way so the answer that he/she found was wrong. The dialogue between the pre-service teacher and the researcher while solving the problem is as follows:

A1: P from below 1 unit to another is somewhere from here, Q from above 1 unit to another is somewhere from here. But I rotated therefore these distances stayed in the folded places. (This dialogue happens while drawing the closed version of the figure) So, I cannot show it on this. Besides, if it was given in this way, I would prefer to open it.

R: So, you are saying that, for the solution I would in any way find the solution by opening the figure, right?

A1: Yes, in the first figure, I wrote 5 units from Pythagorean by counting the square units.

R: The shortest way is being asked. Why do you think of this for the shortest way? Can there be another way from P to Q.

A1: This is the shortest way ... (1)

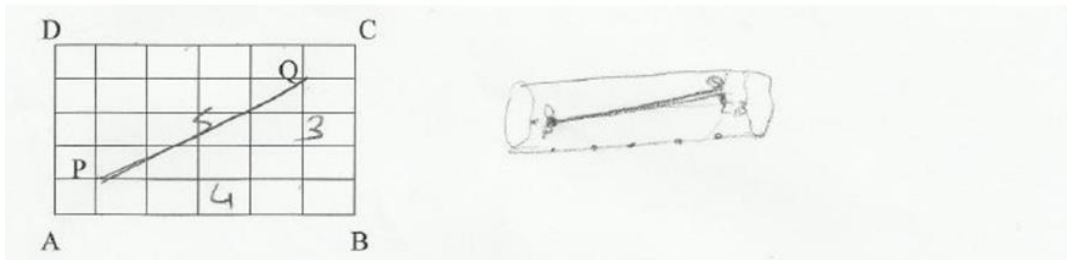


Figure 1. The Solution of A1 regarding the Problem

While reaching from P point to Q, as it can be seen from sentence (1), as the pre-service teacher thought that there is only one way for the shortest without thinking any alternatives and without making any comparisons between the distances, he/she is on 'Uni-structural' level.

The Problem Solution of A2

Although it was stated in the problem that AB and CD sides should be attached, the pre-service teacher attached AD and BC sides. After attaching both sides in this way, he formed the cylinder and placed P and Q points. Then he drew the ant's way. First moved from right to left on the drawing, then from left to right and decided that this was the shortest way. For the solution, the shortest way was calculated as $\sqrt{13}$ by pointing on the rectangular and taking 3 units in vertical and 2 units in horizontal.

A part from the dialogue between the researcher and the pre-service teacher is as follows:

A2: When we rotate in this way, it will pass through from a point from the shortest way. Q point is here, isn't it?

R: Draw this cylinder, if you want. Can you also write which point coincides to other there?

A2: If we think in this way, when we attach like this, for instance A point is here and B point is there. This is C and D point is here. If P point is here, when we rotate in this way, it is the Q point. Then the problem is asking how many units the ant advanced. It should go from the shortest way. It can go from here. How many will be there? If we take a vertical length, then that point for instance is Q point. The distance between these two is 2 units, 3 units, $\sqrt{13}$... (2)

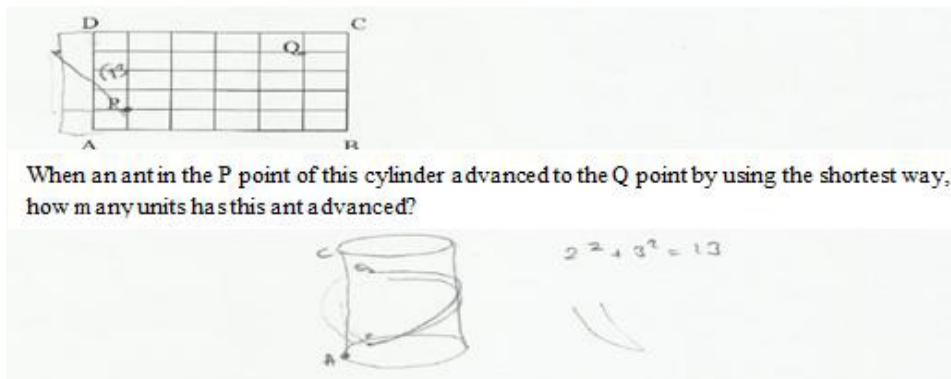


Figure 2. The Solution of A2 regarding the problem.

As it can be seen from the sentence (2), as the student specified more than one ways which the ant can follow and made calculations from different ways but since his/her explanations were superficial and started calculations without thinking deeply and preferred to attach in a wrong way, he /she is at the 'Multi-structural' level.

The Problem Solution of A3

A3 formed the cylinder by attaching AB and CD sides, showed that A and D points and B and C points coincide and pointed P and Q points on the surface. He showed on the rectangular that which distance should be taken between the points for the shortest way. He specified why he took 2 units instead of 3 units and reached the correct solution.

The explanations of A3 during problem solving are as in the following:

A3: If I took P from here, Q would be on the other side, at the back. Then, the shortest way of this will be from the other side; this will not be in fact. It will turn around like this, in fact. (It will come back into the form given in the question. He stated that the shortest way is from the other side rather than first drawing by adding squares under the rectangular.) ... (3)

R: Is this the shortest way?

A3: Yes, at first the other one came into my mind, but this should be like this.

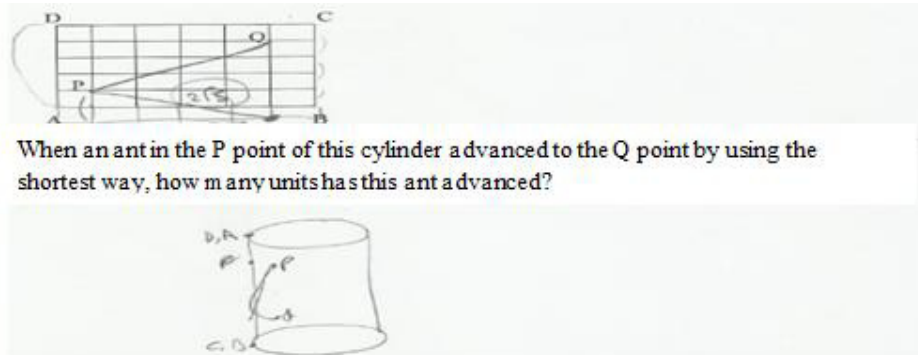


Figure 3. The Solution of A3 regarding the Problem

Since he considered more than one situation and he explained why he chose this way with reasons as it was stated in sentence (3), he is at the 'Relational' level.

According to the responses provided for the current problem, A1 is at the uni-structural level, A2 is multi-structural and A3 is at the relational level. The responses display features in the level of highest relational structure and one way relational structure.

4. Conclusion and discussion

Since the pre-service teacher who has low level spatial visualization skill is at the unistructural level, she focus on the problem, but she uses only relational single data for the solution. She cannot understand the value of data in whole and the relation of this data with the others. According to Hattie and Brown (2004), since the responses at the unistructural level are not profound, it can be said that pre-service teachers at the low level have a superficial learning. When pre-service teachers with middle level spatial visualization skills are analyzed, their responses can be seen at multi-structural level. As the spatial visualization skills increase, level of the pre-service teachers' responses has progressed into a higher level. This can be thought as normal situation because as the level of the students increases, the responses of the students are expected to be at a higher level. Similarly, it was found that pre-service teachers in high levels are at the relational level. In accordance with this information, pre-service teachers in middle or high levels generally use more than one information given in the problem but they cannot fully understand their values in the whole and their relation with the others and it is seen in each 3 situation, pre-service teachers are superficial. Experiences gained in classroom environment for geometrical concepts are developing their spatial skills (Hoover, 1996). From this point it can be concluded that in order to develop spatial skills, the necessity of carrying out various activities comes in to view. There are studies which show that dynamic geometry programs and using abstract materials can be effective at this level. Thus, Baki and Güven (2007), Kösa (2011), Uygan (2011) displayed in their studies that dynamic geometry and Arıcı (2009) in his study that origami can be effective in developing

spatial skills. The fact that in Lian and Idris's (2006) studies the participants are mostly at Unistructural and multi-structural levels and the responses of the pre-service teachers in this part of the study display similarities. Çelik (2007) and Ardiç and others' (2012) studies shows parallelism with the results obtained in a part of this study in terms of the responses of the participants stayed under relational level.

When we look at the general, the responses coming from pre-service primary mathematics teacher for the questions testing spatial visualization skills from two to three-dimension are mostly at relational level and at the least at Unistructural level. Their explanations are through rote learning, they started their calculations without thinking deeply which means there is no in-depth thinking and the questions were analyzed mostly superficially. Therefore, it cannot be said that pre-service teachers are successful.

The previous studies give us the fact that spatial thinking is mostly related with geometry rather than mathematics (Battista, 1990; Grande, 1990; Karaman, 2000). Since space is a more suitable field for the development of students' spatial skill, the importance given for teaching this field should be increased.

The level of students' responses can be determined by moving from various components that are specified by the other researchers of spatial skills and other subjects included in spatial geometry. The data obtained in this study was analyzed according to thinking levels of SOLO model. After evaluating the same data by using a different taxonomy, a comparison between two models can be made so that the subject can be considered from different point of view. In this way, if there are missing or overlooked aspects, they can be added.

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