Supplementary Material for "Reliability estimation of a consecutive k-out-of-n system for non-identical strength components with applications to wind speed data"

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Additional simulation study

When the second parameters are unknown

In this part, we present the performances of Bayes estimates using Lindley's approximation (based on informative and non-informative priors) and ML estimate by plots. Figures 1 and 2 present MSE and ER of estimates for Kumaraswamy and Burr Type XII distributions with sample sizes $m = 50, 75, 100$ and 125. In these figures, the parameters are chosen as $\alpha_{1_i} = 12-(2i/10)$, $\alpha_{2_i} = 6-(i/10)$, $\beta_i = 2+(2i/10)$ and $\lambda_i = 1+(i/10)$, $i = 1, ..., 58$. Figure 1 represents the estimate results of $R_{9,5}$ for Kumaraswamy distribution which takes the values 0.06813 to 0.96244 when the values of system are taken as $(k, n_1, n_2) = (5, 6, 3)$. Figure 2 represents the estimate results of $R_{8,4}$ for Burr Type XII distribution which takes the values 0.09671 to 0.99025 when the system values are taken as $(k, n_1, n_2) = (4, 5, 3)$. The following procedure is used to draw the plots:

Step 1: For given $(\alpha_1, \alpha_2, \beta, \lambda)$, $R_{9,5}(R_{8,4})$ is computed.

Step 2: For given m, samples from Kumaraswamy (Burr Type XII) distribution are generated for the strength and the stress variables.

Step 3: Estimates of $R_{9,5}$ $(R_{8,4})$ are evaluated.

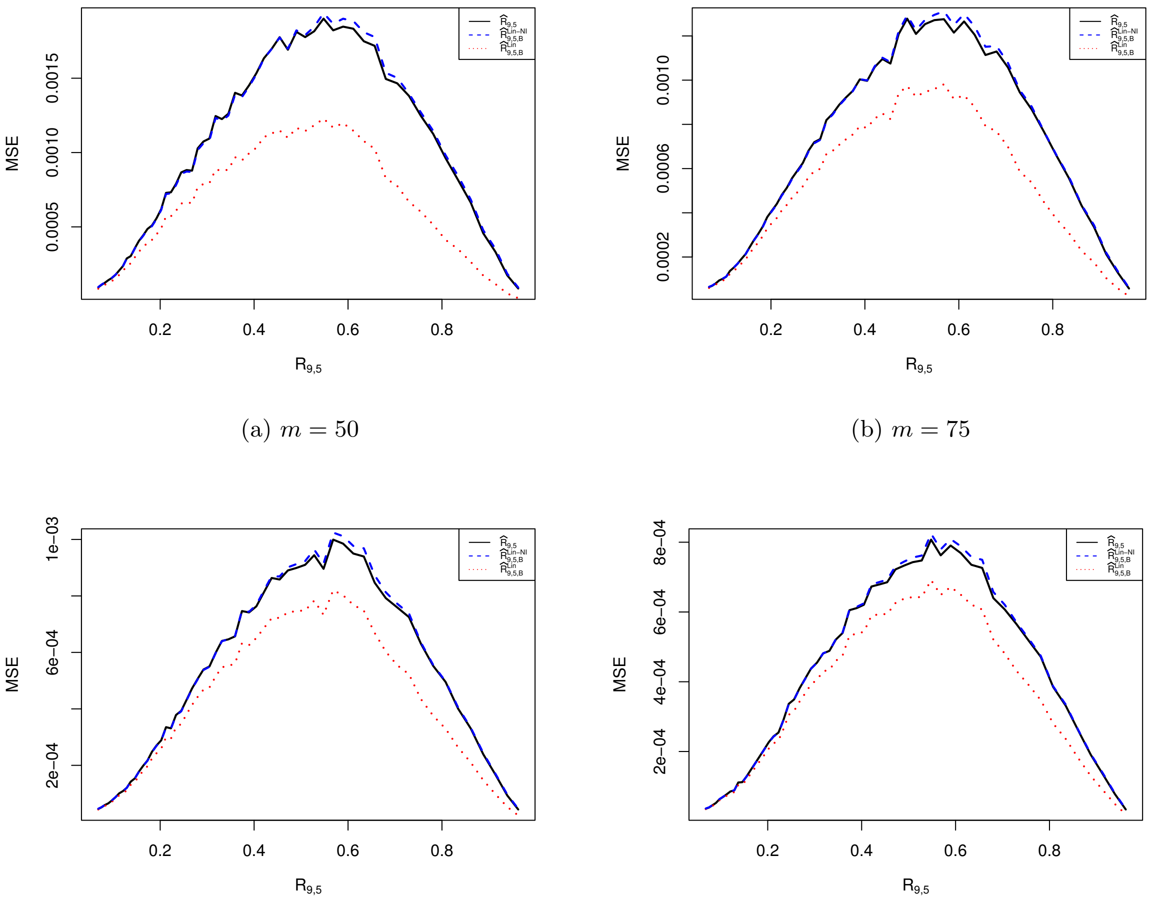
Step 4: Steps 2-3 are repeated $N = 2500$ times, the MSE or ER for estimates of $R_{9,5}$ $(R_{8,4})$ are calculated as by using $\sum_{i=1}^{N}(\widehat{R}_{9,5}^{(i)} - R_{9,5})^2/N$ $\left(\sum_{i=1}^{N}(\widehat{R}_{8,4}^{(i)} - R_{8,4})^2/N\right)$.

From Figures 1 and 2, it is observed that the error values of all estimators are getting bigger when $R_{n,k}$ is getting closer to 0.5 and getting smaller when $R_{n,k}$ is getting closer to the extreme values. The smallest error is obtained by the Bayes estimate based on informative prior. Moreover, ML estimate and Bayes estimate based on non-informative prior show similar performances and are close to each other around extreme values. These observations show that the results obtained from these figures are similar to Tables 1 and 3 in the main document.

When the second parameters are known

In this part, we present some additional tables for the different parameter setups. In Tables 1-2, ML, UMVU, exact Bayes and two approximate Bayes estimates of $R_{n,k}$ are listed. The true values of the parameters are taken as $(\alpha_1, \alpha_2, \beta) = (1.25, 3, 7)$ and $(\lambda_1, \lambda_2, \lambda_3) = (6, 9, 3)$ and $(2, 3, 6)$. In Bayesian case, the informative prior $(a_1, b_1) = (1.25, 1)$, $(a_2, b_2) = (3, 1)$, $(a_3, b_3) = (7, 1)$ and the non-informative prior are used. Bayesian estimates based on MCMC method are computed by using Gibbs sampling with 4000 iterations. In addition, the asymptotic confidence and HPD intervals corresponding to obtained point estimates are given in Table 3.

From Tables 1- 2, MSE, ERs and biases decrease with the increase in sample sizes, as expected. It is observed that ML and UMVU estimates show similar results, and they are getting close to each other as the sample size increases. Bayes estimators based on informative priors show better performance than these classical estimates while Bayes estimates based on non-informative priors give similar results to them. In these tables, Bayes estimates using Lindley's approximation based on informative priors show the best performance in terms of error values. Moreover, it is also observed that approximate Bayes



(a) $m = 50$ (b) $m = 75$

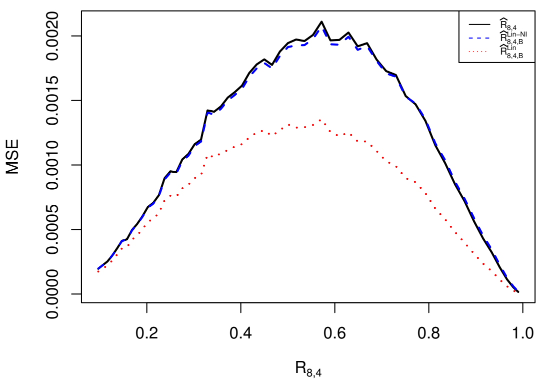
(c) $m = 100$ (d) $m = 125$

Figure 1: MSE (or ERs) of the estimates when $m = 50, 75, 100$ and 125 for Kumaraswamy distribution

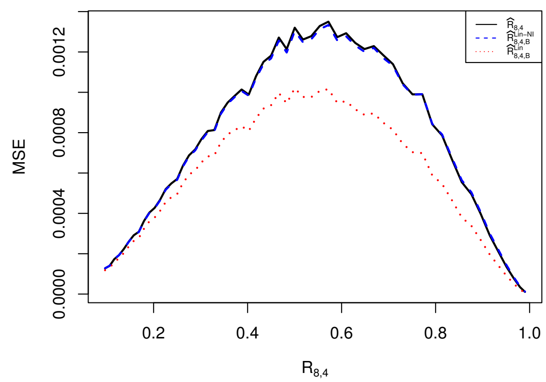
estimates and their corresponding ER values are generally close to the exact Bayes estimate values. From Table 3, asymptotic confidence interval of $R_{n,k}$ has wider length than the HPD credible intervals. HPD credible intervals based on informative priors provide the smallest AL, and the coverage probabilities of all intervals are quite satisfactory.

Moreover, we present some additional tables for the large values of m, n and k. In Tables 4 - 5, ML, UMVU and two approximate Bayes estimates of $R_{n,k}$ are listed. Then, their corresponding interval estimates are given in Table 6. In these tables, the true values of the parameters are taken as $(\alpha_1, \alpha_2, \beta) = (0.75, 2, 20)$ and $(\lambda_1, \lambda_2, \lambda_3) = (6, 9, 3)$. The informative prior $(a_1, b_1) = (0.75, 1)$, $(a_2, b_2) = (2, 1)$, $(a_3, b_3) = (20, 1)$ and the non-informative prior are used in Bayesian case.

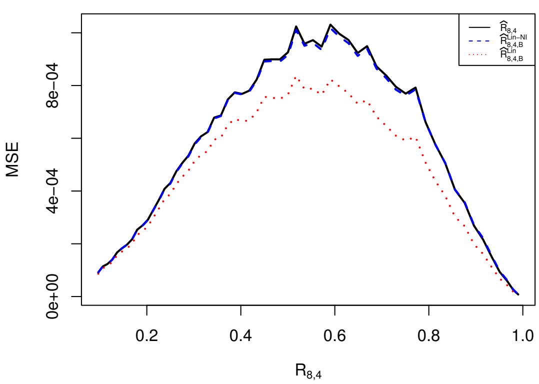
From Tables 4 - 5, it is observed that estimates of $R_{n,k}$ provide similar performances as in the Tables 1- 2. Also, intervals results in Table 6 show similar performances as in the Table 3.



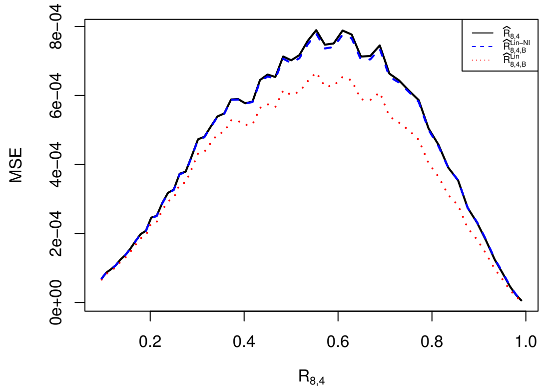
(a) $m = 50$



(b) $m = 75$



(c) $m = 100$



(d) $m = 125$

Figure 2: MSE (or ERs) of the estimates when $m = 50, 75, 100$ and 125 for Burr Type XII distribution

Table 1: Estimates of $R_{n,k}$ for Weibull distribution when $(\alpha_1, \alpha_2, \beta) = (1.25, 3, 7)$ and $(\lambda_1, \lambda_2, \lambda_3) = (6, 9, 3)$

					Bayes (Inf. Prior)			*Bayes* (Non-inf. prior)		
(k, n_1, n_2)	$R_{n,k}$	m	$\widehat{R}^{MLE}_{n,k}$	$\widehat{R}^{U}_{n,k}$	$\widehat{R}_{n,k,B}$	$\widehat{R}^{Lin}_{n,k,B}$	$\widehat{R}^{MC}_{n,k,B}$	$\widehat{R}_{n,k,B}$	$\widehat{R}^{Lin}_{n,k,B}$	$\widehat{R}^{MC}_{n,k,B}$
(2,3,1)	0.88474	10	0.87913	0.8418	0.87298	0.86829	0.87297	0.80163	0.86682	0.86657
			-0.00561	-0.00056	-0.01176	-0.01645	-0.01177	-0.08311	-0.01793	-0.01817
			0.00284	0.00288	0.00148	0.00114	0.00148	0.01015	0.00332	0.00330
		20	0.88304	0.88565	0.87811	0.87648	0.87822	0.84391	0.87669	0.87661
			-0.00170	0.00090	-0.00663	-0.00827	-0.00652	-0.04083	-0.00806	-0.00813
			0.00136	0.00137	0.00092	0.00084	0.00092	0.00319	0.00147	0.00147
		30	0.88340	0.88514	0.87975	0.87918	0.88000	0.85743	0.87912	0.87911
			-0.00134	0.00040	-0.00499	-0.00556	-0.00474	-0.02732	-0.00562	-0.00563
			0.00094	0.00095	0.00072	0.00068	0.00072	0.00178	0.00100	0.00100
		40	0.88381	0.88511	0.88063	0.88059	0.88109	0.86425	0.88058	0.88054
			-0.00094	0.00037	-0.00411	-0.00415	-0.00366	-0.02050	-0.00416	-0.00420
			0.00072	0.00072	0.00060	0.00056	0.00058	0.00120	0.00075	0.00075
(2,2,2)	0.84630	10	0.84208	0.84546	0.83583	0.82290	0.83584	0.83038	0.82839	0.83040
			-0.00422	-0.00084	-0.01047	-0.02340	-0.01045	-0.01592	-0.01790	-0.01590
			0.00418	0.00441	0.00203	0.00170	0.00203	0.00445	0.00461	0.00446
		20	0.84387	0.84561	0.83963	0.83572	0.83960	0.83780	0.83692	0.83779
			-0.00242	-0.00069	-0.00667	-0.01058	-0.00670	-0.00849	-0.00938	-0.00851
			0.00201	0.00206	0.00130	0.00114	0.00130	0.00209	0.00212	0.00209
		30	0.84513	0.84630	0.84192	0.83971	0.84176	0.84105	0.84048	0.84102
			-0.00117	0.00000	-0.00438	-0.00658	-0.00454	-0.00525	-0.00582	-0.00528
			0.00139	0.00142	0.00104	0.00097	0.00103	0.00143	0.00144	0.00143
		40	0.84599	0.84688	0.84330	0.84197	0.84325	0.84310	0.84249	0.84289
			-0.00031	0.00058	-0.00300	-0.00432	-0.00304	-0.00320	-0.00381	-0.00341
			0.00102	0.00103	0.00082	0.00076	0.00080	0.00105	0.00104	0.00103
(3,1,3)	0.52293	10	0.52948	0.52297	0.51859	0.50352	0.51860	0.52063	0.51974	0.52065
			0.00655	0.00004	-0.00434	-0.01941	-0.00433	-0.00230	-0.00319	-0.00228
			0.00849	0.00905	0.00372	0.00262	0.00372	0.00798	0.00801	0.00798
		20	0.52735	0.52403	0.52164	0.51815	0.52161	0.52281	0.52252	0.52281
			0.00442	0.00110	-0.00129	-0.00478	-0.00132	-0.00013	-0.00041	-0.00012
			0.00410	0.00423	0.00248	0.00209	0.00248	0.00396	0.00397	0.00396
		30	0.52555	0.52331	0.52192	0.52043	0.52191	0.52250	0.52234	0.52248
			0.00262	0.00038	-0.00101	-0.00250	-0.00102	-0.00043	-0.00059	-0.00046
			0.00281	0.00287	0.00197	0.00183	0.00197	0.00275	0.00275	0.00274
		40	0.52379	0.52210	0.52119	0.52041	0.52124	0.52146	0.52140	0.52153
			0.00086	-0.00084	-0.00174	-0.00252	-0.00169	-0.00147	-0.00153	-0.00140
			0.00212	0.00216	0.00162	0.00156	0.00162	0.00209	0.00210	0.00209

Table 2: Estimates of $R_{n,k}$ for Weibull distribution when $(\alpha_1, \alpha_2, \beta) = (1.25, 3, 7)$ and $(\lambda_1, \lambda_2, \lambda_3) = (2, 3, 6)$

					Bayes (Inf. Prior)			*Bayes* (Non-inf. prior)		
(k, n_1, n_2)	$R_{n,k}$	m	$\widehat{R}^{MLE}_{n,k}$	$\widehat{R}^{U}_{n,k}$	$\widehat{R}_{n,k,B}$	$\widehat{R}^{Lin}_{n,k,B}$	$\widehat{R}^{MC}_{n,k,B}$	$\widehat{R}_{n,k,B}$	$\widehat{R}^{Lin}_{n,k,B}$	$\widehat{R}^{MC}_{n,k,B}$
(3,5,1)	0.83773	10	0.83471	0.83702	0.82553	0.81859	0.82555	0.66787	0.81983	0.81971
			-0.00302	-0.00071	-0.01220	-0.01915	-0.01218	-0.16986	-0.01790	-0.01802
			0.00414	0.00442	0.00199	0.00142	0.00199	0.03304	0.00462	0.00459
		20	0.83526	0.83643	0.82952	0.82740	0.82952	0.75058	0.82757	0.82753
			-0.00247	-0.00131	-0.00821	-0.01033	-0.00821	-0.08715	-0.01016	-0.01020
			0.00217	0.00224	0.00142	0.00126	0.00142	0.00990	0.00231	0.00231
		25	0.83652	0.83747	0.83149	0.83009	0.83154	0.76889	0.83034	0.83034
			-0.00121	-0.00026	-0.00624	-0.00764	-0.00619	-0.06884	-0.00739	-0.00739
			0.00172	0.00177	0.00120	0.00110	0.00120	0.00657	0.00181	0.00180
		30	0.83908	0.83990	0.82252	0.83317	0.83429	0.76050	0.83382	0.83376
			0.00135	0.00217	-0.01521	-0.00456	-0.00344	-0.07723	-0.00391	-0.00397
			0.00134	0.00137	0.00085	0.00089	0.00096	0.00673	0.00137	0.00137
(3,4,2)	0.80952	10	0.81028	0.81134	0.79993	0.79112	0.80000	0.72111	0.79593	0.79590
			0.00076	0.00182	-0.00958	-0.01839	-0.00952	-0.08840	-0.01359	-0.01362
			0.00478	0.00516	0.00218	0.00153	0.00219	0.01252	0.00504	0.00501
		20	0.80994	0.81046	0.80371	0.80133	0.80383	0.76445	0.80251	0.80249
			0.00042	0.00094	-0.00581	-0.00819	-0.00569	-0.04507	-0.00701	-0.00702
			0.00247	0.00257	0.00156	0.00135	0.00156	0.00456	0.00254	0.00253
		25	0.81042	0.81084	0.80483	0.80346	0.80515	0.77380	0.80443	0.80441
			0.00090	0.00132	-0.00468	-0.00606	-0.00437	-0.03572	-0.00508	-0.00510
			0.00192	0.00198	0.00131	0.00119	0.00131	0.00326	0.00196	0.00196
		30	0.80856	0.80889	0.80039	0.80317	0.80431	0.76833	0.80355	0.80353
			-0.00096	-0.00063	-0.00913	-0.00634	-0.00521	-0.04119	-0.00597	-0.00598
			0.00168	0.00172	0.00109	0.00113	0.00121	0.00291	0.00172	0.00172
(3,3,3)	0.77339	10	0.77561	0.77471	0.76543	0.75262	0.76539	0.76229	0.76180	0.76234
			0.00222	0.00132	-0.00796	-0.02077	-0.00800	-0.01110	-0.01159	-0.01105
			0.00588	0.00640	0.00259	0.00183	0.00260	0.00590	0.00597	0.00590
		20	0.77618	0.77572	0.76986	0.76632	0.76984	0.76924	0.76910	0.76931
			0.00279	0.00233	-0.00353	-0.00707	-0.00355	-0.00415	-0.00429	-0.00408
			0.00301	0.00314	0.00184	0.00159	0.00184	0.00300	0.00301	0.00300
		25	0.77314	0.77272	0.76843	0.76629	0.76845	0.76754	0.76745	0.76759
			-0.00025	-0.00067	-0.00496	-0.00710	-0.00494	-0.00585	-0.00594	-0.00580
			0.00235	0.00244	0.00159	0.00146	0.00158	0.00238	0.00238	0.00237
		30	0.77453	0.77419	0.76990	0.76870	0.77028	0.76672	0.76978	0.76988
			0.00114	0.00080	-0.00349	-0.00469	-0.00311	-0.00667	-0.00361	-0.00351
			0.00207	0.00213	0.00144	0.00137	0.00146	0.00187	0.00208	0.00207

Table 3: AL and CP of $R_{n,k}$ for Weibull distribution when $(\alpha_1, \alpha_2, \beta) = (1.25, 3, 7)$ and $(\lambda_1, \lambda_2, \lambda_3) = (6, 9, 3)$ and $(2, 3, 6)$

			ACI		HPD (Inf. Prior)		HPD (Non-inf. prior)	
m	(k, n_1, n_2)	$R_{n,k}$	AL	CP	AL	CP	AL	CP
10	(2,3,1)	0.88474	0.20544	0.8980	0.16887	0.9832	0.20867	0.9416
20			0.14498	0.9308	0.12932	0.9728	0.14561	0.9452
30			0.11882	0.9332	0.10928	0.9644	0.11883	0.9448
40			0.10299	0.9352	0.09642	0.9552	0.10287	0.9384
10	(2,2,2)	0.84630	0.30142	0.9620	0.20102	0.9796	0.24504	0.9316
20			0.21376	0.9712	0.15618	0.9720	0.17536	0.9392
30			0.17393	0.9744	0.13198	0.9620	0.14320	0.9436
40			0.15094	0.9744	0.11647	0.9588	0.12403	0.9412
10	(3,5,1)	0.83773	0.24973	0.9140	0.20442	0.9848	0.25187	0.9360
20			0.17971	0.9232	0.15898	0.9660	0.17981	0.9408
25			0.16054	0.9208	0.14493	0.9664	0.16037	0.9424
30			0.14564	0.9392	0.13358	0.9716	0.14553	0.9495
10	(3,4,2)	0.80952	0.26954	0.9016	0.22020	0.9840	0.26968	0.9348
20			0.19479	0.9332	0.17209	0.9748	0.19403	0.9448
25			0.17475	0.9284	0.15755	0.9668	0.17389	0.9412
30			0.16084	0.9336	0.14687	0.9632	0.16010	0.9440
10	(3,3,3)	0.77339	0.30503	0.9280	0.24121	0.9852	0.29337	0.9376
20			0.21838	0.9364	0.18808	0.9704	0.21164	0.9408
25			0.19688	0.9412	0.17340	0.9656	0.19164	0.9376
30			0.17940	0.9412	0.16046	0.9660	0.17478	0.9400
10	(3,1,3)	0.52293	0.33973	0.9220	0.28297	0.9768	0.33910	0.9336
20			0.24138	0.9332	0.22070	0.9676	0.24649	0.9440
30			0.19732	0.9296	0.18753	0.9616	0.20289	0.9408
40			0.17097	0.9292	0.16608	0.9536	0.17667	0.9372

Table 4: Estimates of $R_{n,k}$ for Weibull distribution when $(\alpha_1, \alpha_2, \beta) = (0.75, 2, 20)$ and $(\lambda_1, \lambda_2, \lambda_3) = (6, 9, 3)$

					Bayes (Inf. Prior)		*Bayes* (Non-inf. prior)	
(k, n_1, n_2)	$R_{n,k}$	m	$\widehat{R}_{n,k}^{MLE}$	$\widehat{R}_{n,k}^{U}$	$\widehat{R}_{n,k,B}^{Lin}$	$\widehat{R}_{n,k,B}^{MC}$	$\widehat{R}_{n,k,B}^{Lin}$	$\widehat{R}_{n,k,B}^{MC}$
(5,6,3)	0.93166	25	0.93114	0.93193	0.92786	0.92928	0.92771	0.92761
			-0.00051	0.00028	-0.00379	-0.00238	-0.00395	-0.00405
			0.00031	0.00030	0.00007	0.00013	0.00035	0.00035
		50	0.93151	0.93191	0.92982	0.93029	0.92979	0.92975
			-0.00015	0.00025	-0.00183	-0.00137	-0.00187	-0.00190
			0.00016	0.00015	0.00008	0.00009	0.00016	0.00016
		75	0.93138	0.93164	0.93028	0.93051	0.93023	0.93021
			-0.00028	-0.00001	-0.00138	-0.00115	-0.00143	-0.00144
			0.00011	0.00011	0.00007	0.00007	0.00011	0.00011
(5,4,6)	0.87442	25	0.87372	0.87015	0.86862	0.87097	0.86908	0.86899
			-0.00070	-0.00427	-0.00580	-0.00345	-0.00534	-0.00543
			0.00074	0.00080	0.00015	0.00030	0.00080	0.00080
		50	0.87428	0.87048	0.87178	0.87252	0.87194	0.87192
			-0.00014	-0.00394	-0.00263	-0.00190	-0.00248	-0.00250
			0.00037	0.00041	0.00017	0.00022	0.00039	0.00039
		75	0.87503	0.87117	0.87324	0.87363	0.87348	0.87346
			0.00061	-0.00325	-0.00118	-0.00079	-0.00094	-0.00096
			0.00025	0.00027	0.00015	0.00017	0.00025	0.00025
(5,3,6)	0.81516	25	0.81533	0.81123	0.80869	0.81190	0.81020	0.81016
			0.00017	-0.00393	-0.00647	-0.00326	-0.00496	-0.00500
			0.00119	0.00128	0.00024	0.00048	0.00124	0.00124
		50	0.81539	0.81133	0.81230	0.81324	0.81280	0.81279
			0.00023	-0.00383	-0.00286	-0.00191	-0.00236	-0.00237
			0.00062	0.00066	0.00028	0.00036	0.00063	0.00063
		75	0.81491	0.81085	0.81302	0.81344	0.81318	0.81316
			-0.00025	-0.00431	-0.00214	-0.00172	-0.00198	-0.00200
			0.00040	0.00044	0.00024	0.00027	0.00041	0.00041
(5,2,8)	0.76122	25	0.76146	0.74534	0.75352	0.75743	0.75601	0.75598
			0.00024	-0.01588	-0.00770	-0.00379	-0.00521	-0.00524
			0.00154	0.00203	0.00029	0.00061	0.00159	0.00159
		50	0.76179	0.74612	0.75840	0.75946	0.75905	0.75905
			0.00057	-0.01510	-0.00282	-0.00175	-0.00217	-0.00217
			0.00078	0.00113	0.00035	0.00044	0.00079	0.00080
		75	0.76147	0.74592	0.75933	0.75982	0.75964	0.75965
			0.00025	-0.01530	-0.00189	-0.00140	-0.00158	-0.00157
			0.00050	0.00080	0.00030	0.00033	0.00050	0.00050

Table 5: Estimates of $R_{n,k}$ for Weibull distribution when $(\alpha_1, \alpha_2, \beta) = (0.75, 2, 20)$ and $(\lambda_1, \lambda_2, \lambda_3) = (6, 9, 3)$

					Bayes (Inf. Prior)		*Bayes* (Non-inf. prior)	
(k, n_1, n_2)	$R_{n,k}$	m	$\widehat{R}^{MLE}_{n,k}$	$\widehat{R}^{U}_{n,k}$	$\widehat{R}^{Lin}_{n,k,B}$	$\widehat{R}^{MC}_{n,k,B}$	$\widehat{R}^{Lin}_{n,k,B}$	$\widehat{R}^{MC}_{n,k,B}$
(7,9,4)	0.91090	25	0.91033	0.91097	0.90619	0.90799	0.90598	0.90587
			-0.00056	0.00008	-0.00471	-0.00291	-0.00492	-0.00503
			0.00052	0.00052	0.00009	0.00020	0.00057	0.00057
		50	0.91057	0.91089	0.90842	0.90901	0.90837	0.90834
			-0.00033	-0.00001	-0.00248	-0.00188	-0.00253	-0.00255
			0.00026	0.00026	0.00012	0.00015	0.00027	0.00027
		75	0.91057	0.91078	0.90913	0.90942	0.90910	0.90909
			-0.00033	-0.00012	-0.00176	-0.00147	-0.00180	-0.00181
			0.00017	0.00017	0.00010	0.00011	0.00017	0.00017
(7,6,7)	0.84820	25	0.84847	0.84861	0.84167	0.84482	0.84309	0.84302
			0.00028	0.00041	-0.00653	-0.00338	-0.00510	-0.00518
			0.00104	0.00106	0.00018	0.00040	0.00110	0.00110
		50	0.84732	0.84738	0.84454	0.84536	0.84459	0.84457
			-0.00088	-0.00082	-0.00366	-0.00283	-0.00361	-0.00362
			0.00054	0.00054	0.00024	0.00031	0.00056	0.00056
		75	0.84819	0.84824	0.84622	0.84665	0.84639	0.84637
			-0.00001	0.00004	-0.00197	-0.00155	-0.00181	-0.00183
			0.00036	0.00036	0.00021	0.00024	0.00036	0.00036
(7,5,8)	0.79524	25	0.79541	0.79211	0.78815	0.79166	0.78974	0.78970
			0.00018	-0.00312	-0.00708	-0.00357	-0.00549	-0.00553
			0.00142	0.00149	0.00024	0.00054	0.00148	0.00148
		50	0.79483	0.79177	0.79172	0.79265	0.79197	0.79198
			-0.00040	-0.00347	-0.00352	-0.00258	-0.00327	-0.00326
			0.00067	0.00071	0.00030	0.00038	0.00069	0.00069
		75	0.79566	0.79269	0.79349	0.79398	0.79375	0.79375
			0.00043	-0.00254	-0.00174	-0.00126	-0.00149	-0.00149
			0.00047	0.00049	0.00028	0.00031	0.00048	0.00048
(7,4,9)	0.74678	25	0.74725	0.73958	0.73944	0.74334	0.74147	0.74146
			0.00047	-0.00721	-0.00734	-0.00344	-0.00531	-0.00533
			0.00170	0.00186	0.00027	0.00064	0.00174	0.00174
		50	0.74754	0.74049	0.74367	0.74488	0.74463	0.74465
			0.00076	-0.00630	-0.00311	-0.00191	-0.00215	-0.00213
			0.00083	0.00092	0.00035	0.00046	0.00084	0.00084
		75	0.74784	0.74101	0.74538	0.74594	0.74594	0.74590
			0.00106	-0.00578	-0.00140	-0.00084	-0.00084	-0.00089
			0.00060	0.00066	0.00035	0.00039	0.00060	0.00060

Table 6: AL and CP of $R_{n,k}$ for Weibull distribution when $(\alpha_1, \alpha_2, \beta) = (0.75, 2, 20)$ and $(\lambda_1, \lambda_2, \lambda_3) = (6, 9, 3)$

			ACI		HPD (Inf. Prior)		HPD (Non-inf. prior)	
m	(k, n_1, n_2)	$R_{n,k}$	AL	CP	AL	CP	AL	CP
25	(5,6,3)	0.93166	0.06917	0.9276	0.05431	0.9848	0.07083	0.9464
50			0.04885	0.9344	0.04234	0.9736	0.04931	0.9408
75			0.04000	0.9396	0.03607	0.9672	0.04011	0.9456
25	(5,4,6)	0.87442	0.11940	0.9608	0.08317	0.9868	0.10795	0.9464
50			0.08444	0.9664	0.06502	0.9744	0.07557	0.9472
75			0.06880	0.9636	0.05516	0.9656	0.06124	0.9448
25	(5,3,6)	0.81516	0.14534	0.9560	0.10497	0.9840	0.13473	0.9456
50			0.10301	0.9520	0.08213	0.9672	0.09484	0.9304
75			0.08436	0.964	0.07000	0.966	0.07754	0.9480
25	(5,2,8)	0.76122	0.17316	0.9600	0.11944	0.9852	0.15374	0.9416
50			0.12268	0.9708	0.09363	0.9728	0.10849	0.9436
75			0.10034	0.9716	0.07983	0.9700	0.08858	0.9464
25	(7,9,4)	0.91090	0.08895	0.9348	0.06900	0.9892	0.09072	0.9460
50			0.06307	0.9416	0.05428	0.9740	0.06349	0.9460
75			0.05159	0.9468	0.04632	0.9736	0.05166	0.9504
25	(7,6,7)	0.84820	0.13019	0.9364	0.09840	0.9908	0.12810	0.9392
50			0.09300	0.9452	0.07785	0.9772	0.09077	0.9420
75			0.07576	0.9544	0.06615	0.9704	0.07371	0.9516
25	(7,5,8)	0.79524	0.15142	0.9444	0.11393	0.9864	0.14779	0.9456
50			0.10774	0.9600	0.08988	0.9808	0.10459	0.9576
75			0.08787	0.9488	0.07653	0.9676	0.08514	0.9444
25	(7,4,9)	0.74678	0.16789	0.9504	0.12579	0.9844	0.16289	0.9456
50			0.1191	0.9572	0.0991	0.9812	0.1151	0.9512
75			0.09727	0.9484	0.08449	0.9672	0.09392	0.9420