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Axiomatic Design Approach for Nonlinear Multiple Objective Optimizaton Problem and Robustness in Spring Design

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Abstract: *This paper gives general information about multi-objective, axiomatic and robust design approaches and considers a solution model of nonlinear multi-objective optimization problem based on applying a new robust design approach. Both axiomatic and robust design approaches were used complementarily in a case study with distinct multi-objectives. In this case study, the main target was achieving each objective optimum to minimize the mass and the shear stress of a spring by integrating robustness and durability at the design stage due to trade off between objectives. This spring problem was examined using the independence axiom of the axiomatic design methodology. Also, semangularity and reangularity concepts were used and design matrices were formed to find coupled and decoupled solutions. It was observed that there were some acceptable design parameter values for which the design became decoupled. Graphical and numerical results were checked to see if they were compatible with each other. Finally, this decoupled design was given appropriate tolerances by using robust design method. This way, a robust and durable spring was designed which would satisfy the given specifications with minimum cost in the existing literature from the view point of axiomatic design approach.*

Keywords: *Axiomatic design, multi-objective design, robustness.*

1. Introduction

Product design requires satisfaction of multicriteria and nonlinear optimization problems. With multi-objective optimization it is generally more difficult to achieve each objective’s optimum because of trade-off between the various objectives, so the absolutely optimal solution may not exist [1]. The solutions depend on decision makers’ preferences related to the current problem. It is an interactive process of finding solution, providing new preferences, finding another solution which satisfies these preferences [2]. There are many multicritea design approaches to aid in decision-making during product design. Axiomatic design of S u h [3] and robust design of T a g u c h i, C h o w d h u r y and W u [4] are the most acknowledged ones.

In product design, integrating robustness into the product at the design stage is accepted to be the ideal. Although many attempts are made to increase the robustness of products, the issue of durability is left out for the verification and testing stage of the product development in most cases.

In designing of components it is necessary to assign tolerances to all the dimensions and consider the variability of all inputs and outputs. The assigned tolerances should guarantee that the system will behave as expected despite the variation of inputs, at least from a statistical point of view. Usually, there is a conflict between tolerance values assigned by design engineers and those desired by manufacturing engineers. This conflict of interest can be resolved if design engineers consider tolerances as a decision parameter in overall optimization problem. How these tolerances affect the output of the system should be understood so that, those tolerances with a significant impact are kept tighter while other tolerances can be kept loose.

In this paper, a set of mathematical models to integrate robustness and durability at the design stage will be presented, and a spring design problem will be solved as an illustration. An axiomatic design approach and a robust design approach will be formulated in a complementary way with the newly determined multi-objectives.

2. Methods

2.1. Multi-objective design

Defined by multi-objectives nature, the optimization problems are related to huge mathematical computations that are practically impossible to be done in reasonable time without some kind of computer aid. Here comes the use of the software decision support systems to optimize many objectives [2]. To pick a solution which satisfies all the objectives in an appropriate amount requires some considerations. Multi-objective design approach assigns a scalar to each solution based on its value for each design criterion [5]. Each solution is evaluated this way.

2.2. Axiomatic design

In this design method, customer needs are translated into Function Requirements (FRs). Then, each of these FRs is assigned an appropriate Design Parameter (DP) [3]. This assignment is done with respect to the first axiom of the axiomatic design approach. This is the independence axiom which states that in an acceptable design, design parameters and the functional requirements are related in such a way that a specific design parameter can be adjusted to satisfy its corresponding functional requirement without affecting other functional requirements [3]. Zigzagging is used to sort through the FR to DP transformation. It makes use of function hierarchies. Transformation is done at each level of the function tree separately, starting from the highest level and going to the lowest [3]. This alternation between the DP and FR domains reduces any confusion or unnecessary elements in the domains. The second axiom is the information axiom, which states that the best design is a functionally uncoupled design that has the minimum information content [3]. Minimum information means that manufacturing, distribution, and other processes associated with the product are relatively easy to follow. Also, symmetry, interchangeability of parts and etc., reduce the information content. In this method, some mathematical calculations are made to find the best design [3]. These calculations will be presented in the spring example of the following sections.

2.3. Robust design approach

Robust design approach makes up for the variables that deter the system from working at the nominal conditions [6]. In real life, the product may not be operated under conditions for which it was designed. During the design stage, it is important to eliminate or delay failure, disturbances (noise). Noise also encompasses the problems that can arise during the manufacturing and distribution of the product as well as unit-to-unit variability. Robust design approach enables the designer to assign the right tolerances so that the design will function under noise with the least amount of money spent on manufacturing [6]. These design approaches will be used in the following design example.

2.4. Spring design example

To demonstrate these approaches, a compression spring of round music wire with square and ground ends is to be designed. The three Design Parameters (DPs) for the problem are wire diameter (d), mean coil Diameter (D), and number of active coils (N_a). It is planned to use this spring in an application where it should have a free length (L_f) of 44.45 mm with a deflection (δ_{max}) of 12.7 mm under the operating load (P) of 62.3 N.

Design stress (τ_d) should be less than 896.31 MPa, and maximum allowable stress (τ_a) at the solid length of spring should be less than 1034.21 MPa. Spring will be installed in a hole of 15.24 mm diameter ($\overline{D}$) and the frequency of surge waves (f_n) for the spring is required to be at least 100 Hz. Number of inactive coils, Q, is 2. It is also desired that proper coil clearance and pitch angle α are provided. The spring should be designed so that its mass and the shear stress under operating load are minimum while satisfying all other requirements stated in the above. To formulate this problem, the following material properties and constants were used based on Goel and Singh [6]. These are the shear modulus, G (80.85 GPa), mass density, ρ (7888.77 kg/m^3), gravitational constant, g (9.81 m/s^2), and weight density of material, γ (77.389 kN/m^3).

Spring formulas to be used in further calculations are taken from Shigley (see [7]) as:

(1) spring index (C) $C = \frac{D}{d}$,

(2) pitch (p) $p = \frac{(L_f - 2d)}{N}$,

(3) shear modulus (G) $G = \frac{E}{2(1+\nu)}$,

(4) coil clearance (cc) $\text{cc} = \frac{(L_0 - L_s)}{N_a}$,

(5) spring rate (k) [7] $$k = \frac{Gd^4}{8D^3 N_a},$$

(6) max deflection ($\delta_{\max}$) $$\delta_{\max} = L_f - L_s,$$

(7) maximum load ($P_{\max}$) $$P_{\max} = k.\delta_{\max}$$

(8) shear stress ($\tau_{\max}$) $$\tau_{\max} = \frac{8DP}{\pi d^3}.$$

Wahl correction factor [7] is

(9) $$K_w = \frac{4D-d}{4D-4d} + \frac{0.615d}{D}.$$

Corrected max shear stress ($\tau_{\max}'$)

(10) $$\tau_{\max}' = K_w.\tau_{\max}.$$

Spring mass (M)

(11) $$M = 1/4(N+Q)\pi^2 Dd^2\rho.$$

Natural frequency (f_n) [7]

(12) $$f_n = \frac{d}{2\pi D^2 N_a}\sqrt{\frac{G}{2\rho}}.$$

(13) Pitch angle (α) $$\alpha = \mathrm{tg}^{-1}\left(\frac{p}{\pi D}\right).$$

Finally, the constraints of problem statement are stated as [7]

(14) $$\frac{8PD^3N}{Gd^4} = 12.7,$$

(15) $$\frac{d}{2\pi D^2 N}\sqrt{\frac{G}{2\rho}} \geq 100,$$

(16) $$\frac{L_0 - L_s}{N} \geq d/10,$$

(17) $$\frac{8PD}{\pi d^3}\left(\frac{(4D-d)}{(4D-4d)} + \frac{0.651d}{D}\right) \leq 896.31\ \mathrm{N/m^2},$$

(18) $$\frac{d\,G(L_f - dN)}{\pi D^3 N}\left(\frac{(4D-d)}{(4D-4d)} + \frac{0.651d}{D}\right) \leq 1034.21,$$

(19) $$\frac{(N+Q)\pi^2 Dd^2\rho}{4} \leq 385,$$

(20) $$D + \left(\frac{11}{10}\right)d \leq 15.24\ \mathrm{mm},$$

(21) $$D/d \geq \mathrm{cc},$$

where N is a nonnegative integer. Also,

(22) $$d > 1.27\ \mathrm{mm},\ 2d < D < 25.4\ \mathrm{mm}.$$

From the preceding constraints, three functional requirements (FRs) are drawn. These are mass (M), shear stress ($\tau_{\max}'$) and deflection (δ).

3. Mathematical models

In this section, the mathematical models will be presented. Shear stress and mass equations as well as reangularity and semangularity relations based on these FRs will be derived.

Out of the three functional requirements, two are the most critical [7]. First one is the mass of the spring since it affects the cost. The second one is the shear stress caused by twisting, which determines if there will be any failure.

These two critical functional requirements, which are mass (M) and shear stress ($\tau_{\max}'$), are given by

(23) $$M = 1/4 N_T \pi^2 D d^2 \rho,$$

(24) $$\tau_{\max}{}' = \frac{8PD}{\pi d^3}\left(\frac{(4D-d)}{(4D-4d)} + \frac{0.651d}{D}\right).$$

For these two functional requirements, two Design Parameters (DP_1 and DP_2) were selected. They were coil diameter (*D*) and wire diameter (*d*):

(25) $$DP_1 = D,$$

(26) $$DP_2 = d.$$

Axiomatic design equation is written as,

(27) $$\begin{Bmatrix} M' \\ \tau_{\max}{}' \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} . \begin{Bmatrix} x \\ y \end{Bmatrix},$$

where the left-hand side signifies functional requirements in dimensionless form. x and y are the dimensionless forms of wire and coil diameters respectively. The 2×1 matrix on the right-hand side signifies design parameters in dimensionless form. The 2×2 matrix is the design matrix, *A*. Since functional requirements and design parameters are nonlinearly related, elements A_{ij} of matrix *A* may be expressed as

(28) $$A_{ij} = \frac{\partial FR_i}{\partial DP_j}.$$

This makes the elements of *A*:

(29) $$A_{11} = y^2 z,$$

(30) $$A_{12} = 2xyz,$$

(31) $$A_{21} = \frac{K'_w(X,\ Y)}{y^3},$$

(32) $$A_{22} = -3x\frac{K'_w(X,\ Y)}{y^4}.$$

Furthermore, the relation between dimensionless forms of DPs and FRs are calculated as:

(33) $$M' = xy^2,$$

(34) $$\tau'_{\max} = \frac{x}{y^3} K_w(X,\ Y).$$

The aim is to assign such values to *x*, and y such that the design matrix *A* will be reduced to a diagonal (uncoupled) or an upper or lower triangular (decoupled) matrix [3].

To help with finding the optimum solution, there are two concepts to quantize the independence of functional parameters. These are called reangularity and semangularity. Reangularity *R* can be obtained by using the following equation [3]:

(35) $$R = \left[1 - \frac{(A_{11}A_{12} + A_{21}A_{22})^2}{(A_{11}{}^2 + A_{21}{}^2).(A_{12}{}^2 + A_{22}{}^2)}\right]^{1/2}.$$

In addition, semangularity *S*, can be calculated as suggested in [3]:

(36) $$S = \frac{|A_{11}|}{(A_{11}{}^2 + A_{21}{}^2)^{1/2}} . \frac{|A_{22}|}{(A_{12}{}^2 + A_{22}{}^2)^{1/2}}.$$

If *R* and *S* are both equal to 1, the design is an uncoupled design. The uncoupled solution is the best solution but it is rarely achieved. The corresponding design matrix for this solution is a diagonal matrix. When *R* is equal to *S* but not 1, the design approaches a decoupled design. Decoupled design is also an acceptable solution. The corresponding design matrix is an upper or lower triangular matrix [3]. Contour plot of mass versus DPs can be used to see the regions where the solution approaches uncoupled or decoupled state (see Fig. 1). If FR contours are parallel to DP axes but perpendicular to each other, this means that there is an uncoupled solution [4]. It can be seen from Fig. 1 that there is no such solution for this problem. Contours of reangularity and semangularity were plotted on MATLAB with DPs used as axes (see Fig. 2).

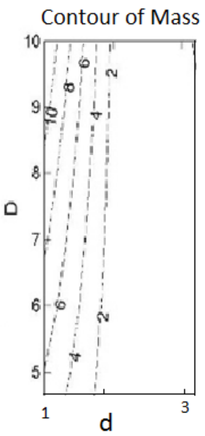

Fig. 1. Contours of Mass (kg) plotted on MATLAB

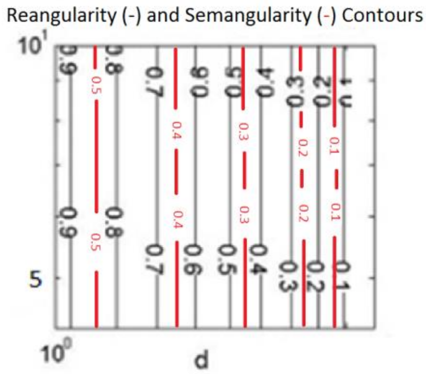

Fig. 2. Contours of reangularity (black line) and Semangularity (red line), log-scale with *D* and *d* in mm

As seen from Fig. 1, the two FRs are coupled in general. However, from Fig. 2 it can be seen that there are certain regions in the DP domain where *S* and *R* approach 1. These are the regions where the two critical FRs become decoupled. It is seen that *S* and *R* are close to equal for values of *d* and *D* around 1.778 mm and 15.24 mm respectively. When checked with design matrix, *A*, of (27) on Fortran, these two values give decoupled solution as well.

For the next step, deflection (δ) and number of active coils (*N*) are added as the third FR and DP respectively.

(37) $$z = N.$$

Dimensionless form of δ is calculated as

(38) $$\delta' = \frac{x^3 z}{y^4}.$$

Corresponding axiomatic design equation along becomes

(39) $$\begin{Bmatrix} M' \\ \tau_{\max}' \\ \delta' \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \cdot \begin{Bmatrix} x \\ y \\ z \end{Bmatrix},$$

where since $A_{ij} = \frac{\partial \mathrm{FR}_i}{\partial \mathrm{DP}_j}$,

(40) $$A_{11} = y^2 z,$$

(41) $$A_{12} = 2xyz,$$

(42) $$A_{13} = xy^2,$$

(43) $$A_{21} = \frac{K'{}_w(X,\ Y)}{y^3},$$

(44) $$A_{22} = -3x\frac{K'{}_w(X,\ Y)}{y^4},$$

(45) $$A_{23} = 0,$$

(46) $$A_{31} = \frac{3x^2 z}{y^4},$$

(47) $$A_{32} = -\frac{4x^3 z}{y^5},$$

(48) $$A_{33} = \frac{x^3}{y^4}.$$

By solving the design matrix of (39) on Fortran, a possible decoupled solution is obtained when *x*=5, *y*=7, and *z*=9. Matrix *A* becomes

(49) $$A = \begin{bmatrix} X & X & X \\ 0 & X & 0 \\ X & X & 0 \end{bmatrix},$$

with X_s signifying nonzero values. Converting the dimensionless *x*, *y*, and *z* values into DPs gives

D = 12.7 mm
d = 1.778 mm and
N = 9.

If (39) is written more symbolically

(50) $$\begin{Bmatrix} \mathrm{FR}_1 \\ \mathrm{FR}_2 \\ \mathrm{FR}_3 \end{Bmatrix} = \begin{bmatrix} X & X & X \\ 0 & X & 0 \\ X & X & 0 \end{bmatrix} . \begin{Bmatrix} \mathrm{DP}_1 \\ \mathrm{DP}_2 \\ \mathrm{DP}_3 \end{Bmatrix},$$

which can also be written as:

(51) $$\mathrm{FR}_1 = C_1\mathrm{DP}_1 + C_2\mathrm{DP}_2 + C_3\mathrm{DP}_3,$$

(52) $$\mathrm{FR}_2 = C_2\mathrm{DP}_2,$$

(53) $$\mathrm{FR}_3 = C_1\mathrm{DP}_1 + C_2\mathrm{DP}_2.$$

Equations (51)-(53) show the independencies between DPs and FRs. It is indicated in (51) that mass is affected by all three of the design parameters. Deflection is affected by both wire and coil diameters. However, it is independent of the number of active coils (53). Also, only wire diameter affects shear stress. It is independent of coil diameter and the number of active coils (52). These independencies gained through this analysis are a great advantage for the design.

4. Results

Solutions for the case were determined by using axiomatic design and multi-objective design approach. Optimal values of design parameters and functional requirements that satisfy constraints from the axiomatic design case are shown in Table 1.

Table 1. Solution of axiomatic design approach

D, mm	*d*, mm	*N*, #	Deflection, mm	Mass, g	Stress, MPa	Frequency, Hz	Spring hole, mm	Spring index
12.7	1.778	9	11.684	22.226	432.92	302.5	14.65	7.14

Generally, spring index ranges from about 6 to 12. This constraint is satisfied. Frequency is desired to be greater than 100 Hz. This constraint is satisfied too. Lastly, spring hole diameter is less than 15.24 mm, which is one of the most important constraints.

To find the appropriate tolerances, robust design method was applied. Different tolerance ranges were tried by calculating the maximum and minimum mass and shear stress values for each range. A reliability of 95% was wanted so the smallest tolerance range, which provided the appropriate deviation from the nominal FR values, was calculated. This tolerance range along with the nominal DP values is given in Table 2. If the DP values stay within this range, FR values show adequate insensitivity to noise factors [5].

Table 2. Solutions of multi-objective axiomatic and robust design approach

Multiobjective axiomatic and robust design solution									
$T_D = 0.1$, $T_d = 0.025$, mm	*D*, mm	*d*, mm	*N*, #	Deflection, mm	Mass, g	Stress, MPa	Frequency, Hz	Spring hole, mm	Spring index
nom	12.70	1.78	9	11.68	22.226	432.922	302.5	14.656	7.14
min	12.60	1.75	9	11.94	21.318	447.883	303.02	14.526	7.19
max	12.80	1.80	9	11.17	23.133	418.718	301.98	14.785	7.10

5. Discussion and conclusion

In this paper, multi-objective, axiomatic and robust design approaches were exemplified by investigating a mechanical spring design. Axiomatic design and multi-objective design approaches were integrated to minimize mass and shear stress of the spring with integration of robustness and durability at design stage.

When multi-objective and axiomatic design methodologies are integrated, design Tolerance for coil Diameter (T_D) is 0.1 mm and Tolerance for wire diameter (T_d) is 0.025 mm. This means that there is no rapid change in FRs if coil diameter ranges between 12.6 and 12.8 mm and if wire diameter is kept between 1.753 and 1.803 mm. Thus, system’s robustness and durability are developed and sensitivity to noise factors is decreased. Also, independence axiom is satisfied which provides versatility to the design. These, along with minimum manufacturing costs, are the advantageous properties provided by a methodological selection of coil and wire dimensions and the number of coils.

For future improvements, different methodologies can be integrated to axiomatic design approach. Multi-objective evolutionary strategy tends to do parallel computing that can solve a sufficient number of solutions distributed on the Pareto Front (PF) and provided to the decision-makers for the next decision [8]. Then, the system will be more robust and insensitive to noise factors. The use of reangularity and semangularity in multi-objective optimization provides to reduce the functional coupling degree of the system. By that way improvements can be made to achieve better designs. It is planned to extend the existing definition of *S* and *R* to include multi-objective optimization problems where the number of design variables exceed the number of objectives [9].

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