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Procedia Social and Behavioral Sciences 15 (2011) 3422–3425

Procedia
Social and Behavioral Sciences

WCES-2011

The investigation of student approach to problem solving about some topics of Number Theory

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Abstract

Number Theory is described as the center of mathematics. Divisibility, parity of numbers, prime and composite numbers are the main topics of Number Theory. The purpose of this study was to investigate the Turkish students' conception and difficulties of divisibility of numbers, prime numbers and odd and even numbers concept. Furthermore this study also explored the differences, if any, of problem solving approach to different problems in three topics. The case study was used in this study. This study showed that each student's conception of same topic is different. The different responses and difficulties were discussed in details. One possible suggestion is to use different representation of problems and emphasizing multiple ways of solutions during the instruction.

Keywords: Number Theory; Prime and Composite Number;

1. Introduction

Elementary concepts of number theory, despite their importance to the field of mathematics, have received scant attention in mathematics education research. In Turkey also, number theory is rarely takes place in researches which are done in mathematics education.

Previous studies have used concepts from elementary number theory as a mathematical context for investigating different issues; for example, Martin and Harel (1989) used notions of divisibility in research on preservice teachers' understanding of mathematical proof. Leron (1985) adapted a theorem on the infinity of prime numbers to illustrate a more constructive approach to indirect proofs. Lester and Mau (1993) used prime factors in research on problem solving in a course for prospective elementary school teachers. Movshovitz-Hadar and Hadass (1990) applied proofs for irrationality of square roots of prime numbers for investigating the pedagogical role of paradox and conflict resolution in the education of prospective mathematics teachers. But in our research, number theory concepts themselves are the primary focus of investigation as in the Zazkis and Campbell's (1996) researches.

Prime numbers take place an important part in number theory. There are two properties in particular that seem to present a mystery to the learner. One is the existence of infinitely many prime numbers, which entails very large primes. Another is the property that prime numbers are not generated by a simple polynomial function. Therefore,

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1877–0428 © 2011 Published by Elsevier Ltd.
doi:10.1016/j.sbspro.2011.04.312

we believe that an understanding of a concept of a prime number by an elementary school teacher should include at least the following:

a) Awareness that any natural number greater than 1 is either prime or composite and the ability to cite and explain the definition of a prime number;
b) Understanding that if a number is represented as a product it is composite unless the factors are 1 and a prime;
c) Awareness that composite numbers have a unique prime decomposition and that the number of primes is infinite (Zazkis and Liljedahl, 2004).

In Mathematics, how the question resolved is as important as solving the question. Every right answer does not mean that it is solved in a right way. Any mistake which is done during solution may result in right answer.

In this study, important point is not teaching number theory but the thinking structure of students and the way which is used by students during problem solving that have learnt number theory.

2. Method

Case study design is used as a model for this study (Yin, 1994). Each document went under a qualitative analysis for identifying patterns and big ideas. In order to increase the trustworthiness of the study, we applied peer debriefing on my analysis and used data triangulation. Triangulation and peer debriefing are important factors in ensuring the quality of a qualitative inquiry (Bogdan & Biklen, 1998; Yin, 1994; Guba & Lincoln, 1989).

2.1. The Aim of the Study

The aim of this study is to examine the mathematic students' knowledge about main topics in number theory such as prime and composite numbers, and also to explore how they use these information when they are solving problems related to number theory. In addition we examined learning styles of students more deeply.

2.2. Questions

These questions are similar to question sets proposed by Zazkis and Liljedahl (2004).

1) How do you describe prime and composite number?
2) Consider F= 151 × 157. Is F a prime number? Circle (YES/NO). Explain.
3) Consider m(2k+1), where m and k are whole numbers. Is this prime number? Can it ever be prime?

First question is initially included in the interview as an easy "warm-up" question. Therefore, first question was analyzed for the ways in which students described prime numbers and we were interested the relationships that participants identify between prime and composite numbers.

In second question, F is given as a product of natural numbers and it requires no work. The important thing in this question is the kind of answers.

Third question is also given as a product of numbers. However, since the product is represented in algebraic notation.

2.3. Data Collection

In our study we used interactive interview techniques because it was more suitable for the nature of study so we understood better "how" and "why" the students solved these problems in this way in their working sheets.

We prepared working sheets which includes the questions and empty space for the answers. The questions in working sheets are composed of prime and composite numbers. We have chosen each question from other researchers' papers which were approved by experts in these topics.

The papers were prepared before and distributed to chosen students and they solved them in the class by supervision of a teacher. After that, papers were collected and according to the students' answer suitable ones for interview were decided and scheduled for interview and individual interactive interview was done at that time. During interview, students explained deeply how they have solved problems. Some more questions were added to interview according to the students' answers or according to students' behaviors. That is why, semi-structured interview was preferred.

3. Analysis

In the first question 50 student were asked to describe the meaning of prime and composite numbers. In Table 1 answers of students are grouped according to description of prime numbers;

Table 1. Descriptions of prime number

Answers	Number of Students
Divisible only by 1 and itself	43
Having exactly 2 factors	5
Undefined	2

As we see in Table 1, %86 of students used definition "divisible only by 1 and itself" and %10 of students used definition "having exactly 2 factors". However, the phrase "divisible only by 1 and itself" created ambiguity in the consideration of the primarity of number 1. It was pointed out to the students that by mathematical convention the number 1 is not considered to be prime, and therefore "having exactly 2 factors" was more accurate indicator of the property of primarity (Zazkis and Liljedahl, 2004).

In Table 2, there are descriptions of composite number and the number of students for the answers are given;

Table 2. Descriptions of composite number

Answers	Number of Students
Have more than two factors	20
Have more than two multiples	6
Have other factors besides 1 and itself	4
Besides prime numbers	2
Undefined	18

As we see in Table 2, %40 of students used the definition "have more than 2 factors". In this question it is interesting that most of students could give the definition of prime number but could not give the definition of composite number.

In second question, it is asked to 73 students to determine whether the number F, given as $F= 151\times157$, was prime and to explain their decision. Table 3 provides a summary of students' answers to second question;

Table 3.Summary of responses to second question

Correct Answers (Justification)	Number of Students
Definition of prime number	30
Definition of composite number	15
Application of algorithm	9
Other	5
Incorrect Answers (Justification)	**Number of Students**
Product of primes is prime	2
Misapplication of an algorithm	4
Undefined	8

As shown in Table 3, a total of 59 out of 73 students have claimed correctly that F was a composite number. However, not all the correct answers were accompanied by correct justifications.

Some students tried to calculate F and could get the right answer. However, the need for the algorithm is somewhat troublesome because it shows that these students could not conclude that F was a composite number from considering its representation as a product.

Third question is similar to second question in that it asks the students to consider the primarity of a number presented as a product but this product is represented in algebraic notation. Solving question has become more difficult because of using algebraic notations. This question is asked to 50 students and 34 out of 50 students used the definition "varies according to the values of m and k". And they tried several times. As a result they could find prime and composite numbers.

10 students listed the factors as 1, m, 2k+1 and m(2k+1). These students' definition is like "This product cannot be prime number because it has more than two factors". 5 out of 50 students could not give any answer for this question. Table 4 provides a summary of students' answers to third question;

Table 4.Summary of responses to third question

Answers	Number of Students
Varies according to the values of m and k	34
Composite number	10
Prime number	1
Undefined	5

4. Results and Suggestion

Study results revealed that prime numbers has been defined as "divisible by itself and 1". 43 students our of 50 students has answered correctly. But when the definition of composite numbers were asked, students responded "numbers that are not prime numbers" or the answers given like "divisible numbers by prime or multiple of prime numbers".

Around 20 students were not able to give the sufficient definition of composite numbers. Students were not able to give new definitions if they have not been seen similar definitions before.

What's more when students were asked if F=151×157 number F is a prime number or not. Some of the responses were "multiplication of two prime numbers is also a prime number". Most of students answers were correct but not enough explanations were given.

When the primarity were asked in abstract form like m(2k+1) the number of wrong answers increased dramatically.

Each problem has its own unique characteristics. Therefore, students should given chance to develop multiple perspectives during problem solving activities. We should teach both side of the coin .In other words when teach the prime number concept we should also discuss what does it mean non-prime numbers.

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