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## An investigation of 5th grade Turkish students' performance in number sense on the topic of decimal numbers

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### Abstract

The purpose of this research is to investigate the number sense of 5th grade Turkish students on the topic of decimal numbers. The research participants are composed of 121 5th grade students from two different elementary schools. All of the students have already completed the 5th grade mathematics curriculum. The data of the research was collected by number sense test about decimal numbers (NSTDN). It contained 16 questions and four components of number sense. The results of this quantitatively analysed research have shown that the 5th grade students' number sense on decimal numbers is low. There was no significant difference in number sense between the genders. These findings are consistent with the previous researches.

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### 1. Introduction

In the Principles and Standards for School Mathematics, the National Council of Teachers of Mathematics [NCTM] (2000) stated that development of number sense is the centre of this Standard. Besides NCTM, other national reports and studies emphasize the importance of number sense in mathematics teaching and learning (Dehanea, 1997; Sowder & Schappelle, 1989; Verschaffel, Greer & De Corte, 2007).

Number sense refers to a person's general understanding of numbers and operations and the ability to handle daily-life situations that include numbers. This ability is used to develop useful, flexible and efficient strategies for handling numerical problems (McIntosh, Reys & Reys, 1992; Reys & Yang, 1998; Yang, 2005). Students with good number sense possess several important but elusive capabilities, including flexible mental computation, numerical estimation, and quantitative judgment (Greeno, 1991). They can move seamlessly between the real world of quantities and the mathematical world of numbers and numerical expressions. They can invent their own procedures for conducting numerical operations and represent the same number in multiple ways depending on the context and purpose of this representation (Case, 1998). The capabilities of these students indicate that why the number sense is so important in mathematics education.

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Number sense is a complex process involving (Yang, Li & Li, 2008) many different components of numbers, operations, and their relationships. Although a satisfactory definition of number sense has not been given in the related studies, an effort has been made to define the basic characteristics about what number sense may be. Based on a review of the number sense literature (McIntosh et. al, 1992; Markovits & Sowder, 1994; NCTM, 2000; Yang & Wu, 2010; Li & Yang, 2010), this study focused on number sense to include:

a. *Understanding the basic meaning of numbers:* This implies to understand the base 10-number system (whole numbers, fractions, and decimals), including place value, pattern of numbers and using multiple ways to represent numbers (McIntosh et. al, 1992).

b. *Recognizing relative number size:* This component determines understanding of relativity of numbers and their absolute magnitudes.

c. *Being able to use a benchmark appropriately:* This implies that person can use the benchmarks which provide mental referents thinking about numbers to solve problems appropriately (McIntosh et. al, 1992).

d. *Being able to judge the reasoning of a computational result:* This component refers to using mental computation strategies in order to solve problems and to judge the reasoning of results.

Gay and Aichele (1997) stated that students should develop number sense about whole numbers, fractions, decimals, and percent. Among these, decimal numbers are the most problematic ones because decimal numbers have been recognized for some time to be a significant source of learning and teaching difficulties (Stacey et al., 2001).

Students and even pre-service teachers have various misconceptions about decimal numbers (Steinle & Stacey, 1998; Stacey et. al, 2001). For example, erroneous rules or misconceptions reveal when students write and read decimals, understand the decimal place value, order the decimal numbers (longer-is-larger, shorter-is-larger), put a decimal on a numerical axis, and understand the meaning of decimal point (Resnick et. al, 1989; Steinly & Stacey, 2003; Bilgin & Akbayır, 2002; Irwin, 2001). The examples of misconceptions indicate that students' number sense is not adequate and they cannot understand the basic meaning of decimal numbers. Helme and Stacey (2000) stated the central problem for many people as lacking the understanding of decimal numeration and they also noted that this is a long-standing problem that appears internationally.

While much is known about students from different grades and it is known that even pre-service teachers have poor number sense (Harç, 2010; Kayhan Altay, 2010; Markovitz & Sowder, 1994; Yang, 2005; Tsao, 2004), little is known about specialized number sense which focuses particular subject like fractions, percent etc. The aim of this study is to investigate the 5th grade students' understanding of number sense related to decimal numbers. Besides this, the study investigates the performance of number sense between genders.

## 2. Method

### 2.1. Participants

The study was conducted with a total of 121 5th grade students in nine different classes of two different elementary schools located in Istanbul. 54 (44.63%) students were female and 67 (55.37%) students were male in the study. The students have just completed the 5th grade curriculum. Their age ranges from 11 to 12. The mathematics achievement of students in the study was at mediocre level and their levels were similar.

### 2.2. Test instrument

So as to collect the data, Number Sense Test about Decimal Numbers (NSTDN) was used. It was adapted from the problems discussed in the related literature. The opinions of the two subject field experts and of the two academicians were taken on the items used in the testing instrument before collecting the data. Firstly, whether or not the questions in the testing instrument represented the various types of number sense components that were defined in the related literature was asked to the experts in order to determine the content validity. Moreover, the

experts examined the items in terms of their difficulty level, whether they caused any misunderstandings or not, to what extent they tested the measures that they aimed to test and whether they were well-expressed or not. A pilot study with 20 5th grade students who are comparable in general academic background to the participants in the larger study was done to ensure that items were clear, appropriate, and that the test time allocated. After the pilot study, four questions were excluded because of difficulty level and item discrimination index. Final version of NSTDN includes 16 questions, 4 different questions from 4 different number sense components. All questions were selected to be multiple choice type each of which had four different answer choices and only one of them was the correct answer. To test the reliability of test, Kuder-Richardson 20 coefficient was examined and its reliability was calculated to be 0.71.

### 2.3. Procedure

Each participant was given a number sense test. The test took about 30 minutes. Before the test started, the researcher read the instructions of the test out loud. Specifically, the following directions were given: 1. Participants were told to estimate or mentally compute and not to carry out a written algorithm to find an exact answer on each item; 2. They should not turn to the next page until they were told to do so. The researcher monitored the test to control the pace and also to validate that the directions, such as not executing a written algorithm, were followed.

### 2.4. Data Analysis

The answers of the participants were examined by using quantitative analyses. Correct answers were labeled as 1 while incorrect ones were labeled as 0. The answers were analyzed quantitatively in terms of percentage, frequency, mean, standard deviation, and t test.

## 3. Result

Table 1 presents the mean scores, standard deviation and percentage of correct answers on the NSTDN, according to the number sense components.

Table 1. Descriptive statics for the students' performance on each number sense component

Components of Number Sense	Number of item	$\bar{X}$	SD	%
<i>Understanding the meaning of decimal numbers</i>	4	1.21	.76	30.37
<i>Recognizing relative size of decimal numbers</i>	4	1.66	1.06	41.74
<i>Being able to use benchmark appropriately</i>	4	2.41	1.03	60.33
<i>Being able to judge the reasoning of a computational result that includes decimal numbers.</i>	4	1.19	1.01	29.73
<i>Total</i>	16	6.48*	2.32	40.54

\*The highest possible score was 16.

For the sample students, the mean score of NSTDN was 6.48 and the mean percentage of correct answers was only about %40 which shows that students were not able to solve more than half of the questions in the entire test. This indicated that these students – generally speaking – did not perform well on the number sense about decimal numbers. The results also showed that the highest correct percentage of the students was in “*being able to use benchmark appropriately*” component and their lowest correct percentage was in “*being able to judge the reasoning of a computational result that includes decimal numbers*” component.

According to the NSTDN results, male students got 6.76 while female students got 6.14 mean score. Although males have higher mean score, according to statistics, this difference was not significant [ $t_{(119)} = -1.44, p > 0.05$ ]. Gender did not seem to be an important factor on the performance in number sense.

Students’ responses indicated some possible reasons of low number sense. It can be said that misconceptions about decimal numbers and rule-based methods are some possible reasons of low number sense. The following sample questions from NSTDN support this idea. It was expected from a student to know that there always exists a number between two different decimal numbers in the conceptual question (Table 2). Approximately half of the students selected choice A, while about %25 of the students selected the correct choice. The percentages showed that half of the students thought that there was no decimal number between 2.52 and 2.53. Hence we concluded that they thought of the decimal numbers like consecutive whole numbers and therefore it was evident that the meaning of decimal numbers was not understood by the students clearly.

Table 2. An example question from NSTDN

Component of Number Sense	Question	Percentages of choices
Understanding the meaning of decimal numbers	A and B were two given points on the number line. How many different decimal numbers are in between A and B?	A: %46.7
	<p style="text-align: center;"> <math>\leftarrow</math> ————   ————   ————   ————   ———— <math>\rightarrow</math>                  2,51    2,52    2,53    2,54                  A            B             </p>	B: %18.5 C: %10.2 D: %24.6
	A) 0            B) 10            C) 100            D) Infinite	

In another question (Table 3), students were asked to compare two decimal numbers whose place number after the decimal point was different. About % 44 of the students who selected choice A, thought that  $k$  is bigger than  $m$ . It can be said that the reason of this selection was comparison of 89 and 9. These students probably used whole number thinking and this misconception name is larger-is-bigger (Steinley & Stacey, 1998).

Table 3. An example question from NSTDN

Component of Number Sense	Question	Percentages of choices
Recognizing relative size of decimal numbers	Given that $k$ is 0.89 and $m$ is 0.9, which of the following ordering is true? A) $k > m$ B) $k = m$ C) $k < m$ D) I can't tell exactly.	A: %43.2 B: %5.3 C: % 37.4 D: %6.5

Table 4 reports another question from NSTDN which is from “being able to judge the reasoning of a computational result that includes decimal numbers” component. More than half of the students who probably used a rule based method chose choice A. The students multiplied two numbers as if they were whole numbers and then placed the decimal point by starting at the right and moving the point the number of places equal to the sum of the decimal places in both numbers multiplied. This question showed that students tend to use rule based methods rather than number sense methods.

Table 4. An example question from NSTDN

Component of Number Sense	Question	Correct Percentage
Being able to judge the reasoning of a computational result that includes decimal numbers.	Without exact calculation, find the result. $0.498 \times 28$	A: %56.7 B: %11.3 C: % 12.8 D: %19.2
	A) 1.394    B) 1394    C) 139.4    D) 13.94	

#### 4. Conclusion

The findings of this study show that the sample 5th graders did not perform well on the number sense on the topic of decimal numbers. This finding is not only consistent with earlier studies (Harç, 2010; Kayhan Altay, 2010) in Turkey but also is similar to international results (Yang, 2005; Tsao, 2004). Misconceptions about decimal numbers and rule based methods are seemed to be a major impediment to development of number sense. In order to develop number sense, the mathematics curricula and textbooks must emphasize on the conceptual understanding and

teaching of number sense.

These sample students did not perform well on each number sense component. Among these components, they performed better on the component of “*being able to use benchmark appropriately*” component. But they performed poorly on *being able to judge the reasoning of a computational result that includes decimal numbers*” component. This finding is similar to earlier studies. The result of this study has also shown that gender does not have a significant effect on students’ performance in number sense. This finding also confirms the statement of earlier studies (Yang, 1995; Aunio, Ee, Hautamaki & Van Luit, 2004; Harç, 2010; Kayhan Altay, 2010). As a final word, this study indicates that the development of number sense needs to be considered an essential topic in school mathematics. For the future studies, it will be useful to analyze the performances of students’ number sense on the specialized subject.

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