
Journal of
Interdisciplinary
Mathematics

Journal of Interdisciplinary Mathematics

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/tjim20

Radiation of sound waves by semi-perforated duct

Burhan Tiryakioglu & Hülya Öztürk

Journal of Interdisciplinary Mathematics
ISSN: 0972-0502 (Print), ISSN: 2169-012X (Online)
DOI : 10.1080/09720502.2021.1959998

Radiation of sound waves by semi-perforated duct

Burhan Tiryakioglu *
Department of Applied Mathematics
Marmara University
Istanbul 34722
Turkey

Hülya Öztürk
Department of Mathematics
Gebze Technical University
Kocaeli 41400
Turkey

Abstract

In this paper, the Wiener-Hopf method is studied to solve the radiation of acoustic waves from a perforated part through an infinite channel. At first the boundary value problem involving a Helmholtz equation with mixed-boundary conditions is formulated into a Wiener Hopf equation by applying the Fourier transform. After that the Wiener-Hopf kernel is split into two functions that are analytic in specified regions of the complex-plane and equation is solved analytically. Finally, numerical results showing the effect of the problem's parameters (channel radius, frequency, etc.) on the radiation phenomenon are graphically displayed.

Subject Classification: *[2010] 78A45, 47A68, 42B10.*

Keywords: *Fourier transform, Wiener-Hopf method, Boundary value problem, Radiation, Acoustics.*

1. Introduction

The duct and pipe structures are commonly used in many technical and industrial devices such as exhaust systems, ventilation systems, jet fans and turbofan engines. Hence, in order to control the harmful and unwanted noise in these systems, the diffraction and radiation of acoustic

**E-mail:* `burhan.tiryakioglu@marmara.edu.tr`

waves in ducts have been treated by several authors through analytical techniques [1, 2]. The first study to describe the sound field of an unflanged duct was made by Levine and Schwinger who considered the radiation of sound from rigid cylindrical duct [3].

The reduction of noise is generally achieved by silencers. The most well-known of such silencers are acoustically absorbent linings, which have been widely investigated in literature [4, 5, 6, 7, 8]. On the other hand, perforated structures affect the engine performance directly. Perforated panel or plate are commonly employed to reduce sound pressure levels across a broad range of applications including industrial installations and propulsion devices [9, 10, 11, 12]. In particular, perforated structures are of special interest and widely used in the exhausts of automobile engines, in modern aircraft jet and turbofan engines, etc [13, 14]. The phenomenon of perforated cylinders has been analyzed by various authors, with or without flow. This consideration is important because perforated cylinders provide some facilities for analyzing of sound radiation. Demir and Cinar considered the propagation of sound in an infinite two-part duct carrying mean flow inserted axially into a larger infinite duct with wall impedance discontinuity [15]. In their study, the perforated cylinder properties were investigated and some numerical results were presented.

In this work, acoustics analysis of infinite duct having perforated part is presented. The geometry of the waveguide problem under consideration is sketched in Fig. 1. The part $z<0$ of the inner cylinder is hard walled while the part $z>0$ is perforated. To solve this boundary value problems, several different methods are available [16, 17]. An alternative and widely used method is the Wiener-Hopf technique [18]. Wiener-Hopf equation is derived by using Fourier transform and the radiated field is determined explicitly. Finally, numerical calculations have been carried out to observe the influence of various parameters such as duct radius, frequency and perforated part on the radiation phenomenon.

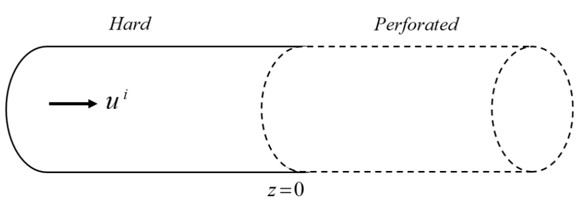

Figure 1

Geometry of the problem.

2. Formulation of the Problem

Consider an infinite cylindrical duct occupying the space $\{\rho = a, z \in (-\infty, \infty)\}$ (see Fig. 1). Let the incident field propagating in the positive z direction is taken to be

$$\psi^i(r,z) = A_0 J_0(j_n r / a) e^{i\alpha_n z} \tag{1a}$$

Here j_n is the n-th root of the equation

$$J_1(j_n) = 0 \tag{1b}$$

and α_n stands for

$$\alpha_n = \sqrt{k^2 - (j_n / a)^2}, \alpha_0 = k \tag{1c}$$

where $k = \omega / c$ denotes the wave number of the medium and c is the speed of sound. A_0 stands for the amplitude of the incident wave. Duct walls are assumed to be infinitely thin. The part $z < 0$ of the inner cylinder is hard walled while the part $z > 0$ is perforated. From the symmetry of the geometry of the problem and of the incident field, the acoustic field everywhere will be independent of θ, where (r, θ, z) are the usual cylindrical polar coordinates. We introduce a scalar potential $\psi(r,z)$ which defines the acoustic pressure and velocity by $p = i\omega\rho_0\psi$ and $\mathbf{v} = grad\,\psi$, respectively. Here ρ_0 is the density of the undisturbed medium. Time dependence is assumed to be $e^{-i\omega t}$ and suppressed throughout this paper, where ω is the angular frequency. For analysis purposes, it is convenient to express the total field as

$$\psi^T(r,z) = \begin{cases} \psi_1(r,z) & , r > a, -\infty < z < \infty \\ \psi_2(r,z) + \psi^i(r,z), & r < a, -\infty < z < \infty \end{cases} \tag{2}$$

ψ_j, $j = 1,2$ which satisfy the Helmholtz equation

$$\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial z^2} + k^2\right]\psi_j(r,z) = 0, j = 1,2 \tag{3}$$

and the following boundary conditions:

$$\frac{\partial}{\partial r}\psi_1(a,z) = 0, z < 0 \tag{4a}$$

$$\frac{\partial}{\partial r}\psi_2(a,z) = 0, z < 0 \tag{4b}$$

Consider now the continuity conditions related to the total field at $r = a,\ \ z > 0$ which are given by

$$\frac{\partial}{\partial r}\psi_1(a,z) - \frac{\partial}{\partial r}\psi_2(a,z) = 0, z > 0 \tag{4c}$$

$$\psi_1(a,z) - \psi_2(a,z) = i\frac{\zeta_p}{k}\frac{\partial}{\partial r}\psi_2(a,z) + \psi^i(a,z), z > 0 \tag{4d}$$

where ζ_p is the specific impedance, describing the acoustic properties of the perforated screen. For stationary media, the empirical formula of the specific acoustic impedance ζ_p is given by [9]

$$\zeta_p = [0.006 - ik(t_w + 0.75d_h)] / \sigma \tag{4e}$$

where t_w is the screen thickness, d_h the perforate hole diameter and σ the porosity.

3. Wiener-Hopf Equation

The unknown fields $\psi_1(r,z)$ and $\psi_2(r,z)$ satisfy (3) for $z \in (-\infty, \infty)$. By taking Fourier transform of these equations, one can obtain the following integral representations

$$\psi_1(r,z) = \frac{k}{2\pi}\int_L A(\alpha)H_0^{(1)}(\lambda kr)e^{-i\alpha kz}d\alpha \tag{5a}$$

$$\psi_2(r,z) = \frac{k}{2\pi}\int_L B(\alpha)J_0(\lambda kr)e^{-i\alpha kz}d\alpha \tag{5b}$$

where L is the integration contour along or near the real axis in the complex α-plane. $A(\alpha)$ and $B(\alpha)$ are spectral coefficients to be determined. J_0 and Y_0 are the usual Bessel functions of the first and second kinds, respectively and $H_0^{(1)}$ is the Hankel function of the first kind. λ is the square-root function defined by

$$\lambda(\alpha) = \sqrt{1-\alpha^2} \quad , \quad Im(\lambda) \geq 0 \tag{6}$$

Branch cuts for λ is taken on the line from 1 to ∞ and from $-\infty$ to –1. As usual in this kind of Wiener-Hopf problem, we will assume that the surrounding medium is slightly lossy and k has a small positive imaginary part. The lossless case can be obtained by letting $Imk \to 0$ at the end of the analysis. Applying the boundary condition (4a) on $r = a$ and taking Fourier transforms gives

$$-A(\alpha)\lambda k H_1^{(1)}(\lambda ka) = \Phi^+(\alpha) \tag{7 a}$$

Similarly, (4a-c) at $r = a$ gives

$$A(\alpha) = B(\alpha)\frac{J_1(\lambda ka)}{H_1^{(1)}(\lambda ka)} \tag{7 b}$$

Now consider the equation (4d) and taking Fourier transform and using (7a,b), one can obtain

$$A(\alpha)H_0^{(1)}(\lambda ka) - B(\alpha)J_0(\lambda ka) = \Phi^-(\alpha) + i\frac{\zeta_p}{k}\Phi^+(\alpha) - \frac{A_0 J_0(j_n)}{i(\alpha_n + k\alpha)} \tag{7 c}$$

where Φ^+ and Φ^- are a function analytic at the upper ($\operatorname{Im}\alpha > 0$ or $\operatorname{Im}\alpha = 0$ and $\operatorname{Re}\alpha > 0$) and lower ($\operatorname{Im}\alpha < 0$ or $\operatorname{Im}\alpha = 0$ and $\operatorname{Re}\alpha < 0$) half plane and defined as

$$\Phi^+(\alpha) = \int_0^\infty \frac{\partial}{\partial r}\psi_1(a,z)e^{i\alpha kz}dz \tag{8 a}$$

$$\Phi^-(\alpha) = \int_{-\infty}^0 [\psi_1(a,z) - \psi_2(a,z)]e^{i\alpha kz}dz \tag{8 b}$$

The substitution of $A(\alpha)$ and $B(\alpha)$ given by (7a,b) into (7c), one obtains the following Wiener--Hopf equation

$$\Phi^+(\alpha)M(\alpha) = \Phi^-(\alpha) - \frac{A_0 J_0(j_n)}{i\alpha_n(1+\alpha)} \tag{9 a}$$

where

$$M(\alpha) = \frac{J_0(\lambda ka)}{\lambda k J_1(\lambda ka)} - \frac{H_0^{(1)}(\lambda ka)}{\lambda k H_1^{(1)}(\lambda ka)} - i\frac{\zeta_p}{k} \tag{9 b}$$

The Wiener-Hopf equation in (9a) and rearrange it using (9b) in the following form

$$\Phi^+(\alpha)M^+(\alpha) = \Phi^-(\alpha)M^-(\alpha) - \frac{A_0 J_0(j_n)M^-(\alpha)}{i(\alpha_n + k\alpha)} \tag{10}$$

Here, $M_+(\alpha)$ and $M_-(\alpha)$ are the split functions regular and free of zeros in the upper and lower half planes, respectively, resulting from the Wiener-Hopf factorization of $M(\alpha)$ as [19]

$$M(\alpha)=\frac{M_+(\alpha)}{M_-(\alpha)} \tag{11}$$

Now consider equation (10). By virtue of classical decomposition procedure for complex term, one gets

$$\Phi^+(\alpha)M^+(\alpha)=-\frac{A_0 J_0(j_n)M^-(-1)}{i(\alpha_n+k\alpha)} \tag{12}$$

4. Analysis of the Radiated Field

The total field in the region $\rho > a$ can be obtained from (5a)

$$\psi_1(r,z)=\frac{k}{2\pi}\int_L A(\alpha)H_0^{(1)}(\lambda kr)e^{-i\alpha kz}d\alpha \tag{13}$$

Inserting $A(\alpha)$ given by (7a) into (13) gives

$$\psi_1(r,z)=-\frac{1}{2\pi}\int_L \frac{H_0^{(1)}(\lambda kr)}{\lambda H_1^{(1)}(\lambda ka)}\Phi^+(\alpha)e^{-i\alpha kz}d\alpha \tag{14}$$

Utilizing the asymptotic expansion of $H_0^{(1)}(\lambda kr)$ as $kr\to\infty$

$$H_0^{(1)}(\lambda kr)=\sqrt{\frac{2}{\pi\lambda kr}}e^{i\lambda kr-i\pi/4} \tag{15}$$

and using the saddle point technique [20], one obtains

$$\psi_1(R,\theta)\sim\frac{i}{\pi}\frac{\Phi^+(-\cos\theta)}{\sin\theta H_1^{(1)}(ka\sin\theta)}\frac{e^{ikR}}{kR} \tag{16}$$

where $\Phi^+(\alpha)$ is given by (12). Here, R and θ are the spherical coordinates defined by

$$r=R\sin\theta, z=R\cos\theta \tag{17}$$

5. Numerical Results

The numerical results based on the mathematical formulation of the proposed study are presented in this section. All graphics are obtained by applying the Matlab programming. In these figures, the Sound Pressure Level (SPL) defined by

$$SPL = 20\log_{10}\left|\frac{p}{2\cdot 10^{-5}}\right|$$

where p denotes the amplitude of the acoustic pressure of the sound wave, with the observation angle θ changing from 0 to π. The far field values are plotted at a distance 46 m away from the duct edge [19]. Parameter values related with the perforated part are taken from the study of [9, 15].

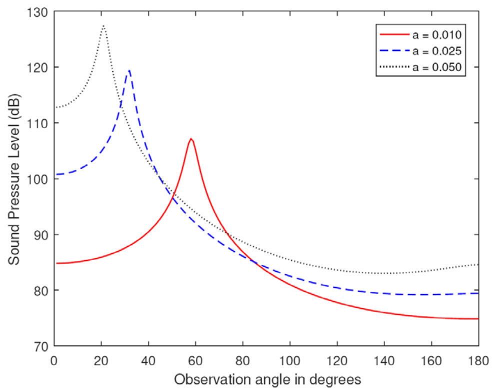

Figure 2

Sound pressure level for different values of duct radius a with f = 1500 Hz, t_w = 0.00081, d_h = 0.0249, σ = 0.057.

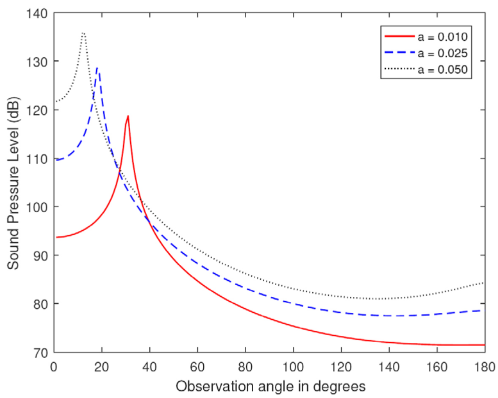

Figure 3

Sound pressure level for different values of duct radius a with f = 2500 Hz, t_w = 0.00081, d_h = 0.0249, σ = 0.057.

First, variations of sound pressure level for different values of radius (a) are presented in Figures 2 and 3. In the graphs, it is seen that as the value of a decreases, the sound pressure level decreases.

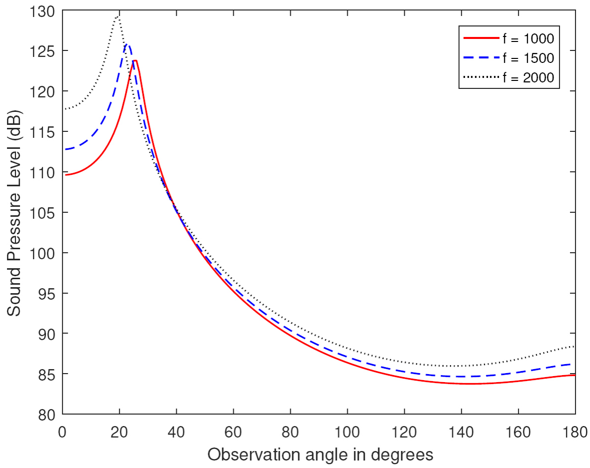

Figure 4

Sound pressure level for different values of frequency f with a = 0.05 m, t_w = 0.00081, d_h = 0.0249, σ = 0.057.

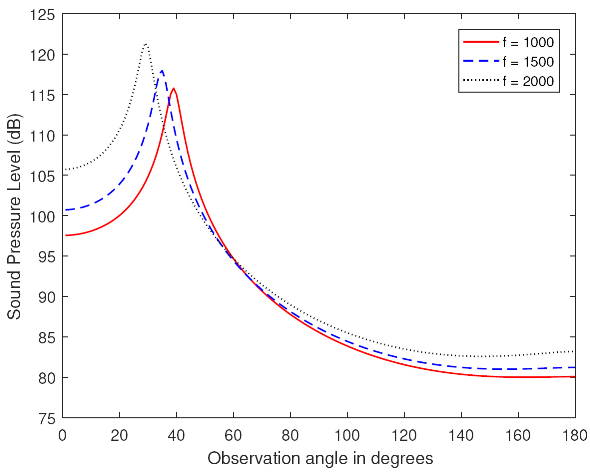

Figure 5

Sound pressure level for different values of frequency f with a = 0.025 m, t_w = 0.00081, d_h = 0.0249, σ = 0.057.

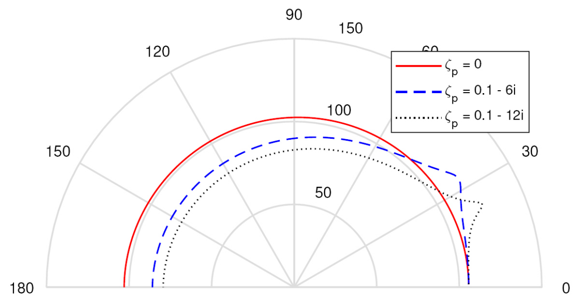

Figure 6

Sound pressure level for open perforated duct with *f* = 1000 Hz, *a* = 0.05 m.

Similar analysis is also carried for different values of frequency. Two different duct radii are considered: 0.05 m and 0.025 m. Both cases are shown in Figures 4 and 5.

In Figure 6, the effect of specific acoustic impedance (ζ_p) on sound pressure level is first studied for two different acoustic impedances and compared to unperforated case. It is observed that existing of perforated part makes contribution to the reduction of sound pressure level and when imaginary coefficient of acoustic impedance decreases the pressure level decreases.

Finally, the validity of the calculated results in this study has to be investigated. In order to investigate, (ζ_p) has been chosen as a small enough value, which is denoted as Case 1. On the other, Case 2 is a work

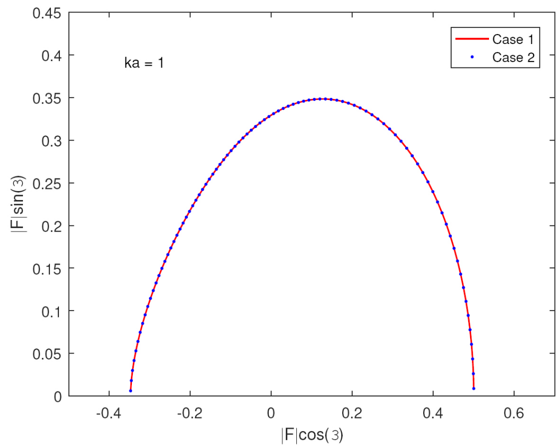

Figure 7

Comparison of the radiated field with the study of [5] for rigid duct.

[5] where the walls of the duct are assumed rigid. Results exhibit excellent agreement as expected. Notice that for Figure 7,

$$\mathcal{F}(\theta)=\frac{i}{\pi}\frac{\Phi^{+}(-\cos\theta)}{\sin\theta H_{1}^{(1)}(ka\sin\theta)}$$

6. Conclusions

The radiation of sound waves in a cylindrical duct, whose parts $z<0$ and $z>0$ are hard walled and perforated, respectively is investigated rigorously by the Wiener-Hopf method. The problem is formulated into a Wiener-Hopf equation. This equation is solved with the help of Wiener-Hopf technique. Numerical results are presented in Section 5, and it is observed that, one can analyze radiation characteristic with some specific values of duct radius (a), frequency (f) and perforated properties (ζ_p). The effects of various values of the parameters on the radiation phenomenon have been shown graphically. The analysis performed in this work shows that one can reduce the sound wave radiation by properly choosing the perforated part properties.

References

[1] A. D. Rawlins, A bifurcated circular waveguide problem, *IMA J. of Appl. Math.*, 54, 59-81, 1995.

[2] R. Nawaz and J. B. Lawrie, Scattering of a fluid-structure coupled wave at a flanged junction between two flexible waveguides, *J. Acoust. Soc. Am.*, 134, 1939-1949, 2013.

[3] H. Levine, J. Schwinger, On the radiation of sound from an unflanged circular pipe, Physical Review, 73, 383-406, 1948.

[4] A. Khalid, S. Younas, I. Khan, R. Manzoor, R. Nawaz and E. M. Sherif, Mode-matching analysis for two-dimensional acoustic wave propagation in a trifurcated lined duct, *Journal of Interdisciplinary Mathematics*, 22(7), 1095-1112, 2019.

[5] B. Tiryakioglu and A. Demir, Radiation analysis of sound waves from semi-infinite coated pipe, *Int. J. Aeroacoust.*, 18(1), 92-111, 2019.

[6] H. Ozturk, Wiener–Hopf approach for the coaxial waveguide with an impedance-coated groove on the inner wall, *J. Eng. Math.*, 124, 75–88, 2020.

[7] A. D. Rawlins, Radiation of sound from an unflanged rigid cylindrical duct with an acoustically absorbing internal surface, Proc. Roy. Soc. Lond. A. 361, 65-91, 1978.

[8] H. Ozturk, G. Cinar and O. Y. Cinar, Reflection and transmission of plane acoustic waves in an infinite annular duct with a finite gap on the inner wall, Math. Meth. Appl. Sci, 34220-230, 2011.

[9] J. W. Sullivan and M. J. Crocker, Analysis of concentric-tube resonators having unpartitioned cavities, *J. Acoust. Soc. Am.*, 64, 207-215, 1978.

[10] B. Nilsson and O. Brander, The Propagation of Sound in Cylindrical Ducts with Mean Flow and Bulk-reacting Lining I. Modes in an Infinite Duct, *J. Inst. Maths. Applics,* 26, 269-298, 1980.

[11] B. Tiryakioglu, Radiation of Acoustic Waves by a Partially Lined Pipe with an Interior Perforated Screen, *J. Eng. Math.*, 122(1), 17-29, 2020.

[12] B. Tiryakioglu, Radiation of sound by a coaxial waveguide with semi-infinite perforated duct, *Waves in Random and Complex Media,* 1-13, 2020. DOI: 10.1080/17455030.2020.1782511.

[13] I. J. Hughes and A. P. Dowling, The absorption of sound by perforated linings, *J. Fluid Mech.*, 218, 299-335, 1990.

[14] C. Lawn, Calculation of acoustic absorption in ducts with perforated liners, *Applied Acoustics,* 89, 211-221, 2015.

[15] A. Demir, O. Y. Cinar, Propagation of sound in an infinite two-part duct carrying mean flow inserted axially into a larger infinite duct with wall impedance discontinuity, Z. Angew. Math. Mech., 89, 454-465, 2009.

[16] S. C. Shiralashetti, M. H. Kantli, A. B. Deshi and P. B. Mutalik Desai, A modified wavelet multigrid method for the numerical solution of boundary value problems, *Journal of Information and Optimization Sciences*, 38(1), 151-172, 2017.

[17] L. Yong, Richardson iteration for linear equations and application in two-point boundary value problem, *Journal of Interdisciplinary Mathematics*, 21(1), 231-242, 2018.

[18] B. Noble, Methods Based on the Wiener-Hopf Techniques, Pergamon Press, London, 1958.

[19] A. Demir, S. W. Rienstra, Sound Radiation from a Lined Exhaust Duct with Lined Afterbody, 16th AIAA/CEAS Aeroacoustics Conference, Stockholm, Sweden, 1-18, 2010.

[20] H. Ozturk, G. Cinar and O. Y. Cinar, Rigorous Analysis of TM Wave Scattering by a Large Circumferential Gap on a Dielectric-Filled Circular Waveguide, U. P. B. Sci. Bull., Series A, 80, 301-310, 2018.

Received January, 2021

Revised May, 2021